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Autor(en): **Chevalier, Jean**

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A New Interpretation of the Clock Paradox in Special Relativity

by **Jean Chevalier**[1])

University of Geneva, Institute for Theoretical Physics

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Abstract. We attempt to clarify the confusion about the clock paradox of special relativity by introducing a symmetrical definition of simultaneity. This definition is well adapted to the postulate of physical equivalence of inertial frames in uniform relative motion. The Langevin effect is then an immediate consequence of the non-transitivity of this simultaneity relation.

1. Introduction

Although the orthodox relativists (justly) consider they have definitively resolved the clock paradox (CP) of special relativity (SR), many papers are continually published on this subject. Several of them even deny the age difference of the famous Langevin twins [1, 2, 3]. In particular, the more or less tedious discussions provoked by M. Sachs's recent paper [3] have convinced us once more that the misunderstandings about the CP are essentially imputable to *language* questions. In this note, we try to reformulate the problem in a language conceived to avoid such mistakes.

2. A Definition of Simultaneity

We generalize Einstein's definition of simultaneity as follows (one-dimensional case):

Definition. Consider two points O, O' in uniform relative motion (URM) with respect to an inertial frame of reference. Two events E, E' occur in O and O' respectively, at the same time as the emission of light signals S and S' respectively. E and E' are simultaneous if and only if S and S' are seen simultaneously by an observer M constantly situated in the middle point of the variable line OO'. This definition is clearly equivalent to the following one:

Suppose two events E, E' are occurring in two inertial frames Σ and Σ' in URM. The identical clocks H and H' of these frames are timed so that they indicate the same time when the points O, O' coincide (E, E' occur in O and O' respectively). E and E' are then simultaneous if their time coordinates are equal.

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Although this definition apparently deprives the simultaneity of its relativistic character, it has the advantage of being more efficiently adapted than the usual one to the postulate of physical equivalence of inertial frames in URM.

3. Conditions of Transitivity of Simultaneity

Definition. Suppose three events E, E', E'' are taking place at points in URM. We say that these events are *transitively simultaneous* if and only if we have:

$$E \; sim \; E' \quad \text{and} \quad E' \; sim \; E'' \Rightarrow E \; sim \; E''.$$

We can then prove the following *theorem*:

Theorem. Consider three inertial systems Σ, Σ', Σ'' in URM, and O, O', O'' their respective origins. In order that three events E, E', E'' be *transitively simultaneous*, it is necessary and sufficient that all three of the points O, O', O'', where these events occur, coincide spatially once in the course of their evolution.

The *sufficiency* of the condition results immediately from our definition. In order to prove its *necessity*, it is useful to observe that this definition can be interpreted geometrically as follows: the simultaneous events E and E' are localized on the same hyperbola $x^2 - c^2t^2 = \text{constant}$, with world straight lines of E and E' passing by O (Fig. 1) (remark due to Prof. G. Wanders). We can then prove the second part of our

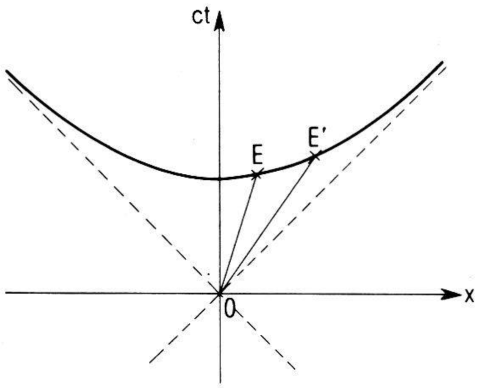

Figure 1
Two simultaneous events E and E'.

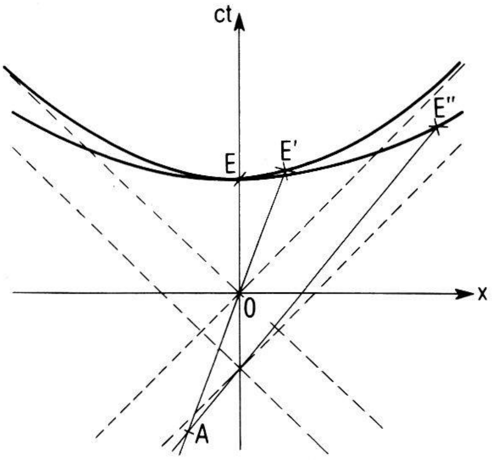

Figure 2
Transitivity of simultaneity.

theorem as follows (Fig. 2). Working for instance in the proper system of E, we suppose $E\,sim\,E'$, $E\,sim\,E''$ (*a priori*, the world straight line of E'' does not necessarily pass by O). If A is the intersection point of the E'- and E''-world straight lines, the hypothesis of the transitivity of simultaneity ($E'\,sim\,E''$) means that E' and E'' should be on an equilateral hyperbola of centre A. But the elementary properties of these curves allow us to establish that this is possible only if $A \equiv O$, which proves the theorem. It is, moreover, clear that the theorem is also true for any number of events of this type.

4. Consequences of the Theorem

The Langevin twins' trip is then interpreted as follows. According to our definition it is clear that no age difference appears as long as the twins are in URM. On the other hand, in order to avoid eventual difficulties due to the traveller's accelerations in O at the start and end of his journey, we shall simply suppose that this twin *passes by O*, going as well as coming back (velocity $+u$, $-u$ respectively). Therefore, we still have to examine what happens at the turning point A (Fig. 3). It is then possible to admit

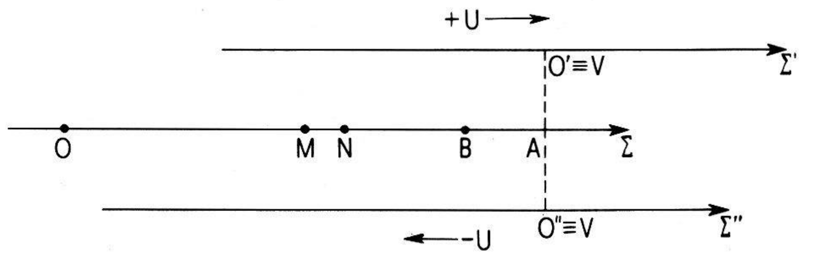

Figure 3
The Langevin twins.

that, at the precise moment when $O' \equiv V$ passes by A, the origin O'' of a frame Σ'' also passes by it (but with a velocity $-u$ with respect to Σ), emitting a light signal S''. From this moment we decide to identify O'' to V. The simultaneous light signals S and S' emitted by O and $O' \equiv V$ at the moment of turning back are seen simultaneously in N, which is clearly on the right side of the middle M of OA. Because of the light velocity invariance, S'' also meets S in N, and during the 'flight time' of S'', $O'' \equiv V$ moves *to the left side* of A, coming in B. According to our definition the inequality $|ON| > |BN|$ implies that the emissions of S and S'' *are not simultaneous*: *the emission of S is earlier than that of S''*, and this time-lag being maintained until the arrival of $O'' \equiv V$ in O, the observer O will be older than V at that very moment. By a quantitative treatment of this question we obtain easily the classical result, $t_V = t_O\sqrt{1-(u^2/c^2)}$.

This description shows clearly that the inertial parts of the trip do not contribute directly to the phenomenon. The time-lag is entirely due to a 'breaking of simultaneity' in A (all three of the points O, O', O'' never coincide): in order to again be 'in simultaneity' with V, O must let pass a certain finite time $\Delta t = t_O[1-\sqrt{1-(u^2/c^2)}]$.

5. Conclusion

In our opinion, the preceding description has the great advantage of avoiding completely, current, ambiguous expressions, such as 'slowness of moving clocks', 'a moving clock changes its rhythm' [4, p. 176], etc. Although the contradiction between these statements and the principle of physical equivalence of inertial frames in URM is indeed specious, it undoubtedly continues to trouble many physicists. On the contrary, our language is, so to say, 'adapted' to this principle. Its essential feature is to associate the Langevin effect to a 'breaking of simultaneity', which is an immediate consequence of the light velocity invariance. Last, but not least, from our viewpoint this problem has clearly nothing to do with general relativity.

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