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About the possibility of supraluminal transmission of information in the Bohm-Bub theory.

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Abstract We present two possible ways for solving the so called Gleason trouble in the framework of Bohm-Bub hidden variable theory: the so-called Wiener-Siegel method and a new method proposed by us. We study the non-local properties exhibited by these methods, and discuss the experimental possibility of measuring these non-local effects.

1 Introduction

The Bohm-Bub (B-B) theory [1] is a hidden variable theory whose goal is to simulate the interaction undergone by a quantum system during the process of measurement. The B-B theory suffers from what is called the "Kochen-Specker disease" or "Gleason trouble" [2,3,4]: the result of a measurement, when the value of the hidden variable is fixed, does depend on the whole decomposition of the set of commuting observables being measured, when this decomposition is complete, and is not defined otherwise, when degeneracies occur.

For instance, in Bell-like spin measurements [5], the result of a Stern-Gerlach experiment realised on one of the two spin 1/2 particles emitted in an entangled state does depend on the choice of the angle of setting of a second Stern-Gerlach apparatus along the trajectory

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of the second particle. If only one apparatus is set, which acts on one particle of the pair, the B-B theory does not allow us to predict the result of the measurement, even if the value of the hidden variable is specified.

Belinfante [3] described a generalisation of B-B theory, in order to solve the Gleason trouble (called the Wiener-Siegel method in [4]). We rediscuss the validity of the concept of dimensionality implicitly used in these works, and propose to consider dimensionality not as an absolute concept but well as a relative concept, which depends on the measurements that we perform on the system (the questions that we, observers, ask to Nature). As a consequence, we propose a new generalisation of B-B theory, in order to solve the Gleason trouble, and show that it is equivalent with Wiener-Siegel's method when a single measurement is performed on the system. We recover standard quantum predictions in the case of successive measurements, but predict the possibility of a supraluminal transmission of information in the case of simultaneous measurements realised in far away distant regions of space. We propose a simple experiment aimed at testing the relevance of this possibility, and another possible experimental test for Wiener-Siegel's method. These experiments can be carried out with the experimental set up used for testing the violation of Bell's inequalities. The basic idea which underlies our approach is the following:

If one accepts that the experimental violation of Bell's inequalities showed the relevance of the projection postulate, our results imply that we can use the same experimental set up as the one used for testing this violation, in a "fine regime", in order to test the relevance of theories, as B-B theory, in which the projection is not instantaneous. In last resort, the experiments that we outline here make it possible to test directly the reality of the "time of projection". Note well that the Loch Ness monster which concerns us in the present approach possesses two faces: the existence of hidden variables at one side and of an absolute, Newtonian spatio-temporal referential at the other side. It is the combination of these two (still presently hypothetical) features that makes it possible to predict discrepancies with quantum standard predictions in the case of Bell-like experiments.

2 The Bohm-Bub theory.

The B-B theory aims at describing the process of measurement of a quantum system of which the state Ψ belongs to a N-dimensional Hilbert space H. Note that the B-B theory is presently not defined in the case of an infinite dimensional Hilbert space, and the passage to the limit where N goes to infinity is a ill-posed problem in this case. The theory assumes that, in addition to the quantum state, the system is described by a hidden variable (sometimes called hidden vector or hidden state), Ξ, randomly distributed in a N-dimensional Hilbert space H^*. In our approach, which is the same as in [3], this distribution is assumed to be isotropic, so to say, if we expand the variable Ξ in an arbitrary orthonormal basis of H^*, the distribution is equal to a function f which depends on Ξ only through its norm $|\Xi|$ ($|\Xi| = \sqrt{\sum_{i=1}^{N} |\xi_i|^2}$ where ξ_i denotes the i-th component of Ξ

in a basis of H^*), and is normalised on H^* relatively to the element of volume given by the product $\Pi_{i=1}^n dRe\xi_i dIm\xi_i$: $\int \Pi_{i=1}^n dRe\xi_i dIm\xi_i\ f(|\Xi|) = 2\pi^N \int \Pi_{i=1}^n |\xi_i| d|\xi_i| = 1$. As it is shown in [3], when f is a Gaussian distribution, we recover the Wiener-Siegel theory, while when f takes non-zero values in an arbitrary small vicinity of the value $|\Xi| = 1$, we recover the Bohm-Bub theory. In order to obtain the most general results, we shall from now on consider that the choice of f is arbitrary, and make in numerical applications the more convenient choices for it.

Let us consider a complete observable O, so to say an observable of which no eigen value is degenerate. Its spectral decomposition is then unambiguously given by N orthonormal vectors of H, that we shall call $\mathbf{e_i}$ $(i : 1...N)$. According to the B-B theory, during the measurement of this observable, the state evolves according to the following equation:

$$\dot{\psi}_i = \gamma\psi_i \cdot \sum_{j=1}^{n} J_j(R_i - R_j),\ i : 1...N. \ (1)$$

where $\psi_i = < \mathbf{e_i}, \Psi >$, $J_j = |\psi_j|^2$, and $R_j = \frac{|\psi_j|^2}{|\xi_j|^2}$, $j : 1...N$. It can be shown [6] that, during a typical time comparable to $\frac{1}{\gamma}$, the quantum state ψ will asymptotically collapse onto the direction $\mathbf{e_j}$ which maximises R_j: $R_j > R_i$, $\forall\, i,\ i \neq j$. It can happen that this direction is not unique, in which case the final state is not always defined. Anyhow, these situations constitute a subset of null measure of the set of hidden variables H^* and can be consistently neglected in what follows. The probability of finding the j^{th} result during the measurement process of O is thus given by the measure of the subset of H^* in which $\frac{|\psi_j|^2}{|\psi_i|^2} > \frac{|\xi_j|^2}{|\xi_i|^2}$, $\forall i, i \neq j$. When ξ is isotropically distributed [3], this probability is equal to $|\psi_j|^2$ and we recover the quantum probability. This property is called the polychotomic algorithm by Belinfante, and we give a sketch of its proof in appendix A.

The Gleason trouble, and a solution of the problem.

A problem occurs when the observable is not complete, so to say, when at least one of its eigenvalues possesses a degeneracy strictly superior to one. Then, we are free to choose different complete bases which diagonalise the observable. Computations show [2,4] that, for a fixed value of the hidden variable Ξ and for a given quantum state Ψ, the result of the measurement is sensitive to the choice of the complete basis that we make in order to apply B-B theory. This ambiguity of the B-B theory is in fact related to deep mathematical reasons and reflects a property of hidden variable theories called contextuality (see [7] for a review), a concept which emerged from the pioneering works of Gleason, J. Bell, Kochen and Specker [8,9,10]. This is why the problem is known in the litterature as the "Gleason trouble", or "Kochen and Specker paradox" [3]. Tutsch proposed a modification of B-B theory [2], which was later corrected by Belinfante (see [4] for a deep analysis of the question), in order to cure the disease (to rise the ambiguity). Let us now describe this technique, that we shall from now on call the Wiener-Siegel method, following the notation adopted in [4].

Consider, for every eigen subspace of the observable O associated to the eigen value o_l

(l : $1...M$, $M < N$), the projection Ψ_l of the quantum state Ψ along this subspace, and impose, for the choice of the orthonormal basis in which the B-B procedure is applied, that the M normalised vectors $\frac{\Psi_l}{||\psi_l||}$ belong to this basis.

We are still free to choose the $N - M$ remaining vectors of the basis, but, for all choices of the hidden variable Ξ, the result of the evolution given in (1) does not depend of this choice. Effectively, the initial components of the quantum state Ψ along these $N - M$ vectors is equal to zero, and remain so for all subsequent times because of the form of the B-B equation of evolution. We also recover the quantum probabilities, essentially because the restriction of an isotropic distribution on H^* to a M-dimensional subspace of H^* is still isotropic on this subspace.

This shows how this supplementary prescription for the choice of the basis which diagonalises the observable O allows us to get rid of the ambiguities of B-B theory.

3 Non-locality of B-B theory.

As we showed it in the previous section, the modified B-B theory allows us to simulate the quantum probability associated to any kind of measurement, for instance to Bell-like spin measurements. In virtue of Bell's theorem, the B-B theory must exhibit non-locality. This point was studied in detail in [11]. The author showed that, during a Bell-like spin measurement, the result of a Stern-Gerlach experiment realised on one of the two particles emitted in an entangled spin state does depend on the angle along which a Stern-Gerlach apparatus is set, along the trajectory of the second particle, and also on the fact that the second particle reaches this Stern-Gerlach magnet before or after, or at the same time as the first one. This time is to be taken relatively to the rest frame of the laboratory. The two measurement processes can occur in space like separated regions, so that we can effectively speak here about non-locality. Note that deep connections exist between non-locality and contextuality [12,7].

In [4], the authors also showed that if two successive measurements are realised (one in let us say the Right region on the first particle, and one in let us say the Left region on the second one), with non-parallel analyzers in both regions, and that the hidden variable Ξ has no time to randomize between both measurements, the B-B theory predicts new correlations between these successive results, which differ from quantum predicted correlations. The reason for this is that, for instance, when we compute in the B-B theory the probability of getting the result spin up in the Left region after having measured the result spin up in the Right region, with orthogonal Stern-Gerlach apparatusses, we sum over a restricted region of H^* (the one associated with the result spin up in the Right region). The distribution of hidden variables is thus no longer isotropic and we do not recover the standard quantum correlations.

3.1. Interpretation of the discrepancies.

Christensen and Mattuck considered that these results are drawbacks of the B-B theory: being given that the temporal order between the measurement processes in space like separated regions is not Lorentz invariant (for sure, because the time in the B-B theory is Newtonian), and being given that space-time geometry of our universe seems to be Minkoskian, they concluded that the B-B theory may not be applied to the description of the measurement of the properties of entangled systems in Bell-like situations.

Our interpretation of their results is fundamentally other, principally because of the three following arguments.

i) Clearly, the question of non-locality [5] is motivated by the contradiction between Minkoskian causality and the projection postulate, in which the act of measurement changes instantaneously the state of the system, eventually over space like intervals. We consider that the experimental violation of Bell's inequalities confirms the projection postulate and shows that, according to Gisin words [13], "something" goes faster than light. Note that this contradiction between the collapse process and Minkoskian causality can lead to paradoxal situations, as in the "paradox of the two bottles" [14].

ii) As it is well known, altough individual collapse processes violate causality, a miracle occurs, due to the fact that quantum mechanics is a stochastic, probabilistic theory. The average values of the results of observables measured by local observers can be shown to be unaffected by what happens in far away regions, so that no information can be carried faster than light. Even if the coexistence between non-locality and quantum mechanics is thus, according to the words of Shimony, "peaceful" [15], we consider that non-locality is a deep and fundamental property of nature which could lead in certain non-miraculous circumstances to the possibility of a supraluminal telegraph. The prejudice against this possibility is so strong that it has been used very often as a theoretical argument to rule out theories in which such a telegraph is permitted. To our knowledge, nobody ever tried experimentally to construct a supraluminal telegraph in a Bell-like situation. We make in the last chapter a proposal in this direction.

iii) The question of the existence of an absolute space-time is still presently open, and the difficulties met in order to conciliate general relativity and quantum mechanics are related to this problem. It might be that the program of quantization of space-time implies the selection of privilegged frames at a physical level, for instance of the rest frame of the laboratory in which experiments are realised. This would in fact constitute a come back to the conceptions of Lorentz and Poincaré about special relativity, where the relativity principle and the existence of an absolute medium (ether) were not considered yet as incompatible concepts.

In conclusion, we consider thus that non-orthodox theories as the B-B theory, in which the measurement is not instantaneous, are guiding examples which allow us to indicate to the experimenter what to measure in order to detect eventual manifestations of the existence of an absolute rest-frame. Instead of rejecting the theory because of its non-standard features, we consider that the theory is interesting in itself, because it replaces the instantaneous projection process by a process continuous in time, which is more physically appealing.

According to us, the discrepancies obtained by Christensen and Mattuck between B-B theory and standard quantum predictions are in last resort interesting novelties which are worth being tested experimentally.

Note that, in B-B theory, the hidden state Ξ remains constant during the time $\frac{1}{\gamma}$ of the measurement, but is assumed to randomize after a time τ_R, so we have necessarily $\frac{1}{\gamma} < \tau_R$ [16]. As it was noticed in [4], if we perform a Bell-like spin measurement, and that the distances d_L and d_R between the two Stern-Gerlach devices (or polarisers in the case of photons) and the sources are such that $\frac{1}{\gamma} < \frac{|d_L - d_R|}{c} < \tau_R$, where c is the speed of the particles sent along opposite directions, the B-B theory predicts correlations which differ from standard quantum theory. These are formally given by a quadrature in appendix B. in function of the distribution function f. Practically, this experiment can be realised if the experimentator carefully controls, in a Bell-like situation, the extent of the source and the distances d_L and d_R, and repeats the measurement of correlations for different values of these distances. This constitutes our first experimental proposal, directly inspired from the results of [4].

Note that, presently, the values of the typical times $\frac{1}{\gamma}$ and τ_R remain an open problem. even if some speculative proposals were made in order to evaluate these times [1.6.16]. Note also that the estimation of the intensity of the expected discrepancies between the B-B theory and the observations can be shown (appendix B) to depend on our choice of the shape of f, the distribution function of $|\Xi|$. We shall come back to these points in the conclusion.

We shall now propose an alternative prescription in order to solve the Gleason trouble and show how it inspires another type of experimental test.

4. The concept of dimensionality.

As we discussed it in the first section, the B-B theory is defined only when the observable under measurement is complete, so to say, when to each result of the measurement only one eigen state is associated. For instance, if the system is a Bell-like system which consists of two entangled spin 1/2 particles, the complete Hilbert space is four dimensional. Bohm and Bub considered in [1] that the state of the hidden variable must be expanded in a space H^* which possesses the same dimensionality as the complete Hilbert space H. and. in the case of degenerate observables, this is the origin of the Gleason trouble.

Our objection to this approach is that the concept of complete space itself is not relevant. practically speaking, for the following reasons.

i) We live in continuous space-time, so the complete space H ought to be infinite dimensional, but in this case B-B theory is not of application.

ii) Even if some approximation of the measurement process in the complete space can be done through a finite dimensional B-B model, which remains to prove. we must also show why the entanglement between the quantum system and the part of the external universe which interacted with him in the past (source, external fields) can be neglected in the model. Untill now, this point was not seriously discussed in the litterature relative to the subject.

iii) Most of all, it is possible in principle that we do not know yet all the degrees of freedom of the particles implied in the experiment. After all, dimensionality is a subjective concept which depends on what we measure and how we measure it. Poincaré discussed these questions in his famous book "La science et l' hypothèse" and arrived to the conclusion that a blind person, which percepts space through the surface of his organism (feet, fingers), and not through his eyes experiences a space of dimensionality comparable to the amount of muscles involved in this perception, which is not a fixed value. and is highly superior to three ! Remarkably, this idea was indirectly confirmed by the mathematician Hadamard who studied some years later the dependence of the solutions of a N+1-dimensional Dalembert like wave propagation problem towards the dimension of space N and showed that the propagation of information is only possible for N equal to one or three. For other values of N, a receiver distant from a ponctual source does not receive the original signal (up to a time delay), but an image of this signal which mixes signals emitted at different times and makes it very difficult for the receiver to recompose the initial information sent at the source. Remark that from our five senses. those which allow to explore far away regions of space (hearing and vision), are based on a 3+1 wave propagation of the kind mentioned here. It is impossible to decide if our image of space-time as a 3 + 1 dimensional continuum is a consequence of the fact that evolution forced us to select perceptional channels which are the most advantageous, informationally speaking, or if the universe is intrinsically 3 + 1 dimensional. The choice that we made here not to consider dimension as an absolute, objective, concept goes in the sense of the first of these alternatives. In some sense, we trust thus the unverifiable intuition according to which the theoretical physics developped by, for instance, ants or fishes (or at least their representation of the world) ought to be radically different from ours.

To conclude, we consider that dimensionality must not be considered as an intrinsic and absolute concept, but rather as a property imposed by the measurement process. The interest of these considerations lies in the fact that they suggest immediately another way of solving the Gleason trouble, that we shall develop in next section.

5. A new generalisation of B-B theory.

If the concept of a complete Hilbert space is not fundamental or even operationally relevant, it is logical to define B-B theory in a space of which the dimension (M) is equal to the amount of different outcome channels of the observable under measurement. which is, when degeneracies occur, smaller than the dimension (N) of the "complete" Hilbert space which describes the system. For instance, in a Bell-like situation. if we use only one Stern-Gerlach apparatus, $M = 2$ while $N = 4$.

We propose thus to assume that the hidden variables Ξ are isotropically distributed in a M-dimensional Hilbert space (H^*). In order to generalise the equation of evolution of B-B, let us rewrite it in a more convenient form. The equation 1 can be rewritten following [1] in terms of one-dimensional projectors:

$$|\mathbf{e_i} > \dot{\cdot} < \mathbf{e_i}, \mathbf{\Psi} > = (\gamma \sum_{j=1}^{n} J_j(R_i - R_j)) \, |\mathbf{e_i} > \cdot < \mathbf{e_i}, \mathbf{\Psi} >, \; i : 1...N. \; (2)$$

where $J_j = < \mathbf{\Psi}, \mathbf{e_j} >< \mathbf{e_j}, \mathbf{\Psi} >$, and $R_j = \frac{J_j}{|\xi_j|^2}$, $j : 1...N$. It is straightforward to write a generalisation of this expression which covers the previous case and the degenerate case as well:

$$|\mathbf{\Pi}_l \dot{\mathbf{\Psi}} > = (\gamma \sum_{j=1}^{M} J_j(R_l - R_j)) \, |\mathbf{\Pi}_l \mathbf{\Psi} >, \; l : 1...M. \; (3)$$

where $\mathbf{\Pi}_l$ is the projector on the eigensubspace of the l^{th} eigen value of O (l: 1...M). $J_j = < \mathbf{\Psi}, \mathbf{\Pi}_j \mathbf{\Psi} >$, and $R_j = \frac{J_j}{|\xi_j|^2}$, $j : 1...M$. We can make use of the fact that the spectral decomposition of an observable is complete ($\sum_{l=1}^{M} \mathbf{\Pi}_l = \mathbf{1}$) to rewrite the previous equation as:

$$|\dot{\mathbf{\Psi}} > = \sum_{l=1}^{M} \gamma \sum_{j=1}^{M} J_j(R_l - R_j) \, |\mathbf{\Pi}_l \mathbf{\Psi} >$$

If we combine this new evolution law and the assumption that the hidden variables Ξ are isotropically distributed in a M dimensional Hilbert space (H^*), we recover the quantum probability. Effectively, it is easy to check that with this evolution law, the projections of the state vector along the eigen subspaces of O remain parallel to their initial value, and that their rate of change is the same as the one we had with the Wiener-Siegel method provided we identify the values of the components of the hidden variables here and there. For what does concern the distribution of the hidden variables, remember that the useful distribution, in the Wiener-Siegel method is the projected distribution (from an N-dim Hilbert space onto a M-dim Hilbert space), of an isotropic distribution. It is trivial to show that the projection (the restriction to a subspace of the whole space) of an isotropic distribution is still isotropic, so that the distribution of hidden variables in the Wiener-Siegel method is precisely the distribution that we postulated here. We have thus, at this level, full agreement between the Wiener-Siegel method and ours.

As we shall show it now, this agreement is lost when we consider multiple simultaneous measurement processes on a same system, provided we make a reasonable assumption of independence of the different processes.

5.1. The new prescription for multiple measurements.

Altough our new prescription is unambiguous when we consider a unique measurement process, it is not obvious how to generalise it when multiple measurements are concerned.

because M differs from measurement to measurement. To remain coherent with our remarks about dimensionality, we must consider that our generalisation of B-B theory really describes the physical interaction with the apparatus. When the experimental set up involves multiple and separated apparatuses, it is natural to assume that the interaction terms due to different apparatusses will cumulate, in an independent manner. We make thus the following assumptions, which are the most simple ones to do in the present context:

i) Assumption of linearity:

The righthand term in the new equation of evolution 4 is now the sum of the K terms associated to a single apparatus evolution:

$$|\dot{\Psi}> = \sum_{k=1}^{K} \sum_{j_k,l_k=1}^{M_k} (\gamma J_{j_k}(R_{l_k} - R_{j_k}))\, |\mathbf{\Pi}_{l_k}\Psi> . \quad (4)$$

where $\mathbf{\Pi}_{\mathbf{l_k}}$ is the projector on the eigensubspace of the l^{th} eigen value of the k^{th} observable O_k (l_k : $1...M_k$), $J_{j_k} = <\Psi, \mathbf{\Pi}_{j_k}\Psi>$, and $R_{j_k} = \frac{J_{j_k}}{|\xi_{j_k}|^2}$ (j_k : $1...M_k$), and where the hidden variables Ξ_k are isotropically distributed in a M_k dimensional Hilbert space (H_k^*).

Note that Tutsch already deduced a similar equation [2], where the different projectors are associated to the different measuring apparatusses involved during the experiment. As it was noticed in [4], the distribution of hidden variables postulated by Tutsch could lead to contradictions with quantum predictions for instance when the observable under measurement is unique (only one apparatus) and degenerate. This does not happen in our case, as we showed it in the previous section.

ii) Assumption of independence:

The hidden variables which appear in each of these terms are independently distributed.

5.2. Successive measurements in a Bell-like situation.

As a consequence of the assumption of independence, when successive measurements are performed on a system, no memory effect can be seen (in other words, no extra-correlation appears between successive measurement results, because the hidden variable is not constant in time, and its randomization at each new measurement is independent of what happened during the previous measurement). In the particular case of Bell-like experiments, when both Stern-Gerlach magnets are placed dissymetrically towards the source (at different distances), a situation where the Wiener-Siegel method was shown to imply the possibility of discrepancies with standard quantum results, no new effect is thus expected in our case.

Nevertheless, it turns out that, when both polarisers are symmetrical, a situation where the Wiener-Siegel method was shown to be in perfect agreement with standard quantum results, our prescription predicts new possible effects, as we show it in the appendix B, where we make a choice for the isotropic distribution f which corresponds to the B-B theory, and give numerical estimations of the discrepancies in this case. Remark that the intensity of the discrepancy is strongly sensitive to our choice for the isotropic distribution of the hidden variables. We shall come back to this point in the conclusion.

These predictions mean that, if the generalisation of the B-B theory that we proposed presents some element of reality, when only one Stern-Gerlach device (or polariser in the case of photons) is placed, acting on one particle of a pair in the singlet state. the probability of getting spin (polarisation) up (down) at the outcome of this aanalyser is 1/2 (1/2), while it will be different, in general, if another analyser is placed at the other side. This means that we can use this device as a supraluminal telegraph. In a sense, it is not so astonishing to predict the possibility of a supraluminal telegraph in the framework of B-B theory, for two main reasons. The first, already mentioned, reason is that the B-B theory is formulated in Newtonian space-time, and not in Minkoskian space-time. The second reason is that it is a non-linear theory. The connections between faster than light transmission of information and non-linear generalisations of Schrödinger equation were clarified in several articles [13,15,17]. Note that, according to our discussion of section 3.1. the prediction of the existence of a supraluminal telegraph is also made possible by the Wiener-Siegel method, provided memory times are long enough to be effectively measured.

6. Conclusions.

The aim of this article was to show that if we decide to describe the projection process as a real physical phenomenon, characterised by a duration time and a memory time instead of the commonly accepted "instantaneous" version, new physical effects are to be expected. We chose to make the computation of these effects in the framework of the B-B theory. because of its simplicity, and also of the fact that this theory is formulated in Newtonian space-time, a natural framework when one wants to put into evidence the possible existence of an absolute medium, suggested by the validation of the projection postulate in numerous Bell-like experiments. We showed how different choices of prescription made to avoid the Gleason trouble lead to non-standard effects: in the case of successive measurements for the Wiener-Siegel method, in the case of simultaneous measurements for ours. These simple models must be considered as useful caricatures of reality, which aim at confirming a physically obvious result: the existence of a non-null collapse time $\frac{1}{\gamma}$ can be measured in a symmetrical ($d_L = d_R$) Bell-like configuration, while the existence of a memory time τ_R can be measured in a dissymmetrical ($d_L \neq d_R$) Bell-like configuration. The fact that theoretically predicted discrepancies are strongly model dependent in the Wiener-Siegel method as well as in ours is not a fundamental problem. Effectively, in our view. these discrepancies must not be considered as exact predictions but only as indications for the experimentator about where he has to look in order to see new effects. We consider that the best thing to do is to realise carefully the experiment, so to say to let variate d_L and d_R and to check if discrepancies with quantum results appear in dissymetric situations. or in symmetric situations. If one of these effects is observed, it will always be time to reconsider the very simple models given here.

Note that the models developped here are also caricatural for what does concern the description of the temporal extension of the wave packets associated to the particles. For instance, we did not take into account the fact that, because of the intrinsic breadth of the energy levels implied during the cascade process of the atomic source of entangled photons, the photon pair possesses a temporal width which is not negligible at all [18]. The B-B theory, as well as quantum theory itself, do not give a precise description of how we ought to take this temporal extension into account in explicit computations. The nature of time and the problem of temporal distributions of probabilities constitute even today an open problem in quantum mechanics [19]. It is interesting to note that all the problematic aspects of the quantum measurement process to which we refered in this work were already mentioned by E. Schrödinger [20 a,b] in a series of articles written in 1935, which were motivated by the famous EPR article [21]. Let us reproduce (in the disorder) some quotations of them:

About entangled systems: *"Best possible knowledge of a whole does not include best possible knowledge of its parts-and that is what keeps coming back to haunt us."* [20 a]

"Measurements on separated systems cannot directly influence each other-that would be magic." [20 a]

"It is rather discomfortimg that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter's mercy in spite of his having no access to it." [20 b]

About the quantum jump: *"...any measurement suspends the law that otherwise governs continuous time-dependence of the ψ-function and brings about in it a quite different change."* [20 a]

"Permit me to repeat that we do not possess a Q.M. whose statements should not be valid for sharply fixed points of time." [20 a]

"That "sharp time" is an anomaly in Q.M., and that beside, more or less independent of that, the special treatment of time forms a serious hindrance to adapting Q.M. to the relativity principle, is something that in recent years, I have brought up again and again, unfortunately without being able to make the shadow of a useful counterproposal." [20 a]

About the EPR paradox: *"The paradox would be shaken, though, if an observation did not relate to a definite moment. But this would make the present interpretation of quantum mechanics meaningless, because at present the objects of its predictions are considered to be the results of measurements for definite moments of time."* [20 b]

The three first sentences show that Schrödinger shared the conviction of Einstein, Podolski and Rosen according to which separated systems cannot be described by entangled states. The following sentences bring a new element to EPR's view, and show that Schrödinger was aware of the fact that the instantaneous character of the quantum jump is incompatible with a relativistic description of the quantum measurement process. By developing in the present paper an approach in which the principle of a non-local interaction ("magics") is not rejected a priori, we could focus on the other paradoxical element of the quantum measurement emphasised by Schrödinger: its sharpness.

Before closing this conclusion, it is useful to recall a first experiment which was realised by Papaliolos in 1967 [22] in order to test the validity of the B-B theory. Papaliolos assumed that the B-B theory was aimed at describing the passage of a photon through a polarising device, and that the randomization time of the hidden variable Ξ was not infinitely small. He let pass low intensity pulses prepared in a given polarisation state through two successive, very close to each other, polarisers. If the distance between these two polarisers is smaller than the randomization time of the hidden variable times the speed of light, new correlations are predicted by the B-B theory (departure from the Malus law). Papaliolos did not see any such effect even for extremely short distances. Tutsch [2] later criticised the relevance of these negative results by noting that

i) It is not the same photon which passes through both polarisers.

ii) Maybe that the hidden variable is not attached to the quantum system only, but that its behavior (and thus its randomization) depend also on the measuring apparatus (note that this view was developped in the framework of axiomatic probability in [23,24,25,26]).

The experiments proposed here help to prolongate the polemics and to put it again in the field of experimentation. The experiment with a dissymetric configuration aimed at checking the presence of a memory time is very close in principle to Papaliolos experiment. Furthermore, the first objection of Tutsch is no longer valid in this case because the second (strongly entangled) photon keeps its identity during the first experiment. The experiment with a symmetric configuration aimed at checking the presence of a measurement time is built "sur mesure" in order to meet the second objection of Tutsch, because it relies on the assumption that hidden variables are associated to the measuring apparatus. Note that it is possible to find a simpler experimental configuration aimed at checking the presence of a memory time in the B-B theory, in the case where hidden variables are associated to the measuring apparatus and where the system is, as in Papaliolos case, a two level system. This experiment implies one polariser and two successive particles while the experiment of Papaliolos implies two successive polarisers and one particle [27]. This experiment is presently in realisation at Paris Nord (J. Robert et al), in an experimental configuration where the two level system is concretized by atomic spins. We are presently working on the treatment of the numerical data collected during the experiment.

7. Appendix A.The B-B dynamics and statistics.

For reasons of simplicity, let us consider the simplest situation (N = 2), which contains the essential features of the model. The equation 1 now reads:

$$\dot{\psi_1} = \gamma\psi_1 \cdot |\psi_2|^2 \cdot (\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_2|^2}{|\xi_2|^2})$$

$$\dot{\psi_2} = \gamma\psi_2 \cdot |\psi_1|^2 \cdot (\frac{|\psi_2|^2}{|\xi_2|^2} - \frac{|\psi_1|^2}{|\xi_1|^2}). \quad (5)$$

From the evolution equation, it is straightforward to deduce the following equation, which implies that the norm of the state vector is preserved by the evolution.

$$< \Psi, \dot{\Psi} > = \psi_1^* \dot{\psi}_1 + \psi_2^* \dot{\psi}_2 = 0. \quad (6)$$

The initial phases of ψ_1 and ψ_2 are preserved by the evolution, and do not influence it, so that we can consistently assume that ψ_1 and ψ_2 are positive real numbers.

Then, if $\frac{|\psi_1|^2}{|\xi_1|^2} < \frac{|\psi_2|^2}{|\xi_2|^2}$ at time $t = 0$, the time derivative of ψ_1 is negative, so ψ_1 decreases while ψ_2 increases and the inequality is preserved for all times. Integrating, we obtain that ψ_1 asymptotically vanishes, so to say, that the state collapses onto the second basis vector $|\mathbf{e}_2>$. Similarily, if initially $\frac{|\psi_1|^2}{|\xi_1|^2} > \frac{|\psi_2|^2}{|\xi_2|^2}$, the state asymptotically collapses on the first basis vector $|\mathbf{e}_1>$. These properties are only briefly sketched here, but they are discussed in great detail in [2], where the stability criterion of Lyapunov is applied to the general case (N dimensions). The slowest collapse time is shown in [6] to be proportional to $\frac{1}{\gamma}$.

Another important property of the model is that the regions of H^* which will asymptotically collapse to $|\mathbf{e}_1>$ ($|\mathbf{e}_2>$), the "attractor bassins" of these experimental outcomes, are invariant under norm dilations in the sense that, if, for an initial state vector Ψ, the hidden vector Ξ causes a collapse on the outcome $|\mathbf{e}_1>$ ($|\mathbf{e}_2>$), then the hidden vector $\lambda\Xi$ causes a collapse on the outcome $|\mathbf{e}_1>$ ($|\mathbf{e}_2>$), for all complex non-null factors λ. This is due to the fact that if we simultaneously multiply the state vector Ξ by a complex factor λ and the time by the positive dilation factor $|\lambda|^2$, the equation 1 is invariant. Now, the attractive bassins in H^* relative to the outcomes $|\mathbf{e}_1>$ ($|\mathbf{e}_2>$) refer to the asymptotic behaviour of the state vector, so that they are the same if time is dilated.

This property is important because it explains why all isotropic distributions yield the same distribution of final outcomes. Whatever the choice that we make for the distribution function f introduced in section 2, when we integrate it over the attractor bassins in H^* in order to find the probability associated to the different possible outcomes of the experiment (here $|\mathbf{e}_1>$ and $|\mathbf{e}_2>$), the integration over $|\Xi|$ always factorises and provides a trivial normalisation factor $\frac{1}{(2\pi)^N}$ (here $N = 2$). The non-trivial contribution to the integral comes from the angles and does not depend on the choice of the function f. We shall now use advantageously this property in order to check that the isotropic distributions for the hidden vectors Ξ, combined with the equation 1, exactly simulate the quantum probability. The freedom of choice for f allows us to make for it a choice similar to Wiener-Siegel's one [3], so to say $f(|\Xi|) = \frac{1}{(\pi)^N} \cdot exp(-|\Xi|^2)$. For $N = 2$, the probability of getting the outcome $|\mathbf{e}_1>$ ($|\mathbf{e}_2>$) is thus equal to:

$$\frac{1}{(\pi)^2} \cdot \int_{\Omega_{1(2)}} dRe\xi_1 dIm\xi_1 dRe\xi_2 dIm\xi_2 \; exp(-\xi_1^2 - \xi_2^2),$$

where $\Omega_{1(2)}$ denotes the region of H^* in which $\frac{|\psi_1|^2}{|\xi_1|^2} > (<) \frac{|\psi_2|^2}{|\xi_2|^2}$ at time $t = 0$, while Re and Im refer to the real and imaginary parts of the components of the hidden vector in the basis ($|\mathbf{e}_1>$, $|\mathbf{e}_2>$). This integration becomes trivial if we introduce the polar coordinates by $\xi_i = |\xi_i| \cdot expi\theta_i$, $i = 1, 2$. For instance, the probability of getting the outcome $|\mathbf{e}_1>$

is equal to:

$$4\int_0^{\infty} d|\xi_1|\ |\xi_1|\ exp(-|\xi_1|^2) \int_{\sqrt{\frac{|\psi_2|^2|\xi_1|^2}{|\psi_1|^2}}}^{\infty} d|\xi_2|\ |\xi_2|\ exp(-|\xi_2|^2)$$

This reduces to:

$$2\int_0^{\infty} d|\xi_1|\ |\xi_1|\ exp(-|\xi_1|^2) \cdot exp(-\frac{|\psi_2|^2|\xi_1|^2}{|\psi_1|^2})$$

which is equal, thanks to the normalisation of the state vector ($|\psi_1|^2 + |\psi_2|^2 = 1$), to $|\psi_1|^2$. Similarily, we obtain that the probability of realising the outcome $|\mathbf{e}_2>$ is equal to $|\psi_2|^2$, in agreement with standard quantum predictions, as it must. The generalisation of this result to the general case $N > 2$ is entirely similar to the previous treatment, and leads also to perfect agreement with standard quantum results.

8. Appendix B: Discrepancies with standard Q.M.

Before we discuss the discrepancies which appear between standard quantum predictions and the probabilities computed on the basis of isotropic distributions, in the case of the Wiener-Siegel method (with memory time) and in ours (with simultaneous separated measurements), let us establish the framework in which we shall work , so to say a Bell-like situation. The quantum system under study consists then of a pair of spin 1/2 particles, moving along opposite directions, with spins in the so-called singlet state. Such a state can be described in a four dimensional Hilbert space, the tensorial product of two two-dimensional Hilbert spaces which represent the respective spins of the Left and Right particles. A basis of this space can always be written as: $(|+\rangle_L \otimes |+\rangle_R, |-\rangle_L \otimes |+\rangle_R, |+\rangle_L \otimes |-\rangle_R, |-\rangle_L \otimes |-\rangle_R)$ where $|\pm\rangle_{L,R}$ describe mutually exclusive states of spin of the particles in the distant regions Left (L) and Right (R) along a conventional axis (the same in the two regions) perpendicular to the axis of propagation of the pair. Remark that in real experiments, the particles under study are photons, as for instance in the so called Orsay experiments [28]. These experiments were realised in Paris in 1981, and are equivalent to EPR-Bohm-Bell-like experiments, excepted that they involve polarised photons instead of spin 1/2 particles. The relevant property is thus not the spin of the particle but its linear polarisation, but this does not affect the results that we shall obtain here. When the source of photons is an atomic cascade, and when the detectors are situated on the same straight line passing through the origin (the two photons are emitted along opposite directions), the constraints imposed by the conservation of total angular momentum are such that the amplitudes of the linear polarisations along parallel polarisers in the Left and in the Right zone are the same. Computations show [25] that the state is then described, for what concerns the polarisations of the photons by the singlet state:

$$\frac{1}{\sqrt{2}}\Big(|+\rangle_L \otimes |+\rangle_R + |-\rangle_L \otimes |-\rangle_R\Big). \quad (7)$$

8.1. Experimental test of Wiener-Siegel's prescription: Successive measurements in Bell-like situation.

Let us assume that the initial state is the singlet state, and that we firstly measure the spin (or polarisation in the case of photons) in the right analyser, which lies closer to the source than the left one. The observable under measurement possesses two eigenspaces: the space spanned by $|+\rangle_L \otimes |+\rangle_R$ and $|-\rangle_L \otimes |+\rangle_R$ and the space spanned by $|+\rangle_L \otimes |-\rangle_R$ and $|-\rangle_L \otimes |-\rangle_R$, and is clearly degenerate.

Then, the Wiener-Siegel method implies that we must choose a basis which contains the projection of the initial state onto the eigen spaces of the observable under measurement. For the singlet state, this basis is still the same ($|+\rangle_L \otimes |+\rangle_R$, $|-\rangle_L \otimes |+\rangle_R$, $|+\rangle_L \otimes |-\rangle_R$, $|-\rangle_L \otimes |-\rangle_R$). The evolution is given by the equation 1, which, with our choice of basis, gives:

$$\dot{\psi_1} = \gamma \cdot \psi_1 \cdot (|\psi_2|^2(\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_2|^2}{|\xi_2|^2}) + |\psi_3|^2(\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_3|^2}{|\xi_3|^2}) + |\psi_4|^2(\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_4|^2}{|\xi_4|^2}))$$

$$\dot{\psi_2} = \gamma \cdot \psi_2 \cdot (|\psi_1|^2(\frac{|\psi_2|^2}{|\xi_2|^2} - \frac{|\psi_1|^2}{|\xi_1|^2}) + |\psi_3|^2(\frac{|\psi_2|^2}{|\xi_2|^2} - \frac{|\psi_3|^2}{|\xi_3|^2}) + |\psi_4|^2(\frac{|\psi_2|^2}{|\xi_2|^2} - \frac{|\psi_4|^2}{|\xi_4|^2}))$$

$$\dot{\psi_3} = \gamma \cdot \psi_3 \cdot (|\psi_1|^2(\frac{|\psi_3|^2}{|\xi_3|^2} - \frac{|\psi_1|^2}{|\xi_2|^2}) + |\psi_2|^2(\frac{|\psi_3|^2}{|\xi_3|^2} - \frac{|\psi_2|^2}{|\xi_2|^2}) + |\psi_4|^2(\frac{|\psi_3|^2}{|\xi_3|^2} - \frac{|\psi_4|^2}{|\xi_4|^2}))$$

$$\dot{\psi_4} = \gamma \cdot \psi_4 \cdot (|\psi_2|^2(\frac{|\psi_4|^2}{|\xi_4|^2} - \frac{|\psi_2|^2}{|\xi_2|^2}) + |\psi_3|^2(\frac{|\psi_4|^2}{|\xi_4|^2} - \frac{|\psi_3|^2}{|\xi_3|^2}) + |\psi_1|^2(\frac{|\psi_4|^2}{|\xi_4|^2} - \frac{|\psi_1|^2}{|\xi_1|^2}))(8)$$

When the state is initially prepared in the singlet state, ψ_2 and ψ_3 remain frozen to their initial values zero by the evolution, and the evolution is much simpler:

$$\dot{\psi_1} = \gamma \cdot \psi_1 \cdot (|\psi_4|^2(\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_4|^2}{|\xi_4|^2})), \; \dot{\psi_4} = \gamma \cdot \psi_4 \cdot (|\psi_1|^2(\frac{|\psi_4|^2}{|\xi_4|^2} - \frac{|\psi_1|^2}{|\xi_1|^2}))(9)$$

This is exactly the same equation that we studied in the appendix A, in the case of a non-degenerate measurement with two outcomes ($N = 2$). We can thus apply the results obtained there to conclude that the result $|+\rangle_R$ will be observed with probability 1/2, the same probability as for the result $|-\rangle_R$. This is in perfect agrement with standard quantum predictions, as it must, because after all this was the goal of the Wiener-Siegel method.

What does happen if now we measure the left spin (or polarisation) ? The initial state is, after the collapse of the singlet state equal to $|+\rangle_L \otimes |+\rangle_R$ when a positive spin (polarisation) has been observed in the Right analyser, $|-\rangle_L \otimes |-\rangle_R$ when a negative spin (polarisation) has been observed. Let us now assume that we rotate the Left analyser by an angle θ. We mean by this that the eigen spaces of the positive (negative) spins (polarisations) observed in it are respectively spanned by the vectors

$(cos\frac{\theta}{2}|+\rangle_L + i\,sin\frac{\theta}{2}|-\rangle_L)\otimes|+\rangle_R$ and $(cos\frac{\theta}{2}|+\rangle_L + i\,sin\frac{\theta}{2}|-\rangle_L)\otimes|-\rangle_R$ $((i\,sin\frac{\theta}{2}|+\rangle_L + cos\frac{\theta}{2}|-\rangle_L)\otimes|+\rangle_R$ and $(i\,sin\frac{\theta}{2}|+\rangle_L + cos\frac{\theta}{2}|-\rangle_L)\otimes|-\rangle_R))$.

It can be shown [3] that there are good reasons for imposing that the hidden vectors transform as dual vectors of the state vectors. Then, we obtain [2] the following relation between the squared components ($|\xi_1'|^2$, $|\xi_2'|^2$, $|\xi_3'|^2$, $|\xi_4'|^2$) of the hidden vector in the new basis and the components in the old basis:

$$\begin{aligned}
|\xi_1|^2 &= |\xi_1'|^2 cos^2\frac{\theta}{2} + |\xi_2'|^2 sin^2\frac{\theta}{2} - |\xi_1'||\xi_2'|\,sin\theta\,sin\alpha,\\
|\xi_2|^2 &= |\xi_2'|^2 cos^2\frac{\theta}{2} + |\xi_1'|^2 sin^2\frac{\theta}{2} + |\xi_1'||\xi_2'|\,sin\theta\,sin\alpha,\\
|\xi_3|^2 &= |\xi_3'|^2 cos^2\frac{\theta}{2} + |\xi_4'|^2 sin^2\frac{\theta}{2} - |\xi_3'||\xi_4'|\,sin\theta\,sin\beta,\\
|\xi_4|^2 &= |\xi_4'|^2 cos^2\frac{\theta}{2} + |\xi_3'|^2 sin^2\frac{\theta}{2} + |\xi_3'||\xi_4'|\,sin\theta\,sin\beta,
\end{aligned} \tag{10}$$

where α (β) is the relative phase between ξ_1' and ξ_2' (ξ_3' and ξ_4'). The observable associated to the Left analyser is degenerate, and we must apply the Wiener-Siegel method once again. If the Right measurement gave the result $|+\rangle_R$, the basis selected in virtue of the Wiener-Siegel prescription contains the vectors $|+'\rangle_L\otimes|+\rangle_R = (cos\frac{\theta}{2}|+\rangle_L + i\,sin\frac{\theta}{2}|-\rangle_L)\otimes|+\rangle_R$ and $|-'\rangle_L\otimes|+\rangle_R = (i\,sin\frac{\theta}{2}|+\rangle_L + cos\frac{\theta}{2}|-\rangle_L)\otimes|+\rangle_R$ because these are the projections of the initial state on the subspaces of the observable. The choice of the two last basis vectors is irrelevant because the component of the state along these directions remains frozen to the value zero by the evolution. If there is no memory effect, so to say, if the initial distribution of the hidden vectors is isotropic, then, a treatment similar to the one followed in the case of the Right measurement allows us to compute the probability for observing the result let us say $|+'\rangle_L$ through the following integral: $\int_{\Omega_1} dRe\xi_1'dIm\xi_1'dRe\xi_2'dIm\xi_2'\; f(|\xi_1'|^2 + |\xi_2'|^2)$, where Ω_1 denotes the region of H^* in which $\frac{cos^2\frac{\theta}{2}}{|\xi_1'|^2} < \frac{sin^2\frac{\theta}{2}}{|\xi_2'|^2}$, while Re and Im refer to the real and imaginary parts of the components of the hidden vector in the new basis. By a treatment equivalent to the one followed in the appendix A, we obtain that the probability for observing the result $|+'\rangle_L$ ($|-'\rangle_L$) in the Left analyser is $cos^2\frac{\theta}{2}$ ($sin^2\frac{\theta}{2}$), in accordance with quantum predictions. Now, if the hidden vector stays the same during both measurements (Right and Left), a constraint appears on the values that it can take when the result of the first measurement is, let us say, $|+\rangle_R$. Instead of integrating on the previous domain, we must restrict the domain to $\Omega_{1-reduced}$, the region of H^* in which simultaneously $\frac{cos^2\frac{\theta}{2}}{|\xi_1'|^2} < \frac{sin^2\frac{\theta}{2}}{|\xi_2'|^2}$ and $|\xi_1|^2 > |\xi_4|^2$, and we must renormalise, so to say, divide the integral on the reduced domain by the integral of the distribution of hidden vectors on the region $|\xi_1|^2 > |\xi_4|^2$ (equal to 1/2). This renormalisation is due to the fact that we condition on the result $|+\rangle_R$ for the first measurement. The $|\xi_i'|^2$ are given in function of the $|\xi_i|^2$ in equation 10 so that the integral to compute is, formally, well defined. It is :

$$2\int_0^\infty d|\xi_3'|\,|\xi_3'|\int_0^{2\pi} d\theta_3\int_0^\infty d|\xi_4'|\,|\xi_4'|\int_0^{2\pi} d\theta_4\int_0^\infty d|\xi_1'|\,|\xi_1'|\int_0^{2\pi} d\theta_1\int_0^{2\pi} d\theta_2\,\cdot$$

$$\int_{\sqrt{\frac{\sin^2\frac{\theta}{2}|\xi_1'|^2}{\cos^2\frac{\theta}{2}}}}^{\infty} d|\xi_2'|\ |\xi_2'|\ f(|\xi_1'|^2 + |\xi_2'|^2 + |\xi_3'|^2 + |\xi_4'|^2)\cdot$$

$$(H(|\xi_1'|^2 \cos^2\frac{\theta}{2} + |\xi_2'|^2 \sin^2\frac{\theta}{2} - |\xi_1'||\xi_2'| \sin\theta \sin\alpha - |\xi_4'|^2 \cos^2\frac{\theta}{2} - |\xi_3'|^2 \sin^2\frac{\theta}{2} - |\xi_3'||\xi_4'| \sin\theta \sin\beta)).$$

where the function $H(x)$ (the Heaviside function) is equal to 1 when x is positive, to 0 otherwise. The problem is that the relative phases now play a role, because interference effects cannot be neglected in the change of basis, so that we cannot compute the integral explicitly. Anyhow, the result of the integral generally differs from $\cos^2\frac{\theta}{2}$. Note also that the memory effect depends on the choice that we make for the isotropic distribution f.

8.2. Experimental test of the new prescription: Simultaneous measurements in Bell-like situation.

The evolution is now given by the equation 4 which in a Bell-like situation, with simultaneous measurements, yields:

$$\begin{aligned}
\dot{\psi_1} &= \gamma\cdot\psi_1\cdot(|\psi_2|^2 + |\psi_4|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_3|^2)}{|\xi_1^L|^2} - \frac{(|\psi_2|^2 + |\psi_4|^2)}{|\xi_2^L|^2}\Big) + \\
&\quad \gamma\cdot\psi_1\cdot(|\psi_3|^2 + |\psi_4|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_2|^2)}{|\xi_1^R|^2} - \frac{(|\psi_3|^2 + |\psi_4|^2)}{|\xi_2^R|^2}\Big) \\
\dot{\psi_2} &= \gamma\cdot\psi_2\cdot(-)(|\psi_1|^2 + |\psi_3|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_3|^2)}{|\xi_1^L|^2} - \frac{(|\psi_2|^2 + |\psi_4|^2)}{|\xi_2^L|^2}\Big) + \\
&\quad \gamma\cdot\psi_2\cdot(|\psi_3|^2 + |\psi_4|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_2|^2)}{|\xi_1^R|^2} - \frac{(|\psi_3|^2 + |\psi_4|^2)}{|\xi_2^R|^2}\Big) \\
\dot{\psi_3} &= \gamma\cdot\psi_3\cdot(|\psi_2|^2 + |\psi_4|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_3|^2)}{|\xi_1^L|^2} - \frac{(|\psi_2|^2 + |\psi_4|^2)}{|\xi_2^L|^2}\Big) + \\
&\quad \gamma\cdot\psi_3\cdot(-)(|\psi_1|^2 + |\psi_2|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_2|^2)}{|\xi_1^R|^2} - \frac{(|\psi_3|^2 + |\psi_4|^2)}{|\xi_2^R|^2}\Big) \\
\dot{\psi_4} &= \gamma\cdot\psi_4\cdot(-)(|\psi_1|^2 + |\psi_3|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_3|^2)}{|\xi_1^L|^2} - \frac{(|\psi_2|^2 + |\psi_4|^2)}{|\xi_2^L|^2}\Big) + \\
&\quad \gamma\cdot\psi_4\cdot(-)(|\psi_1|^2 + |\psi_2|^2)\cdot\Big(\frac{(|\psi_1|^2 + |\psi_2|^2)}{|\xi_1^R|^2} - \frac{(|\psi_3|^2 + |\psi_4|^2)}{|\xi_2^R|^2}\Big)(11)
\end{aligned}$$

where $\xi_i^{L(R)}$ represents the ith component (1: 1, 2) of the hidden variable associated to the Left (Right) measurement process.

We shall firstly consider that initially the state is superposition of $|+\rangle_L \otimes |+\rangle_R$ and $|-\rangle_L \otimes |-\rangle_R$: $|\psi\rangle = \psi_1|+\rangle_L \otimes |-\rangle_R + \psi_4|+\rangle_L \otimes |-\rangle_R$. Then, ψ_2 and ψ_3 remain frozen to their initial values (zero) by the evolution, and the evolution is much simpler:

$$\dot{\psi_1} = \gamma \cdot \psi_1 \cdot |\psi_4|^2 \cdot (|\psi_1|^2 \cdot (\frac{1}{|\xi_1^L|^2} + \frac{1}{|\xi_1^R|^2}) - |\psi_4|^2 \cdot (\frac{1}{|\xi_2^L|^2} + \frac{1}{|\xi_2^R|^2}))$$

$$\dot{\psi_4} = \gamma \cdot \psi_4 \cdot |\psi_1|^2 \cdot (|\psi_4|^2 \cdot (\frac{1}{|\xi_2^L|^2} + \frac{1}{|\xi_2^R|^2}) - |\psi_1|^2 \cdot (\frac{1}{|\xi_1^L|^2} + \frac{1}{|\xi_1^R|^2}))(12)$$

This evolution is very close to the B-B equation in the case of a measurement inside a Hilbert space of dimension 2 which is given in the appendix A:

$$\dot{\psi_1} = \gamma \cdot \psi_1 \cdot (|\psi_4|^2(\frac{|\psi_1|^2}{|\xi_1|^2} - \frac{|\psi_4|^2}{|\xi_4|^2})),\ \dot{\psi_4} = \gamma \cdot \psi_4 \cdot (|\psi_1|^2(\frac{|\psi_4|^2}{|\xi_4|^2} - \frac{|\psi_1|^2}{|\xi_1|^2})).$$

in which case (ξ_1, ξ_4) is distributed isotropically inside a 2-dimensional Hilbert space. The single difference lies in the fact that instead of $\frac{1}{|\xi_1|^2}$ and $\frac{1}{|\xi_4|^2}$, the equation induced by our prescription contains $(\frac{1}{|\xi_1^L|^2} + \frac{1}{|\xi_1^R|^2})$ and $(\frac{1}{|\xi_2^L|^2} + \frac{1}{|\xi_2^R|^2})$ respectively.

The fact that the distributions of (ξ_1^L, ξ_2^L) and (ξ_1^R, ξ_2^R) are isotropic does not, in general, imply that the distribution of the vector $(\sqrt{\frac{1}{(\frac{1}{|\xi_1^L|^2} + \frac{1}{|\xi_1^R|^2})}}, \sqrt{\frac{1}{(\frac{1}{|\xi_2^L|^2} + \frac{1}{|\xi_2^R|^2})}})$ is isotropic.

We have thus a discrepancy between standard quantum results and our predictions in this case. In the case where $f(|\Xi^L|)$ and $f(|\Xi^R|)$ are centered around the value $|\Xi| = 1$ (the Bohm-Bub choice), the probability of finding the outcome let us say $|+\rangle_L \otimes |+\rangle_R$ is then equal to the following integral:

$$\int_0^1 d|\xi_1^L| \int_0^1 d|\xi_1^R| H[(|\psi_1|^2(\frac{1}{|\xi_1^L|^2} + \frac{1}{|\xi_1^R|^2}) - |\psi_4|^2 \cdot (\frac{1}{1-|\xi_1^L|^2} + \frac{1}{1-|\xi_1^R|^2}))]$$

where the function $H(x)$ is, as before, equal to 1 when x is positive, to 0 otherwise, and where $|\psi_i|$ (i=1, or 4) refers to the initial component of the entangled state of the pair in the product basis ($|\psi_1|^2 + |\psi_4|^2 = 1$). Numerical computations show for instance that when standard quantum probability $|\psi_1|^2$ is equal to 0.33, this integral is equal to 0.40. We have thus a discrepancy between standard quantum results and our predictions if we prepare the initial state in the superposition of $|+\rangle_L \otimes |+\rangle_R$ and $|-\rangle_L \otimes |-\rangle_R$. Unfortunately, the symmetries of the integration domain cancel this discrepancy in the case of the singlet state, which, in a laboratory, is the most easily available entangled state available with an atomic cascade as source. Nevertheless, discrepancies of the kind predicted here could be observed with other kinds of source, for instance parametric down converters [29] or neutral kaons sources [30]. Note that we could also predict discrepancies between standard quantum predictions and ours, even with the singlet state, if we let rotate one of the linear polarisations by an angle θ. This can be done by placing a rotating medium on the way of the photon (a solution containing achiral molecules for instance). This breaks the symmetry of the singlet state and transforms the initial components $(\psi_1, \psi_2, \psi_3, \psi_4) = (\frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}})$ into $(\frac{1}{\sqrt{2}}cos\theta, \frac{1}{\sqrt{2}}sin\theta, \frac{1}{\sqrt{2}}sin\theta, \frac{1}{\sqrt{2}}cos\theta)$ up to irrelevant phase factors. The study of the explicit expression of the probabilities in this case are out of the scope of the present work, because it requires the study of the equations 11 which, in the case of simultaneous measurements, cannot be put in a form similar to the already known B-B equations (8). Nevertheless, we still have that the four eigen states of the observable under measurement are equilibrium states as in the B-B equation. Obviously, in this situation of broken symmetry, the probabilities relative to these outcomes are not the same as standard predictions. We could also break the symmetry by rotating one of the polarisers, but this implies a modification of equation 11 that we prefear not to study here, for reasons of simplicity. As for what concerns other numerical or formal results obtained here, we think that they constitute at most an indication for the experimenter about where he has to look at.

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