Zeitschrift: Helvetica Physica Acta

Band: 71 (1998)

Heft: 6

Artikel: Superconductivity in the attractive Hubbard model : the double Hubbard-I approximation

Autor: Rodríguez-Núñez, J.J. / Ghosh, H.

DOI: https://doi.org/10.5169/seals-117127

Helv. Phys. Acta 71 (1998) 658 – 666
0018-0238/98/060658-09 $ 1.50+0.20/0

Helvetica Physica Acta

Superconductivity in the Attractive Hubbard Model: The Double Hubbard–I Approximation

By J.J. Rodríguez-Núñez

Departamento de Física, Universidade Federal de Santa Maria, 97105-900 Santa Maria/RS Brazil.
e-mail: jjrn@ccne.ufsm.br

H. Ghosh

Instituto de Física, Universidade Federal Fluminense, Av. Litorânea S/N, Boa Viagem, 24210-340 Niterói/RJ, Brazil. e-mail: hng@if.uff.br

(29.V.98)

Abstract.

Using the Dyson equation of motion for both the diagonal one-particle Green function, $G(\vec{k},\omega)$ and off–diagonal Green function, $F(\vec{k},\omega)$, at the level of the Hubbard–I decoupling scheme, we have found that they have four poles symmetric in pairs, justifying a more elaborated calculation done by the Zürich group by means of the T-Matrix approach (Pedersen et al, Z. Physik B **103**, 21 (1997)) and the moment approach of Nolting (Z. Physik **255**, 25 (1972)). We find that the energy spectra and the weights of $G(\vec{k},\omega)$ and $F(\vec{k},\omega)$ have to be calculated self-consistently. $G(\vec{k},\omega)$ satisfies the first two moments while $F(\vec{k},\omega)$ the first sum rule. Our *order parameter* $\alpha(T)$ is given by $1/N_s \sum_{\vec{k}} \varepsilon(\vec{k})\Delta(\vec{k})$. Due to the fact that we have a purely local attractive interaction $\Delta(\vec{k})$ can be of *any* s–type wave. However, for a *pure s–wave*, for which $\alpha(T) = 0$, we go back to the mean–field BCS results, with a renormalized chemical potential. In this case, the off–diagonal Green function, $F(\vec{k},\omega)$, satisfies the first two off–diagonal sum rules. We explicitly state the range of validity of our approximation.

Pacs numbers: 74.20.-Fg, 74.10.-z, 74.60.-w, 74.72.-h

After the discovery of the high-T_c materials [1], the study of correlations has gained interested due to the fact that there is the belief[2] that the normal properties of these materials could be explained in the framework of the Hubbard model[3, 4], since electron correlations are strong, i.e., the on-site electron-electron repulsions U are much larger than the energies associated with the hybridization of atomic orbitals belonging to different atoms[5]. This Hamiltonian is a kind of minimum model[6] which takes into account quantum mechanical motion of electrons in a solid, and nonlinear repulsion between electrons. Even though this model is too simple to describe solids faithfully, serious theoretical studies have revealed that to understand the various properties of it is a very difficult task. Its study will prove useful in developing various notions and techniques in statistical physics of many particle physics.

Since the high-T_c superconducting materials are extreme type II superconductors, with a short coherence volume, one might take this fact as an indication of tightly bound pairs or/and that correlations effects strongly affect the properties of such materials. However, this scenario has been challanged by recent tunneling measurements[7] which lead to the conclusion that, for example, underdoped $Ba_2Sr_2CaCu_2O_{8-\delta}$ is described by intermediate coupling interaction, because the pairing fluctuations persist up to T^*[8], where T^* ($T^* > T_c$) is the temperature of pair fluctuations and T_c is the superconducting critical temperature.

One of the simplest model featuring superconductivity and allowing a systematic study of correlations is the attractive Hubbard model[9] which we adopt in this paper as the counterpart of the usual Hubbard model. This model has been used to explore qualitative features of the superconducting phase transition[10]. Ref.[10] is mainly a review of the analytical work done on this model. Recently, Huscroft and Scalettar[11] find that the superconducting order parameter is more stable that the charge density wave order at half-filling in the presence of disorder. Then, due to these considerations, we concentrate in the superconducting properties leaving outside any treatment of charge density wave order.

We will use the Dyson equation of motion technique[12] for both the diagonal, $G(\vec{k},\omega)$, and off-diagonal one-particle Green function, $F(\vec{k},\omega)$, at the level of Hubbard–I decoupling scheme[4]. The main theoretical conclusion of this paper is that both the diagonal and off diagonal one-particle Green functions have four poles. These poles are symmetric in pairs verifying a more elaborated calculation of the Zürich group[13].

The model we study is the Hubbard model[9]

$$H = t_{\vec{i},\vec{j}} c^{\dagger}_{\vec{i}\sigma} c_{\vec{j}\sigma} + \frac{U}{2} n_{\vec{i}\sigma} n_{\vec{i}\bar{\sigma}} - \mu c^{\dagger}_{\vec{i}\sigma} c_{\vec{i}\sigma} \quad , \tag{0.1}$$

where $c^{\dagger}_{\vec{i}\sigma}$ ($c_{\vec{i}\sigma}$) are creation (annihilation) electron operators with spin σ. $n_{\vec{i}\sigma} \equiv c^{\dagger}_{\vec{i}\sigma} c_{\vec{i}\sigma}$. $U = -|U|$ is the local attractive interaction and μ the chemical potential (we work in the grand canonical ensemble). We have adopted Einstein convention for repeated indices, i.e., for the N_s sites $\vec{i}$, the z nearest-neighbor sites $\vec{j}$ and for spin up and down ($\sigma = -\bar{\sigma} = \pm 1$). $t_{\vec{i},\vec{j}} = -t$, for n.n. and zero otherwise. Other types of hopping, i.e., $t' \neq 0$ between next nearest neighboors ($n.n.n.$) could be considered in our formalism[14]. In this paper, we restrict ourselves to $n.n.$ hoping.

We need to evaluate the equation of motion[12] for the operators $c_{\vec{i}\sigma}$ and $c^{\dagger}_{\vec{i}\sigma}$. They are

$$\frac{\partial c_{\vec{i}\sigma}}{\partial \tau} = [c_{\vec{i}\sigma}, H]_- \quad ; \quad \frac{\partial c^{\dagger}_{\vec{i}\sigma}}{\partial \tau} = \left[c^{\dagger}_{\vec{i}\sigma}, H\right]_- \quad , \tag{0.2}$$

where the sign $-$ means the commutator. Combining Eqs. (0.1,0.2) we get

$$\frac{\partial c_{\vec{i}\sigma}}{\partial \tau} = +t_{\vec{i},\vec{l}} c_{\vec{l}\sigma} + U c_{\vec{i}\sigma} n_{\vec{i}\bar{\sigma}} \quad ; \quad \frac{\partial c^{\dagger}_{\vec{i}\sigma}}{\partial \tau} = -t_{\vec{i},\vec{l}} c^{\dagger}_{\vec{l}\sigma} - U c^{\dagger}_{\vec{i}\sigma} n_{\vec{i}\bar{\sigma}} \quad . \tag{0.3}$$

Next, the one-particle Green's function is defined as

$$G_{\sigma}(\vec{i}, \vec{j}; \tau) \equiv - \left\langle \left\langle T_{\tau} c_{\vec{i}\sigma}(\tau); c^{\dagger}_{\vec{j}\sigma}(0) \right\rangle \right\rangle \quad , \tag{0.4}$$

where T_{τ} means time ordering. Combining Eqs. (0.3,0.4), and Fourier analyzing the time and space variables we end up with the following equation for $G(\vec{k}, \omega)$

$$(\omega - \varepsilon_{\vec{k}}) G(\vec{k}, \omega) = 1 + U \Gamma^{(2)}(\vec{k}, \omega) \quad , \tag{0.5}$$

where $\Gamma^{(2)}(\vec{k}, \omega)$ is the Fourier transform of the doubly occupied Green function[15]

$$\Gamma^{(2)}(\vec{k}, \omega) \equiv \left\langle \left\langle n_{\vec{i}\bar{\sigma}}(\tau) c_{\vec{i}\sigma}(\tau); c^{\dagger}_{\vec{j}\sigma}(0) \right\rangle \right\rangle (\vec{k}, \omega) \quad . \tag{0.6}$$

As we see from Eq. (0.6), the one–particle Green function and the doubly occupied Green function are connected thruout the equation of motion. We get the doubly occupied Green function mainly due to the presence of four operators in the Hubbard interaction. In Eq. (0.6), $n_{i\bar{\sigma}} \equiv c^{\dagger}_{i\bar{\sigma}} c_{i\bar{\sigma}}$ is the occupation number operator. Our next step is to apply the Dyson equation to $\Gamma^{(2)}(\vec{k}, \omega)$, which is given by

$$(\omega - U)\, \Gamma^{(2)}(\vec{k}, \omega) = \rho_{\bar{\sigma}} + t_{\vec{i},\vec{l}} \left\langle \left\langle n_{\vec{i}\bar{\sigma}}(\tau) c_{\vec{l}\sigma}(\tau); c^{\dagger}_{\vec{j}\sigma}(0) \right\rangle \right\rangle (\vec{k}, \omega) \quad , \tag{0.7}$$

which we have obtained under the assumption that

$$\frac{\partial n_{\vec{i},\sigma}}{\partial \tau} = 0 \quad . \tag{0.8}$$

Eq. (0.8) is certainly an approximation which reproduces the Hubbard–I solution in the equation of motion approach and it is *only* valid at the level of Eq. (0.7). We leave for the future[16] the study of the effect of going beyond the approximation given by Eq. (0.8). Now we perform a decoupling in the spirit of Hubbard[4] as follows

$$\left\langle \left\langle n_{\vec{i}\bar{\sigma}}(\tau) c_{\vec{l}\sigma}(\tau); c^{\dagger}_{\vec{j}\sigma}(0) \right\rangle \right\rangle \approx \rho_{\bar{\sigma}} G_{\vec{l},\vec{j}}(\tau) - \frac{1}{U} \Delta_{\vec{l},\vec{i}} F^{\dagger}_{\vec{i},\vec{j}} \quad . \tag{0.9}$$

Combining Eqs. (0.7,0.9), we arrive to the following expression for $\Gamma_2(\vec{k},\omega)$

$$(\omega - U)\,\Gamma^{(2)}(\vec{k},\omega) = \rho_{\bar{\sigma}} + \rho_{\bar{\sigma}}\varepsilon_{\vec{k}} G(\vec{k},\omega) - \frac{\alpha(T)}{U} F^{\dagger}(\vec{k},\omega) \quad , \tag{0.10}$$

where $\alpha(T) = 1/N_s \sum_{\vec{k}} \varepsilon(\vec{k})\Delta(\vec{k})$. Combining Eqs. (0.5,0.7,0.10) we get

$$\left[(\omega - \varepsilon_{\vec{k}})(\omega - U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right] G(\vec{k},\omega) = \omega - U(1 - \rho_{\bar{\sigma}}) - \alpha(T) F^{\dagger}(\vec{k},\omega) \quad . \tag{0.11}$$

Thus, from Eqs. (0.11), we see that G and F are coupled and that G reduces to the Hubbard–I solution, as it should be, when $\alpha(T) = 0$. Let us pause for a while to explain the notation. The parameter $\alpha(T)$ is going to be our order parameter, in complete analogy with the decoupling scheme in mean–field treatments (BCS one, for example). Next, we have to find the time evolution for $F^{\dagger}(\vec{k},\omega)$. A similar analysis, i.e., another Hubbard–I decoupling scheme for $F^{\dagger}(\vec{k},\omega)$, shows that

$$\left[(\omega + \varepsilon_{\vec{k}})(\omega + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right] F^{\dagger}(\vec{k},\omega) = \alpha^{*}(T) G(\vec{k},\omega) \tag{0.12}$$

Eqs. (0.11,0.12) produce for $G(\vec{k},\omega)$ and $F^{\dagger}(\vec{k},\omega)$ the following solutions,

$$\begin{aligned} G(\vec{k},\omega) &= \frac{[\omega - U(1-\rho_{\bar{\sigma}})]\left[(\omega + \varepsilon_{\vec{k}})(\omega + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right]}{\left[(\omega - \varepsilon_{\vec{k}})(\omega - U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right]\left[(\omega + \varepsilon_{\vec{k}})(\omega + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right] + \mid \alpha(T) \mid^2} \quad , \\ F(\vec{k},\omega) &= \frac{\alpha(T)\,[\omega - U(1-\rho_{\bar{\sigma}})]}{\left[(\omega - \varepsilon_{\vec{k}})(\omega - U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right]\left[(\omega + \varepsilon_{\vec{k}})(\omega + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}\right] + \mid \alpha(T) \mid^2} \end{aligned} \tag{0.13}$$

From Eqs. (0.13) we conclude that, for $\alpha(T) \neq 0$,

$$G(\vec{k},\omega) = \sum_{j=1}^{4} \frac{\hat{\alpha}_j(\vec{k})}{\omega - \hat{\Omega}_j(\vec{k})} \quad , \quad F(\vec{k},\omega) = \sum_{j=1}^{4} \frac{\hat{\beta}_j(\vec{k})}{\omega - \hat{\Omega}_j(\vec{k})} \quad , \tag{0.14}$$

i.e., the one-particle Green functions have four poles (lifetime effects are neglected here) which turn out to be

$$\hat{\Omega}_1(\vec{k}) = -\,\hat{\Omega}_2(\vec{k}) = \omega_o(\vec{k}) \quad , \quad \hat{\Omega}_3(\vec{k}) = -\,\hat{\Omega}_4(\vec{k}) = \omega_1(\vec{k}) \quad , \tag{0.15}$$

where

$$\begin{aligned} \omega^2_{o,1}(\vec{k}) &\equiv \frac{1}{2}\left[C(\vec{k}) \pm \left[C^2(\vec{k}) - 4\left(\mid \alpha(T) \mid^2 + (1-\rho_{\bar{\sigma}})^2 U^2 \varepsilon^2_{\vec{k}}\right)\right]^{1/2}\right] \quad , \\ C(\vec{k}) &\equiv U^2 + \varepsilon^2_{\vec{k}} + 2U\varepsilon_{\vec{k}} \quad ; \quad \varepsilon_{\vec{k}} \equiv \varepsilon(\vec{k}) - \mu \quad , \end{aligned} \tag{0.16}$$

with $\varepsilon(\vec{k}) = -2t \sum_{j=1}^{d} \cos(k_j)$ and d the lattice dimension. Taking a closer look to $\omega^2_{o,1}(\vec{k})$ in Eqs. (0.16) we conclude that these poles have almost the form of the poles for $G(\vec{k},\omega)$ and

$F(\vec{k},\omega)$ obtained in Ref.[13], since they give four solutions, symmetric in pairs, respecting what we call the *BCS symmetry*. In Ref.[13], we interpreted these two symmetric solutions as corresponding to the *BCS* solution (the opening of the *BCS* gap around the chemical potential) and to the *pair physics* (the correlation gap), respectively. In other words, our two Hubbard–I decouplings have given an additional contribution, which is due to the presence of *pair fluctuations* above T_c and which remain for $T < T_c$. These *pair fluctuations* are the electrons which are not in the Meissner state. The only qualitative difference with respect to the results of Ref.[13] is that here we do not have lifetime effects.

The spectral weights (Eqs. (0.14)) are given by

$$\begin{aligned}
\hat{\alpha}_1(\vec{k}) &= \frac{[\omega_o(\vec{k}) - U(1-\rho_{\bar{\sigma}})][(\omega_o(\vec{k}) + \varepsilon_{\vec{k}})(\omega_o(\vec{k}) + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}]}{2\omega_o(\vec{k})(\omega_o^2(\vec{k}) - \omega_1^2(\vec{k}))} \quad , \\
\hat{\alpha}_2(\vec{k}) &= \frac{[\omega_o(\vec{k}) + U(1-\rho_{\bar{\sigma}})][(\omega_o(\vec{k}) - \varepsilon_{\vec{k}})(\omega_o(\vec{k}) - U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}]}{2\omega_o(\vec{k})(\omega_o^2(\vec{k}) - \omega_1^2(\vec{k}))} \quad , \\
\hat{\alpha}_3(\vec{k}) &= \frac{[\omega_1(\vec{k}) - U(1-\rho_{\bar{\sigma}})][(\omega_1(\vec{k}) + \varepsilon_{\vec{k}})(\omega_1(\vec{k}) + U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}]}{2\omega_1(\vec{k})(\omega_1^2(\vec{k}) - \omega_o^2(\vec{k}))} \quad , \\
\hat{\alpha}_4(\vec{k}) &= \frac{[\omega_1(\vec{k}) + U(1-\rho_{\bar{\sigma}})][(\omega_1(\vec{k}) - \varepsilon_{\vec{k}})(\omega_1(\vec{k}) - U) - \rho_{\bar{\sigma}} U \varepsilon_{\vec{k}}]}{2\omega_1(\vec{k})(\omega_1^2(\vec{k}) - \omega_o^2(\vec{k}))} \quad . \qquad (0.17)
\end{aligned}$$

and

$$\begin{aligned}
\hat{\beta}_1(\vec{k}) &= \frac{\alpha(T)[\omega_o(\vec{k}) - U(1-\rho_{\bar{\sigma}})]}{2\omega_o(\vec{k})(\omega_o^2(\vec{k}) - \omega_1^2(\vec{k}))} \equiv \alpha(T)\bar{\beta}_1(\vec{k}) \quad , \\
\hat{\beta}_2(\vec{k}) &= \frac{\alpha(T)[\omega_o(\vec{k}) + U(1-\rho_{\bar{\sigma}})]}{2\omega_o(\vec{k})(\omega_o^2(\vec{k}) - \omega_1^2(\vec{k}))} \equiv \alpha(T)\bar{\beta}_2(\vec{k}) \quad , \\
\hat{\beta}_3(\vec{k}) &= \frac{\alpha(T)[\omega_1(\vec{k}) - U(1-\rho_{\bar{\sigma}})]}{2\omega_1(\vec{k})(\omega_1^2(\vec{k}) - \omega_o^2(\vec{k}))} \equiv \alpha(T)\bar{\beta}_3(\vec{k}) \quad , \\
\hat{\beta}_4(\vec{k}) &= \frac{\alpha(T)[\omega_1(\vec{k}) + U(1-\rho_{\bar{\sigma}})]}{2\omega_1(\vec{k})(\omega_1^2(\vec{k}) - \omega_o^2(\vec{k}))} \equiv \alpha(T)\bar{\beta}_4(\vec{k}) \quad . \qquad (0.18)
\end{aligned}$$

Now it is an easy matter to convince ourselves that the following relations are satisfied

$$\begin{aligned}
\hat{\alpha}_1(\vec{k}) + \hat{\alpha}_2(\vec{k}) + \hat{\alpha}_3(\vec{k}) + \hat{\alpha}_4(\vec{k}) &= 1 \quad , \\
\omega_o(\vec{k})(\hat{\alpha}_1(\vec{k}) - \hat{\alpha}_2(\vec{k})) + \omega_1(\vec{k})(\hat{\alpha}_3(\vec{k}) - \hat{\alpha}_4(\vec{k})) &= \varepsilon_{\vec{k}} + \rho_{\bar{\sigma}} U \quad , \\
\hat{\beta}_1(\vec{k}) + \hat{\beta}_2(\vec{k}) + \hat{\beta}_3(\vec{k}) + \hat{\beta}_4(\vec{k}) &= 0 \quad , \qquad (0.19)
\end{aligned}$$

which are the first three sum rules for the moments[13, 17]. The second off-diagonal moment is not satisfied because our order parameter is not $\Delta(T)$ but $\alpha(T)$. A discussion of the *failure* of a Hubbard I-type solution (for $\alpha(T) = 0$), as the one presented in Eqs. (0.13), has

been pointed out by Laura Roth[18] many years ago, where she remarked that the diagonal Hubbard I solution only satisfied the first two sum rules for the moments. We should point out that the moment solution and the Hubbard I-type solution are different approaches, which turn out to be approximately equal due to the fact that some of the first sum rules are satisfied. Laura Roth's criticism to the Hubbard–I solution is still valid in the present calculation since we should have a correlation gap for any *finite* value of U/t. In order to close this gap, lifetime effects are called for.

From the spectral theorems we have[19]

$$\left\langle c_{i\sigma}c^{\dagger}_{j\sigma}\right\rangle = \int_{-\infty}^{+\infty} \frac{A_{i,j}(\omega)d\omega}{exp(\beta\omega)+1} \quad ; \quad \langle c_{i\uparrow}c_{j\downarrow}\rangle = \int_{-\infty}^{+\infty} \frac{B_{i,j}(\omega)d\omega}{exp(\beta\omega)+1} \quad . \tag{0.20}$$

Then, we have

$$\rho = \frac{1}{N_s}\sum_{\vec{k}} \int_{-\infty}^{+\infty} \frac{A(\vec{k},\omega)d\omega}{exp(\beta\omega)+1} \quad , \quad \alpha(T) = \frac{1}{N_s}\sum_{\vec{k}} \int_{-\infty}^{+\infty} \frac{\varepsilon(\vec{k})B(\vec{k},\omega)d\omega}{exp(\beta\omega)+1} \quad , \tag{0.21}$$

where $\beta = 1/T$ is the inverse of the temperature and $\alpha(T)$ has been defined just after Eq. (0.10). $A(\vec{k},\omega)$ and $B(\vec{k},\omega)$ are the one–particle spectral densities given as

$$A(\mathbf{k},\omega) \equiv -\frac{1}{\pi}\lim_{\delta\to 0^+} Im[G(\mathbf{k},\omega+i\delta)] \quad ; \quad B(\mathbf{k},\omega) \equiv -\frac{1}{\pi}\lim_{\delta\to 0^+} Im[F(\mathbf{k},\omega+i\delta)] \quad . \tag{0.22}$$

In consequence, by combining Eqs. (0.20,0.21,0.22) with our Green functions (Eqs. (0.13)), we obtain the following self-consistent equations

$$\begin{aligned} \frac{1}{U} &= \int_{-4}^{+4} \varepsilon N(\varepsilon)\left[\frac{\bar{\beta}_1(\varepsilon)+\bar{\beta}_2(\varepsilon)exp(\beta\omega_o(\varepsilon))}{exp(\beta\omega_o(\varepsilon))+1} + \frac{\bar{\beta}_3(\varepsilon)+\bar{\beta}_4(\varepsilon)exp(\beta\omega_1(\varepsilon))}{exp(\beta\omega_1(\varepsilon))+1}\right]d\varepsilon \quad ; \\ \rho &= \int_{-4}^{+4} N(\varepsilon)\left[\frac{\hat{\alpha}_1(\varepsilon)+\hat{\alpha}_2(\varepsilon)exp(\beta\omega_o(\varepsilon))}{exp(\beta\omega_o(\varepsilon))+1} + \frac{\hat{\alpha}_3(\varepsilon)+\hat{\alpha}_4(\varepsilon)exp(\beta\omega_1(\varepsilon))}{exp(\beta\omega_1(\varepsilon))+1}\right]d\varepsilon \quad , \end{aligned} \tag{0.23}$$

where $N(\varepsilon)$ is the 2D density of states, $t = 1$, and we have chosen $\rho = \rho_\sigma = \rho_{\bar{\sigma}}$, i.e., we are in the paramagnetic phase.

We indicate that our Eqs. (0.23) will respect particle-hole symmetry[20]. ¿From the analysis of the first equation of Eq. (0.23) and the definition of the our *order parameter* we can conclude that it allows any type of s–type of wave symmetry, since when performing the $\vec{k}$–integration of $\Delta(k) \times \epsilon(\vec{k})$, we see that $\alpha(T) \neq 0$ only if $\Delta(\vec{k})$ is of s–type. So, in this case, Eq. (0.23) can give rise to an order parameter of symmetry different from *pure* s-wave, a conclusion which was reached in a previous work[21] using the sum rules both for the diagonal and the off–diagonal one–particle spectral functions. However, for a *pure s–wave*, i.e., $\Delta(\vec{k}) = const.$, we get $\alpha(T) \equiv 0$. So, our approximation fails[22] and we must go back to Eq. (0.8) and directly perform our approximation in $\Gamma_2(\vec{k},\omega)$. We arrive to the mean–field BCS results, where the chemical potential gets renormalized by the Hartree shift, i.e., ρU. Due to our lazyness of languaje, we have used the word *order parameter* in this paragraph

to denote $\Delta(\vec{k})$, even though we defined $\alpha(T)$ just after Eq. (0.11) as the order parameter of our theory. Naturally, $\alpha(T)$ is the integral of $\Delta(\vec{k})$ weighted with $\varepsilon(\vec{k})$. So, they are related in some way, i.e., if $\Delta(\vec{k}) \equiv 0$, then $\alpha(T) = 0$.

In short, using the Hubbard–I decoupling scheme for both the diagonal and off-diagonal one-particle Green functions we have shown that these Green functions have four poles, symmetric in pairs, which qualitatively *verify* the more elaborated calculation of Ref.[13], as it has been previously discussed. Our one–particle Green functions satisfy sum rules for the moments and we have obtained other symmetries than a *pure* s–wave order parameter. The range of validity of our approximation is contained in Eqs. (0.7, 0.8). Now we solve our Eqs. (0.23) in a low order approximation: We fix the value of our *order parameter*, $\alpha(T)$, and find the chemical potential using the first of Eqs. (0.23). For $\rho = 0.01$, $U/t = -8.0$ and $\frac{\alpha(T)}{t^2} = 0.1$, we find $\mu/t \approx -2.825$. We should say that our approach is valid for $|U| \geq W$, where $W = 8t$ is the bandwidth in two dimensions. Our calculation have been performed for $\beta = 1/T = 100.0$. We leave for the future[16] the numerical evaluation of the critical temperature, T_c, and the order parameter as function of temperature for different values of U/t and electron concentration.

In order to deal with d–wave superconductivity, we should study a nearest neighboor ($n.n.$) attractive interaction. This model has been considered few years ago, at the mean field level, by Meintrup, Schneider and Beck[23] (See, also Ref.[24]). We mention, while leaving, that lifetime effects can be included in a natural way both in $G(\vec{k}, \omega)$ and $F(\vec{k}, \omega)$, as it has been previously done in Ref.[25] for the self–energy. Work along these lines is in progress.

Acknowlegments

We would like to thank CNPq–Brazil (project No. 300705/95-6), FAPERGS–Brazil, the Swiss National Science Foundation, and CONICIT–Venezuela (project F-139). Prof. Mucio Continentino indicated us that the study of this problem would be a rewarding task. We thank María Dolores García for reading the manuscript.

References

[1] J. Bednorz and K.A. Müller, Z. Phys. B **64**, 189 (1986)

[2] P.W. Anderson, Science **235**, 1196 (1987); *Frontiers and Borderlines in Many Particle Physics.* North Holland, Amsterdam (1988); ibid., *The Theory of Superconductivity in the High–T_c Cuprates.* Princeton Series in Physics (1997); J. González, M.A. Martín–Delgado, G. Sierra and A.H. Vozmediano, *Quantum Electron Liquids and High–T_c Superconductivity.* Lecture Notes in Physics m 38. Springer–Verlag (1995)

[3] J. Hubbard, Proc. R. Soc. London A **276**, 238 (1963)

[4] J. Hubbard, Proc. R. Soc. London A **281**, 401 (1964)

[5] Peter Fulde, *Electron Correlations in Molecules and Solids.* Springer-Verlag (1993). 2nd Edition

[6] Hal Tasaki, *The Hubbard Model: Introduction and Some Rigorous Results.* Preprint Sissa (1995); ibid., J. Statistical Phys. **84**, 535 9(1996)

[7] N. Miyakawa, J.F. Zasadzinski, L. Ozyuzer, P. Guptasarma, D.G. Hinks, C. Kendziora and K.E. Gray, cond-mat/9809398

[8] M. Randeria, cond–mat/9710223; M. Randeria and J.-C. Campuzano, cond–mat/9709107, Varenna Lectures; M. Randeria, "Photoemission Spectroscopy", in *X Trieste Workshop on Open Problems in Strongly Correlated Electron Systems.* ICTP–Italy, July 20–31, 1998

[9] R. Micnas, J. Ranninger and S. Robaszkiewicz, Rev. Mod. Phys. **62**, 113 (1990); A. Alexandrov, J. Ranninger and S. Robaszkiewicz, Phys. Rev. B **33**, 4526 (1986); M. Rice and L. Sneddon, Phys. Rev. Lett. **47**, 689 (1981). The last two references were the first ones to use this model for the study of the bismuthate superconductors. See also, P.H.J. Denteneer et al, Europhys. Lett. **16**, 5 (1991). The last authors compare the understanding of correlations in the attractive Hubbard model to the role played in transitions and critical phenomena by the Ising model

[10] M. Randeria in *Bose Einstein Condensation*, A. Griffin et al (Eds.). Cambridge University Press (1994)

[11] C. Huscroft and R.T. Scalettar, cond–mat/9606038

[12] A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems.* McGraw-Hill, New York (1971)

[13] M.H. Pedersen, J.J. Rodríguez–Núñez, H. Beck, T. Schneider and S. Schafroth. Z. Phys. B **103**, 21 (1997). However, the *BCS* symmetry is broken if we include *double fluctuations* in the T–matrix formulation, as it has been done by S. Schafroth and J.J. Rodríguez–Núñez, Z. Phys. B **102**, 493 (1997)

[14] R. Kirchhofer, Diplôme. Université de Neuchâtel (1997, unpublished); R. Kirchhofer, R. Frésard, H. Beck and J.J. Rodríguez–Núñez, Physica B (1998, accepted), SCES'98; ibid., unpublished

[15] E.V. Anda, "The Metal Insulator Transition in the Hubbard Model". *The Physics and Mathematical Physics of the Hubbard Hamiltonian.* Ed. D. Campbell and F. Guinea. Plenum Press, 1994

[16] J.J. Rodríguez–Núñez, in preparation

[17] W. Nolting, Z. Physik **255**, 25 (1972); W. Nolting, *Grundkurs: Theoretische Physik. 7 Viel-Teilchen-Theorie.* Verlag Zimmermann-Neufang (Ulmen–1992); T. Hermann and W. Nolting, cond–mat/9702022 and J. Magn. Magn. Mater. **170**, 253 (1997); T. Schneider, M.H. Pedersen and J.J. Rodríguez–Núñez, Z. Phys. B **100**, 263 (1996)

[18] Laura Roth, Phys. Rev. **184**, 451 (1969)

[19] G.D. Mahan, *Many–Particle Physics.* Second Edition. Plenum (1990)

[20] M.P. Sørensen and J.J. Rodríguez–Núñez, Physica C **274**, 323 (1997). The authors discuss the $t - J$ model within the moment approach

[21] J.J. Rodríguez–Núñez, C.E. Cordeiro and A. Delfino, Physica A **232**, 408 (1996). The results of this paper are in agreement with the ones of the double fluctuation calculations of Ref.[13]

[22] Due to our approximation (Eq. (0.8)), $\Delta(\vec{k}) = const. \Longrightarrow \alpha(T) = 0$. There is the possibility of still going beyond a mean–field treatment if Eq. (0.8) is not satisfied

[23] Th. Meintrup, Ph. D. thesis, Université de Neuchâtel, 1995 (unpublished); Th. Meintrup, T. Schneider and H. Beck, Europhys. Lett. **31**, 231 (1995)

[24] Q. Chen, I. Kosztin, B. Jankó and K. Levin, cond–mat/9807414. They have considered the "pairing approximation" to explain some superconducting properties in underdoped to overdoped cuprates. See also, H.-J. Kwon and A.T. Dorsey, cond–mat/9809225; H.-J. Kwon, cond–mat/9811059

[25] J.J. Rodríguez–Núñez and S. Schafroth, J. Phys.: Condens. Matter **10**, L391 (1998); S. Schafroth and J.J. Rodríguez–Núñez, to be submitted; J.J. Rodríguez–Núñez, in preparation