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Stochastic Quantisation of Proca's Model

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Abstract. Using the phase space stochastic quantization scheme the quantization of Proca's model in the extended space are investigated.

1 Introduction

Canonical quantisation of systems with first class constraints was formulated along general lines by Dirac [1].The corresponding analysis in the path integral approach was initiated by Faddeev [2] for gauge theories. It was extended by Fradkin and collaborators [3] within the broader framework of preserving BRST invariance[4]. The quantisation of systems with second class constraints, on the contrary, poses problems.In this case it is necessary to replace the canonical Poisson brackets by their corresponding Dirac brackets.The conversion of the Dirac brackets to quantum commutators is, in general, plagued with severe factor ordering problems.Moreover, the abstraction of the canonically conjugate variables is highly nontrivial.Consequently the quantisation of second class systems, either in the canonical or in the path integral formalisms,is
problematic [5,]. For these reasons it is very interesting to derive some of these results within a completely different formalism, namely using the stochastic quantization scheme of Parisi and Wu[8].Stochastic quantization was introduced in 1981 as a novel method for quantizating field theories and provides a remarkable connection between statistical mechanics and quantum field theory [8].

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The plan of this paper is as follows.
In Sec.2 we present the conversion of first class constraints to second class constraints for the Proca's model.
In Sec.3 an exact solution for Proca's model , in the momentum space, was done.
In Sec.4 an exact solution for the ghost sector was obtained.
In Sec.5 we present our conclusions.

2 The Proca's model

The Proca's model is described by the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A^{\mu}A_{\mu} \tag{1}$$

The systems possess two second class constraints

$$\phi_1 = \pi_0 \approx 0, \phi_2 = \partial_i \pi^i + m^2 A_0 \approx 0 \tag{2}$$

which satisfies the following relations:

$$\{\phi_1, \phi_2\} = -m^2 \delta(x-y) \tag{3}$$

The form for the hamiltonian becoms:

$$H_c = \int \left[\frac{\pi^2}{2} + \frac{1}{4}F_{ij}^2 + \frac{m^2}{2}\left(A_0^2 + A_i^2\right)^2 - A_0\phi_2\right] d^3x \tag{4}$$

Using the Batalin-Tyutin formalism [5] the new set of constraints are found to be

$$\tau_1 = \phi_1 + m^2\rho, \tau_2 = \phi_2 + \pi_\rho, \tag{5}$$

which are strongly involutive [5].

$$\{\tau_1, \tau_2\} = 0 \tag{6}$$

Recall that ρ, π_ρ are the extra fields satisfying $\{\rho, \pi_\rho\} = 1$ all the following commutations become zero. The next step is to obtain the involutive Hamiltonian.

After a long and tedious calculations the complete expression for the desired Hamiltonian is

$$\star H = \star H^1 + \star H^2 \tag{7}$$

with

$$\star H^1 = \int d^3x \left[\left(\partial_i A^i\right) m^2 \rho - \frac{\pi_\rho}{m^2}\left(\partial_i \pi^i + m^2 A_0\right)\right] \tag{8}$$

$$\star H^2 = \int d^3x \left[\frac{-1}{2m^2}\pi_\rho^2 - \frac{m^2}{2}(\partial_i\rho)\left(\partial^i\rho\right) \right] \tag{9}$$

which ,by construction,is strongly involutive [5] $\{\star H, \tau_1\} = 0$ and $\{\star H, \tau_2\} = 0$ We define the canonical action by

$$S = \int dt \left[\pi_i \dot{A}^i + \pi_\rho \dot{\rho} + - \star H + \lambda\tau_1 + \mu\tau_2 \right] \tag{10}$$

denoting the Lagrange multipliers by λ and μ.

3 Phase space stochastic quantization

Stochastic quantization proceeds by introducing an extra stochastic time s dependence for coordinates and momenta $q_i(t,s), p_i(t,s)$ subjected to the Langevin equations [7-11].

$$\frac{\partial q_l}{\partial s} = \frac{i\delta S}{\delta q_l} + \xi_l, q_l(t,0) = 0 \tag{11}$$

and

$$\frac{\partial p_l}{\partial s} = \frac{i\delta S}{\delta p_l} + \eta_l, p_l(t,o) = 0 \tag{12}$$

where $q_l = A_l$ and $p_l = \pi_l$ for l=0,1,2,3. By standard consideration the noises are defined as

$$\langle \xi_i(t,s)\xi_l(t',s')\rangle = \langle \eta_i(t,s)\eta_l(t',s')\rangle = 2\delta_{il}\delta(t-t')\delta(s-s') \tag{13}$$

$$\langle \xi_i(t,s)\eta_l(t',s')\rangle = 0 \tag{14}$$

The constraints from (5) are implemented within the stochastic scheme according to [11,12] by imposing for all fictitious times s

$$\tau_\alpha(t,s) = 0 \tag{15}$$

with α =1,2 and τ_α are the constraints from (5).
The consistency conditions are [6]

$$\frac{\partial \pi_0}{\partial s} + m^2\frac{\partial \rho}{\partial s} = 0 \tag{16}$$

$$\partial^i\frac{\partial \pi_i}{\partial s} + m^2\frac{\partial A_0}{\partial s} + \frac{\partial \pi_\rho}{\partial s} = 0 \tag{17}$$

The consistency conditions (16,17) allow to determine the Lagrange multipliers

$$\lambda = M\left(-\dot{A}_0 + m^4\partial^i A_i - m^2\dot{\pi}_\rho - m^4\partial_i\partial^i\rho + i\eta_0 + im^2\xi_\rho\right) \tag{18}$$

$$\mu = M(-\dot{\rho} + m^2\dot{\pi}_0 - \partial^i\dot{A}_i - \partial^i\partial_i A_0 + \frac{1}{m^2}\partial^i\partial_i\pi_\rho + \partial^i\pi_i + i\partial^i\eta_i + i\eta_\rho + im^2\xi_0) \tag{19}$$

In the configuration space we have the following equations of motion

$$\frac{\partial A_0}{\partial s} = -i\dot{\pi}_0 + i\partial^i\pi_i(1+M) + im^2\left(A_0 - M\partial^i\partial_i A_0\right) + i\left(\pi_\rho + M\partial^i\partial_i\pi_\rho\right)$$
$$+im^2M\left(-\dot{\rho} + m^2\dot{\pi}_0 - \partial^i\dot{A}_i - \partial^i\dot{A}_i + i\eta_\rho\right) + \xi_0\left(-m^2M+1\right) \quad (20)$$

$$\frac{\partial A_i}{\partial s} = -i\dot{\pi}_i - im^2A_i + im^2\partial_i\rho - i\partial_j^2A_i + i\partial j\partial_i A_j + \xi_i \quad (21)$$

$$\frac{\partial\rho}{\partial s} = -im^2\partial^iA_i + i\dot{\pi}_\rho + im^2\partial_i\partial^i\rho +$$
$$im^2M\left(-\dot{A}_0 + m^4\partial^iA_i - m^2\dot{\pi}_\rho - m^4\partial_i\partial^i\rho + i\eta_0 + im^2\xi_\rho\right) + \xi_\rho \quad (22)$$

$$\frac{\partial\pi_0}{\partial s} = im^4M\dot{A}_0 + iM\left(m^4\partial^iA_i - m^2\dot{\pi}_\rho - m^4\partial_i\partial^i\rho + im^4\eta_0 + im^2\xi_\rho\right) \quad (23)$$

$$\frac{\partial\pi_i}{\partial s} = -i\partial_iA_0 + i\dot{A}_i - i\pi_i - \frac{i}{m^2}\partial_i\rho + \eta_i \quad (24)$$

$$\frac{\partial\pi_\rho}{\partial s} = -i\dot{\rho} + iM\left(-\dot{\rho} + m^2\dot{\pi}_0 - \partial^i\dot{A}_i + \frac{1}{m^2}\partial^i\partial_i\pi_\rho + \partial^i\pi_i + i\eta_\rho + im^2\xi_0\right) \quad (25)$$

where $i = 1, 2, 3$.

In the momentum space, using the matrix form, after some calculations, we have the following equations of motion:

$$\dot{A} = BA + D \quad (26)$$

where

$$A = \begin{pmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ \rho \\ \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_\rho \end{pmatrix}$$

, The matrix form for the matrix B are presented in the Appendix.

$$D = \begin{pmatrix} -Mm^2\eta_\rho + M\xi_0 - im^2Mk^i\eta_i \\ -\xi_1 \\ \xi_2 \\ \xi_3 \\ \eta_0 - M\xi_\rho \\ -m^2M\xi_\rho - Mm^4\eta_0 \\ \eta_1 \\ \eta_2 \\ -\eta_3 \\ -M\eta_\rho - m^2M\xi_0 - iMk^i\eta_i \end{pmatrix}$$

Here $M = \frac{1}{m^4+1}$. Therefore we get the solution of (26) formally

$$A = \int d\tau exp^{B(s-\tau)} D \tag{27}$$

4 Stochastic quantization of Proca's model in the extended space

In this section an exact solution for the ghost sector will be investigated. In the extended space the hamiltonian has the form [13]:

$$H = H_0 + \frac{1}{2}\left(\eta^1 P^1 + \eta^2 P^2\right) \tag{28}$$

Here P^1, P^2 represent the momentum conjugate to ghosts η^1 and η^2 . From(28) we can deduce immediately that a complete solution has the form

$$\star A = A + A_{gh} \tag{29}$$

A is the matrix from (27) and A_{gh} is a matrix which describes the ghosts . We are interested to obtaining an exact solutions for A_{gh}.
Using the equations of motion in momentum space ,after some calculations,
a formal solution for the ghost sector becomes

$$A_{gh} = \int d\tau exp^{M_{gh}(t-\tau)} D_{gh} \tag{30}$$

where A_{gh} and D_{gh} has the following expressions:

$$A_{gh} = \begin{pmatrix} \eta^1 \\ \eta^2 \\ \eta^3 \\ P^1 \\ P^2 \end{pmatrix}$$

$$D_{gh} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \delta_1 \\ \delta_2 \end{pmatrix}$$

ξ_1, ξ_2,δ_1, δ_2 are the noises.
We are interested to obtaining a diagonal form for the matrix M_{gh}.

From the equations of motion for the ghost sector we can deduce the matrix form M_{gh}:

$$M_{gh} = \begin{pmatrix} 0 & 0 & -k & i \\ 0 & 0 & -i & -k \\ -k & i & 0 & 0 \\ -i & -k & 0 & 0 \end{pmatrix} \tag{31}$$

On the other hand we have the following relation:

$$expM_{gh}\,(t-\tau) = expC^{-1}M_{gh_d}C\,(t-\tau) = C^{-1}expM_d\,(t-\tau)C \tag{32}$$

We know that

$$M_{gh_d} = \begin{pmatrix} exp\lambda_1\,(t-\tau) & 0 & 0 & 0 \\ 0 & exp\lambda_2\,(t-\tau) & 0 & 0 \\ 0 & 0 & exp\lambda_3\,(t-\tau) & 0 \\ 0 & 0 & 0 & exp\lambda_4\,(t-\tau) \end{pmatrix} \tag{33}$$

and the matrix C, after some calculations, has the following form:

$$C = \begin{pmatrix} -2ik & -\frac{k+1}{k-1} & -i & 1 \\ \frac{i(3k-1)}{k-1} & \frac{k+1}{k-1} & -i & 1 \\ \frac{i(3k+1)}{k+1} & \frac{1-k}{1+k} & i & 1 \\ -\frac{i(3k+1)}{k+1} & \frac{k-1}{k+1} & i & 1 \end{pmatrix} \tag{34}$$

From (31,33,34) the solution for the ghost sector is obtained.
The final solution for Proca's model , in the extended space, is obtained from (29).

5 Concluding remarks

In this paper we have made an investigation of the Batalin-Tyutin method for the Proca's model converting second class system into first class. The Langevin equation may be investigated and an exact solution may be obtained in the extended space. Because in the Batalin-Tyutin formalism we have and the extra fields the physical interpretation of these fields was done using the stochastic quantization method in the extended space[7]. We belived that our nonabelian exercise may provide fresh insight in the field of quatum gravity. These problems are under investigation.

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Appendix

The matrix B of Eq. (26) has the following form:

$$
\begin{pmatrix}
iMm^2\vec{k}^2+im^2 & -iMm^2k^1\omega & -iMm^2k^2\omega & -iMm^2k^3\omega & \omega Mm^2 & -Mm^4\omega & iMm^2k^1+ik^1 & iMm^2k^2+ik^2 & iMm^2k^3+ik^3 & -i\vec{k}^2M+i \\
0 & -im^2-ik_1^2 & -im^2-ik_2k_1 & -im^2-ik_3k_1 & -m^2k_1 & 0 & -\omega & 0 & 0 & 0 \\
0 & -im^2-ik_2k_1 & -im^2-ik_2^2 & -im^2-ik_2k_3 & -m^2k_2 & 0 & 0 & -\omega & 0 & 0 \\
0 & -im^2-ik_1k_3 & -im^2-ik_2k_3 & -im^2-ik_3^2 & -m^2k_3 & 0 & 0 & 0 & -\omega & 0 \\
-Mm^2\omega & Mm^2k^1 & Mm^2k^2 & Mm^2k^3 & -iMm^2\vec{k}^2 & 0 & 0 & 0 & 0 & M\omega \\
Mm^4\omega & -Mm^4k^1 & -Mm^4k^2 & Mm^4k^3 & m^4\vec{k}^2 & 0 & 0 & 0 & 0 & 0 \\
k_1 & -im^2 & 0 & 0 & -m^2k_1 & 0 & -i & 0 & 0 & \frac{k_1}{m^2} \\
k_2 & 0 & -im^2 & 0 & -m^2k2 & 0 & 0 & -i & 0 & \frac{k_2}{m^2} \\
k_3 & 0 & 0 & -im^2 & -m^2k_3 & 0 & 0 & 0 & -i & \frac{k_3}{m^2} \\
iM\vec{k}^2+1 & Mk^1\omega & Mk^2\omega & Mk^3\omega & -M\omega & Mm^2 & \frac{-M-1}{m^2}k^1 & \frac{-M-1}{m^2}-k^2 & \frac{-M-1}{m^2}k^3 & -i\frac{M}{m^2}\vec{k}^2+\frac{i}{m^2}
\end{pmatrix}
$$

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