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The Quantization of the Space-time Defects in the Early Universe

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Abstract. In Riemann-Cartan manifold U_4, a new topological invariant is obtained by means of the torsion tensor. In order to describe the space-time defects (which appear in the early universe due to torsion) in an invariant form, the new topological invariant is introduced to measure the size of defects and it is interpreted as the dislocation flux in internal space. Using the so-called ϕ-mapping method and the gauge potential decomposition, the dislocation flux is quantized in units of the Planck length. The quantum numbers are determined by the Hopf indices and the Brouwer degrees. Furthermore, the dynamic form of the dislocations is also studied by defining an identically conserved current.

1. Introduction

As is well known, torsion is a slight modification of the Einstein theory of relativity (proposed in the 1922-23 by Cartan[1]), but is a generalization that appears to be necessary when one tries to conciliate general relativity with quantum theory. If we consider the quantum theory in curved instead of flat Minkowsky space-time, we have some very important new

effects (as, for instance, neutron interferometry[2]). Moreover, when we go to a microphysical level, that is when we are concerned with elementary-particle physics, we realize that the role of gravitation becomes very important and necessary and this happens in the first place when we consider the early universe or the Planck era. In fact, elementary particles are characterized not only by mass but also by spin which occurs in units of $\hbar/2$. A mass distribution in a space-time is described by the energy-momentum tensor and connected with the curvature of space-time. The dynamical relation between the stress-energy-momentum tensor and curvature is expressed in general relativity by Einstein equations. One feels here the need for an analogous dynamical relation including spin density tensor. Since this is impossible in the framework of the general relativity, we are forced to introduce this new geometrical property that we call torsion. Thus, when we deal with a microphysical realm we find that the torsion comes into play and then has to be considered as the source of a gravitational field.

In recent years, a great deal of work on spin and torsion have been done by many physicists[3-6]. Though it has been common to include intrinsic spin with gravitation[7-9], and to relate spin to the torsion tensor[10-12], the quantization of the gravitational field (that is to quantize the Riemann-Cartan space-time itself) and the mechanism of production of torsion in physics and geometry[13] are not very clear. In some recent papers, Ross[14] and Sabbata[15] investigated these problems by the viewpoint of defects of space-time, which may be important in the early universe because of spontaneous symmetry breaking[16]. However, in their framework, the description of the space-time defects is not invariant under coordinate transformations and the quantization of space-time is only an assumption. In order to overcome these shortages, we will restudy the problems in terms of vierbein theory in this paper.

2. The formulation of the space-time defects in invariant form

Recently, some physicists studied the early universe by the viewpoint of defects of the

Riemann-Cartan manifold U_4, in which there exists the non-zero torsion

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{[\mu\nu]}, \quad \mu, \nu, \lambda = 1, 2, 3, 4,$$

where $\Gamma^{\lambda}_{\mu\nu}$ is an asymmetric affine connection. In the discussions of the importance of spin and torsion in the early universe[15], Sabbata proposed an integral

$$l^{\lambda} = \oint T^{\lambda}_{\mu\nu} dx^{\mu} \wedge dx^{\nu} \tag{1}$$

to represent the defects (dislocations) in space-time. By analogy with the well-known Bohr-Sommerfeld relation $\oint pdq = n\hbar$, the author assumed that the integral l^{λ} is quantized in units of the Planck length L_p, i.e.

$$l^{\lambda} = \oint T^{\lambda}_{\mu\nu} dx^{\mu} \wedge dx^{\nu} = nL_p, \qquad L_p = (\hbar G/c^3)^{1/2}, \tag{2}$$

and defined time in the quantum geometric level through the fourth component as

$$t = \frac{1}{c} \oint T dA = nT_p, \qquad T_p = (\hbar G/c^5)^{1/2}, \tag{3}$$

where n is an integer and c the velocity of light. We think that the hypothesis (2) is reasonable because it is based on the fact that, being torsion linked to spin and being the spin quantized, the Planck length L_p enters through the minimal unit of spin, or action $\hbar$. (In fact, we have revealed that the quantization of spin can also be derived from that of torsion, and the manuscript of which is in preparation.) However, to be an observable physical quantity, we learn that it must be invariant under both coordinate and gauge transformations. But the definition (1) does not has the property, that means l^{λ} is not invariant under coordinate transformations and then, is dependent on the choice of coordinate system. Furthermore, the quantization of l^{λ} in (2) is an assumption after all and the quantum number n is not determined. In order to formulate the space-time defects in an invariant form and quantize them naturally, let us investigate the problems in vierbein theory, in which the torsion can be expressed by

$$T^{A}_{\mu\nu} = D_{\mu} e^{A}_{\nu} - D_{\nu} e^{A}_{\mu}, \quad \mu, \nu, A = 1, 2, 3, 4 \tag{4}$$

where e^{A}_{μ} is the vierbein field and

$$D_{\mu} = \partial_{\mu} - \omega_{\mu}(x), \quad \omega_{\mu} = \frac{1}{2} \omega^{AB}_{\mu} I_{AB}$$

is the gauge covariant derivative, $\omega_\mu^{AB}(x)$ stands for the spin-connection and I_{AB} the generator of the Lorentz group.

As in Ref. [17], we can define a gauge parallel vector in internal space, whose existence is closely related to the geodesic $\gamma(S)$

$$\frac{dU^\lambda}{dS} + \Gamma^\lambda_{\mu\nu} U^\mu U^\nu = 0, \qquad U^\mu = \frac{dx^\mu}{dS}, \tag{5}$$

which can be further written in the covariant derivative notation[18]

$$\nabla_\mu U^\lambda = \partial_\mu U^\lambda + \Gamma^\lambda_{\mu\nu} U^\nu = 0, \tag{6}$$

where dS is the element of length of $\gamma(S)$. Using

$$\omega_\mu^{AB} = (\nabla_\mu e_\nu^A) e^{\nu B},$$

(6) multiplied by e_λ^B gives

$$D_\mu U^A = 0, \qquad U^A = e_\lambda^A U^\lambda,$$

which means $U^A(x)$ is a gauge parallel vector along the geodesic $\gamma(S)$. Though the vector U^λ is defined only at points of $\gamma(S)$, it can be extended to a vector field on a neighborhood of any point of $\gamma(S)$, which leads to $U^A(x)$ also a gauge parallel vector field on this neighborhood[19,20]. On the other hand, it is well-known that any integral curve of ordinary differential equations (5) is determined by a point $p_0(x_0^1, \cdots, x_0^4)$ and a direction at p_0[21]. If, at the same point p_0, we give four linearly independent directions $U_{(i)}^\lambda(p_0) = (\frac{dx^\lambda}{dS_i})_{p_0}$ with

$$g_{\mu\nu} U_{(i)}^\mu(p_0) U_{(j)}^\nu(p_0) = \delta_{(ij)}, \qquad i, j = 1, 2, 3, 4,$$

we obtain four geodesics and four corresponding linearly independent gauge parallel vectors marked by the index (i) $(i = 1, 2, 3, 4)$

$$U_{(i)}^A = e_\lambda^A U_{(i)}^\lambda, \qquad U_{(i)}^A U_{(j)}^A = \delta_{(ij)},$$

which are called the gauge parallel basis in internal space. The projection of the torsion tensor (4) on the basis will be[17]

$$T_{\mu\nu(i)} = T_{\mu\nu}^A U_{(i)}^A = \partial_\mu A_{\nu(i)} - \partial_\nu A_{\mu(i)}, \qquad i = 1, 2, 3, 4, \tag{7}$$

where

$$A_{\mu(i)} = e_\mu^A U_{(i)}^A$$

is the $U(1)$ gauge potential. This shows that $T_{\mu\nu(i)}$ can be expressed in terms of $A_{\mu(i)}$ just as the curvature on $U(1)$ principal bundle with base manifold U_4 (i.e. the $U(1)$ gauge field strength), which is invariant for the $U(1)$-like gauge transformation

$$A'_{\mu(i)}(x) = A_{\mu(i)}(x) + \partial_\mu \Lambda_{(i)}(x) \tag{8}$$

where $\Lambda_{(i)}(x)$ is an arbitrary function.

Now, let us investigate the total projection of the torsion on a surface, which will be shown that, in topology, it is associated with the Chern class of the Riemann-Cartan manifold, i.e.

$$l_{(i)} = \int_{\Sigma(\lambda,s)} \frac{1}{2} T_{\mu\nu(i)} dx^\mu \wedge dx^\nu = constant, \tag{9}$$

where $\Sigma(\lambda, s)$ is the 2-dimensional surface determined by two parameters λ and s in the 4-dimensional manifold U_4. The intrinsic coordinates of $\Sigma(\lambda, s)$ are $u = (u^1, u^2)$, that is, for $x \in \Sigma(\lambda, s)$,

$$x^\mu = x^\mu(u^1, u^2), \qquad \mu = 1, 2, 3, 4.$$

It must be pointed out here that the integral $l_{(i)}$ in (9) is quite different from that of Sabbata in (1). Since the index (i) is neither the coordinate nor the group index, $l_{(i)}$ is invariant under general coordinate transformations as well as local Lorentz transformation and, thus, is independent of the coordinate system, but l^λ is not. Furthermore, in $l_{(i)}$ there is another $U(1)$-like gauge invariance for (8). In fact, $l_{(i)}$ is a new topological invariant and relates to the Winding Numbers, which will be seen later. So, we suggest to use the projection $T_{\mu\nu(i)}$ and the new topological invariant $l_{(i)}$ to measure the size of defects of the Riemann-Cartan manifold. It is obvious that $l_{(i)}$ has the dimension of length, which leads us to call $l_{(i)}$ the total projection of dislocation along the i-th gauge parallel base $U_{(i)}^A$ on the surface $\Sigma(\lambda, s)$. The invariant time t is defined in analogy with (3) as

$$t := \frac{1}{c} \int_{\Sigma(\lambda,s)} \frac{1}{2} T_{\mu\nu(4)} d\dot{x}^\mu \wedge dx^\nu. \tag{10}$$

Since on $\Sigma(\lambda, s)$ a $U(1)$ gauge transformation is equivalent to a two-dimensional rotation, $A_{\mu(i)}(x)$ corresponds to the $SO(2)$ gauge connection $\omega_{\mu(i)}^{ab}(x)$. This relationship can be

expressed as follows:

$$\omega^{ab}_{\mu(i)}(x) = -\frac{2\pi}{L_p} A_{\mu(i)} \epsilon^{ab}, \qquad a, b = 1, 2, \tag{11}$$

where $L_p = \sqrt{\hbar G/c^3}$ is the Planck length that is introduced to make both sides of Eq.(11) have the same dimension. The corresponding 2-dimensional gauge parallel vectors $n^a_{(i)}$ on $\Sigma(\lambda, s)$ with respect to $\omega^{ab}_{\mu(i)}$ can be derived from $U^A_{(i)}$ by[22]

$$D_\mu n^a_{(i)} = \partial_\mu n^a_{(i)} - \omega^{ab}_{\mu(i)} n^b_{(i)} = 0, \qquad a, b = 1, 2, \tag{12}$$

$$\omega^{ab}_{\mu(i)} = -\frac{2\pi}{L_p} A_{\mu(i)} \epsilon^{ab} = -\frac{2\pi}{L_p} \epsilon^{ab} e^A_\mu U^A_{(i)}$$

satisfying

$$n^a_{(i)} \partial_\mu n^a_{(i)} = 0, \qquad n^a_{(i)} n^a_{(i)} = C$$

for fixed (i), where C is an arbitrary constant and

$$n^a_{(i)} = n^a_{(i)}(x^\mu(u^1, u^2), \lambda, s) = n^a_{(i)}(u^1, u^2, \lambda, s),$$

in which λ and s are the parameters to determine the surface while u^1 and u^2 the intrinsic coordinates of $\Sigma(\lambda, s)$. Let us consider $C = 1$, that is $n^a_{(i)}$ $(i = 1, 2, 3, 4)$ are unit vectors which can, in general, be written in the form[22]

$$n^a_{(i)} = \frac{\phi^a_{(i)}}{\|\phi_{(i)}\|}, \qquad \|\phi_{(i)}\| = \sqrt{\phi^a_{(i)} \phi^a_{(i)}}, \tag{13}$$

where $\phi^a_{(i)}$ $(a = 1, 2)$ is a vector field on $\Sigma(\lambda, s)$, i.e.

$$\phi^a_{(i)} = \phi^a_{(i)}(x^\mu(u^1, u^2), \lambda, s) = \phi^a_{(i)}(u^1, u^2, \lambda, s). \tag{14}$$

In the opinion of the decomposition of $U(1)$ and $SO(2)$ gauge potential[23], from (12) we get

$$\omega^{ab}_{\mu(i)} = n^b_{(i)} \partial_\mu n^a_{(i)} - n^a_{(i)} \partial_\mu n^b_{(i)}, \qquad A_{\mu(i)} = \frac{L_p}{2\pi} \epsilon_{ab} n^a_{(i)} \partial_\mu n^b_{(i)}$$

for fixed (i). (9) can be changed into

$$l_{(i)} = \frac{L_p}{2\pi} \int_{\Sigma(\lambda,s)} \epsilon_{ab} \partial_\mu n^a_{(i)} \partial_\nu n^b_{(i)} dx^\mu \wedge dx^\nu$$

or, in terms of the intrinsic coordinates $u = (u^1, u^2)$ of $\Sigma(\lambda, s)$,

$$l_{(i)} = \frac{L_p}{2\pi} \int_{\Sigma(\lambda,s)} \epsilon_{ab} \partial_A n^a_{(i)} \partial_B n^b_{(i)} du^A \wedge du^B = \int_{\Sigma(\lambda,s)} \rho_{(i)} du^1 du^2, \tag{15}$$

where

$$\rho_{(i)} = \frac{L_p}{2\pi}\epsilon^{AB}\epsilon_{ab}\partial_A n^a_{(i)}\partial_B n^b_{(i)} \quad A, B = 1, 2 \tag{16}$$

is called the dislocation density projection. In the following, we will interpret the total projection $l_{(i)}$ as the dislocation flux through the surface $\Sigma(\lambda, s)$ and quantize it naturally by means of the so-called ϕ-mapping method.

3. The topological quantization of the dislocation flux

At first, by the train of thought of Ref. [24], we can extend the dislocation density projection $\rho_{(i)}$ to a topological current of dislocations

$$j^A_{(i)} = \frac{L_p}{2\pi}\epsilon^{ABC}\epsilon_{ab}\partial_B n^a_{(i)}\partial_C n^b_{(i)}, \qquad A, B, C = 0, 1, 2, \tag{17}$$

in which $\epsilon^{012} = +1$, $\partial_0 = \partial/\partial u^0$ with $u^0 = \lambda$ or s. (For convenience and without loss of generality, we choose $u^0 = \lambda$.) They are instant conclusions from (17) that the component $j^0_{(i)}$ is just the dislocation density projection $\rho_{(i)}$ in (16), and $j^A_{(i)}$ is identically conserved, i.e.

$$\partial_A j^A_{(i)} = 0.$$

The relevant conserved quantity to $j^A_{(i)}$ is in (15), which means that the total projection $l_{(i)}$ is independent of the surface $\Sigma(\lambda, s)$ on the condition that its boundary $\partial\Sigma$ surrounds the system of dislocations. We note that this property of $l_{(i)}$ is quite similar to that of magnetic flux in cosmical electrodynamics[25]. In fact, $j^A_{(i)}$ just can take the same form as the current density in electrodynamics or hydrodynamics, which will be shown later, and $l_{(i)}$ can be considered as the corresponding dislocation flux along the i-th direction in internal space. Using (13) and

$$\partial_A n^a_{(i)} = \frac{\partial_A \phi^a_{(i)}}{\|\phi_{(i)}\|} - \frac{\phi^a_{(i)}\partial_A\|\phi_{(i)}\|}{\|\phi_{(i)}\|^2}, \qquad \frac{\partial}{\partial\phi^a_{(i)}}\ln\|\phi_{(i)}\| = \frac{\phi^a_{(i)}}{\|\phi_{(i)}\|^2},$$

$j^A_{(i)}$ can be expressed by

$$j^A_{(i)} = \frac{L_p}{2\pi}\epsilon^{ABC}\epsilon_{ab}\frac{\partial}{\partial\phi^c_{(i)}}\frac{\partial}{\partial\phi^a_{(i)}}\ln\|\phi_{(i)}\| \cdot \partial_B\phi^c_{(i)}\partial_C\phi^b_{(i)}.$$

By defining the Jacobian determinants $D^A_{(i)}(\frac{\phi_{(i)}}{u})$ as

$$\epsilon^{ab} D^A_{(i)}(\frac{\phi_{(i)}}{u}) = \epsilon^{ABC} \partial_B \phi^a_{(i)} \partial_C \phi^b_{(i)}, \tag{18}$$

in which

$$D^0_{(i)}(\frac{\phi_{(i)}}{u}) = D_{(i)}(\frac{\phi_{(i)}}{u})$$

is the usual Jacobian determinant of $\phi_{(i)}$ with respect to u, and making use of Laplacian relation in $\phi_{(i)}$-space

$$\partial_a \partial_a \ln \|\phi_{(i)}\| = 2\pi \delta^2(\vec{\phi}_{(i)}), \qquad \partial_a = \frac{\partial}{\partial \phi^a_{(i)}},$$

we obtain the δ-like topological current of dislocations

$$j^A_{(i)} = L_p D^A_{(i)}(\frac{\phi_{(i)}}{u}) \delta^2(\vec{\phi}_{(i)}). \tag{19}$$

The dislocation density projection $\rho_{(i)}$ and the flux $l_{(i)}$ are given by

$$\rho_{(i)} = j^0_{(i)} = L_p D_{(i)}(\frac{\phi_{(i)}}{u}) \delta^2(\vec{\phi}_{(i)}),$$

$$l_{(i)} = L_p \int_{\Sigma(\lambda,s)} D_{(i)}(\frac{\phi_{(i)}}{u}) \delta^2(\vec{\phi}_{(i)}) du^1 du^2. \tag{20}$$

It is obvious that $j^A_{(i)}$, $\rho_{(i)}$ and $l_{(i)}$ are non-zero only when $\vec{\phi}_{(i)} = 0$.

Suppose that the vector fields $\phi^a_{(i)}$ in (14) for fixed (i) possess N zeroes, according to the deduction of Ref. [26] and the implicit function theorem[27], when the Jacobian determinant

$$D_{(i)}(\frac{\phi_{(i)}}{u}) \neq 0,$$

the solutions of $\vec{\phi}_{(i)}(u^1, u^2, \lambda, s) = 0$ can be expressed in terms of $u = (u^1, u^2)$ as

$$u^1 = a^1_{l(i)}(\lambda, s), \quad u^2 = a^2_{l(i)}(\lambda, s), \quad l = 1, \cdots, N,$$

and

$$\phi^a_{(i)}(a^1_{l(i)}(\lambda, s), a^2_{l(i)}(\lambda, s), \lambda, s) \equiv 0, \quad a = 1, 2, \tag{21}$$

where the subscript $l(= 1, \cdots, N)$ represents the l-th zero of the (i)-th vector field.

In the following, we will discuss the dynamic form of the dislocation current $j^A_{(i)}$ and

study the topological quantization of the dislocation flux $l_{(i)}$ through the Winding Numbers[28] $W_{l(i)}$ of $\vec{\phi}_{(i)}$ at $a_{l(i)}$

$$W_{l(i)} = \frac{1}{2\pi}\int_{\partial\Sigma_{l(i)}} d\arctan[\frac{\phi^2_{(i)}}{\phi^1_{(i)}}],$$

where $\partial\Sigma_{l(i)}$ is the boundary of a neighborhood $\Sigma_{l(i)}$ of $a_{l(i)}$ on the surface $\Sigma(\lambda, s)$ with $a_{l(i)} \notin \partial\Sigma_{l(i)}$, $\Sigma_{l(i)} \cap \Sigma_{m(i)} = \emptyset$. It is well-known that the Winding Numbers[29] $W_{l(i)}$ are corresponding to the first homotopy group $\pi[S^1] = Z$ (the set of integers). By making use of (13), it can be precisely proved that

$$W_{l(i)} = \frac{1}{2\pi}\int_{\partial\Sigma_{l(i)}} n^*_{(i)}(\epsilon_{ab} n^a_{(i)} dn^b_{(i)}), \tag{22}$$

where $n^*_{(i)}$ is the pull back of map $n_{(i)}$. This is another definition of $W_{l(i)}$ by the Gauss map $n_{(i)} : \partial\Sigma_{l(i)} \longrightarrow S^1$. In topology it means that, when the point $u = (u^1, u^2)$ covers $\partial\Sigma_{l(i)}$ once, the unit vector $n^a_{(i)}$ will cover S^1 $W_{l(i)}$ times, which is a topological invariant and is also called the degree of Gauss map[30,31]. Using the Stokes' theorem in the exterior differential form and (22), one can deduce that

$$W_{l(i)} = \frac{1}{2\pi}\int_{\partial\Sigma_{l(i)}} \epsilon_{ab} n^a_{(i)} \partial_B n^b_{(i)} du^B = \frac{1}{2\pi}\int_{\Sigma_{l(i)}} \epsilon_{ab} \partial_A n^a_{(i)} \partial_B n^b_{(i)} du^A \wedge du^B.$$

It is noticed that this formula differs from that of (15) only in the domain of integration and the constant L_p. Then, by duplicating the above process, we have

$$W_{l(i)} = \int_{\Sigma_{l(i)}} D_{(i)}(\frac{\phi_{(i)}}{u})\delta^2(\vec{\phi}_{(i)}) du^1 du^2. \tag{23}$$

Since

$$\delta^2(\vec{\phi}_{(i)}) = \begin{cases} +\infty, & for \quad \vec{\phi}_{(i)} = 0 \\ 0, & for \quad \vec{\phi}_{(i)} \neq 0 \end{cases} = \begin{cases} +\infty, & for \quad u = a_{l(i)}, \\ 0, & for \quad u \neq a_{l(i)}, \end{cases}$$

it can be supposed that (by analogy with the procedure of deducing $\delta(f(x))$)

$$\delta^2(\vec{\phi}_{(i)}) = \sum_{l=1}^{N} c_{l(i)} \delta(u^1 - a^1_{l(i)}(\lambda, s))\delta(u^2 - a^2_{l(i)}(\lambda, s)), \tag{24}$$

where the coefficients $c_{l(i)}$ must be positive, i.e. $c_{l(i)} = |c_{l(i)}|$. Substituting (24) into (23) and calculating the integral, we get

$$c_{l(i)} = \frac{|W_{l(i)}|}{|D_{(i)}(\frac{\phi_{(i)}}{u})_{a_{l(i)}}|} \tag{25}$$

easily. Let $|W_{l(i)}| = \beta_{l(i)}$, then from (24) and (25) we have

$$\delta^2(\vec{\phi}_{(i)}) = \sum_{l=1}^{N} \frac{\beta_{l(i)}}{|D_{(i)}(\frac{\phi_{(i)}}{u})_{a_{l(i)}}|} \delta(u^1 - a^1_{l(i)}(\lambda, s))\delta(u^2 - a^2_{l(i)}(\lambda, s)), \tag{26}$$

where the positive integer $\beta_{l(i)}$ is called the Hopf index[32] of map $u \longrightarrow \phi_{(i)}$. Making use of (26), the dislocation current $j^A_{(i)}$ in (19) can be expressed as

$$j^A_{(i)} = L_p \sum_{l=1}^{N} \beta_{l(i)}\eta_{l(i)}\delta(u^1 - a^1_{l(i)}(\lambda, s))\delta(u^2 - a^2_{l(i)}(\lambda, s)) \frac{D^A_{(i)}(\frac{\phi_{(i)}}{u})}{D_{(i)}(\frac{\phi_{(i)}}{u})}|_{a_{l(i)}},$$

in which $A = 0, 1, 2$ and

$$\eta_{l(i)} = sign D_{(i)}(\frac{\phi_{(i)}}{u})|_{a_{l(i)}} = \pm 1$$

is called the Brouwer degree[32] of map $u \longrightarrow \phi_{(i)}$. On the other hand, from the equations (21) one can prove[33] that the generalized velocity of zero of $\phi^a_{(i)}$ is given by

$$V^A_{(i)} = \frac{du^A}{du^0} = \frac{D^A_{(i)}(\frac{\phi_{(i)}}{u})}{D_{(i)}(\frac{\phi_{(i)}}{u})}, \qquad V^0_{(i)} = 1,$$

where $u^0 = \lambda$ and $D^A_{(i)}(\frac{\phi_{(i)}}{u})$ is defined in (18). Then

$$j^A_{(i)} = L_p \sum_{l=1}^{N} \beta_{l(i)}\eta_{l(i)}\delta(u^1 - a^1_{l(i)}(\lambda, s))\delta(u^2 - a^2_{l(i)}(\lambda, s))V^A_{(i)},$$

and

$$\rho_{(i)} = j^0_{(i)} = L_p \sum_{l=1}^{N} \beta_{l(i)}\eta_{l(i)}\delta(u^1 - a^1_{l(i)}(\lambda, s))\delta(u^2 - a^2_{l(i)}(\lambda, s)), \tag{27}$$

which give

$$j^A_{(i)} = \rho_{(i)} V^A_{(i)}, \qquad A = 0, 1, 2.$$

That is the dislocation current $j^A_{(i)}$ of the Riemann-Cartan manifold exactly takes the same form as the current density in classical electrodynamics or hydrodynamics. From (20), (10) and (27), we get the dislocation flux $l_{(i)}$ and the invariant time t in the topological quantum level as

$$l_{(i)} = \sum_{l=1}^{N} n_{l(i)} L_p, \qquad t = \sum_{l=1}^{N} n_{l(4)} T_p, \qquad i = 1, 2, 3, 4, \tag{28}$$

where $n_{l(i)} = \beta_{l(i)}\eta_{l(i)}$ for fixed l and (i), $T_p = \frac{L_p}{c} = \sqrt{\frac{\hbar G}{c^5}}$. So, with torsion, we have minimum units of length and, especially, time $\neq 0$! This in fact would give us the smallest definable

unit of time as $T_p \approx 10^{-43}s$. In the limit of $\hbar \Longrightarrow 0$ (classical geometry of general relativity) or $c \Longrightarrow \infty$ (Newtonian case), we would recover the unphysical $L_p, T_p \Longrightarrow 0$ of classical cosmology or physics.

At the end of this section, it must be pointed out that the quantizations of length and time are natural and rigorous results in our discussions. But what was dealt with in Ref. [15] can only be looked upon as an assumption and the author can not tell us how to determine the quantum numbers. On the contrary, from (28), we see that the quantum numbers are given by the Hopf indices and the Brouwer degrees.

4. Conclusion

In this paper, we obtain a new topological invariant in Riemann-Cartan manifold U_4 in terms of the torsion tensor. It is invariant under general coordinate transformations as well as local Lorentz transformation and, thus, is independent of the coordinate system. In fact, it only depends on the Winding Numbers of a smooth vector field at its zeroes, which are also topological invariants. Meanwhile, there is another $U(1)$-like gauge invariance in it. In order to describe the space-time defects in the early universe or the Planck era in invariant form, we use the new topological invariant to measure the size of defects and interpret it as the dislocation flux in internal space. Using the so-called ϕ-mapping method and the decomposition of $U(1)$ and $SO(2)$ gauge potential, the dislocation flux is quantized naturally and rigorously, which is the quantizations of length and time in Riemann-Cartan manifold. The quantum numbers are determined by the Hopf indices and the Brouwer degrees, i.e. the Winding Numbers. The Planck length L_p and $T_p = \sqrt{\frac{\hbar G}{c^5}} \approx 10^{-43}s$ play the roles of the elementary length and the unit time respectively. As mentioned above, this result is considered to be reasonable because of the fact that, being torsion linked to spin and being the spin quantized, the Planck length L_p enters through the minimal unit of spin, or action $\hbar$. Furthermore, by extending to an identically conserved current, the dynamic form of the dislocations is also obtained and it takes the same form as the current density in classical eletrodynamics or hydrodynamics, which may be important for the production and interac-

tion of the space-time defects in the early universe and will be detailed in other papers.

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