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Higgs-free Massive Nonabelian Gauge Theories[1]

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Abstract. We analyze nonabelian massive Higgs-free theories in the causal Epstein-Glaser approach. Recently, there has been renewed interest in these models. In particular we consider the well-known Curci-Ferrari model and the nonabelian Stückelberg models. We explicitly show the reason why the considered models fail to be unitary. In our approach only the asymptotic (linear) BRS-symmetry has to be considered.

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The discription of massive gauge bosons favoured today is the Higgs-Kibble mechanism: It is the only mechanism of mass generation known so far which leads to a normalizable and unitary theory of massive nonabelian gauge bosons. One introduces new spin-zero-particles (Higgs-fields) with unknown mass and couplings into the theory for which there are no experimental evidence so far. But, for instance, the measured ratio of the W- and Z-boson masses for example is at least a phenomenological indication that these masses are generated by spontaneously symmetry breaking. In recent years the classical Higgs field has become available for a geometrical interpretation as generalized connection in the context of noncommutative geometry [1], which makes the models more attractive.
However, the nondiscovery of Higgs bosons and the well-known shortcomings in this approach (for example the hierarchy problem) lead to continued attempts to construct alternative massive nonabelian gauge theories (see [2] for a review). Two prominent approaches are the

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Curci-Ferrari model [3] and the nonabelian generalization of the massive abelian Stückelberg gauge theory [4,5]. The findings suggest that the properties of perturbative normalizability and of physical unitarity are mutually exclusive. Moreover, gauge invariance and gauge independence are described as necessary but not sufficient conditions for physical unitarity, i.e. for decoupling of unphysical degrees of freedom in the theory [2]. Nevertheless, there has been renewed interest in these models. In fact, Periwal [6] has proposed a nonperturbative condition on a 1PI distributions for physical unitarity which fixes the gauge parameter ξ in a nonlinearly gauged Curci-Ferrari model. But the nonunitarity of this Curci-Ferrari model, for arbitrary values of the parameters of the theory, was quite recently reassured by improving Ojima's proof [7] of this statement [8].

Because of the frequent questioning of the non-unitarity results we want to give a brief re-analysis of these models using the Epstein-Glaser methods in this letter.

The causal Epstein-Glaser formalism [9,see also 10] represents a general framework for perturbative quantum field theory. The method allows for a clear and simplified analysis of these models which accurately spells out the reasons for the absence of unitarity in these models. The analysis is simplified by the fact that only the asymptotic (linear) part of the BRS-transformations is relevant in this approach.
In the causal approach the technical details concerning the well-known UV- and IR-problem in quantum field theory are separated and reduced to mathematically well-defined problems, namely the causal splitting and the adiabatic switching of operator-valued distributions.
The S-matrix is directly constructed in the well-defined Fock space of free asymptotic fields in the form of a formal power series

$$S(g) = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 ... d^4x_n \, T_n(x_1, ..., x_n) g(x_1) ... g(x_n), \tag{1}$$

where $g(x)$ is a tempered test function which switches the interaction. Only well-defined free field operators occur in the whole construction. The central objects are the n-point distributions T_n. They may be viewed as mathematically well-defined time-ordered products. The defining equations of the theory in the causal formalism are the fundamental (anti-)commutation relations of the free field operators, their dynamical equations and the specific coupling of the theory $T_{n=1}$. The n-point distributions T_n in (1) are then constructed inductively from the given first order $T_{n=1}$. Epstein and Glaser present an explicit inductive construction of the general perturbation series in the sense of (1) which is compatible with causality and Poincare invariance.
The causal formalism allows for a comprehensive discussion of massless Yang-Mills theories in four (3+1) dimensional space time (see [11,12,13] for details). It was shown that the whole analysis of nonabelian gauge symmetry can be done in the well-defined Fock space of free asymptotic fields. The LSZ-formalism is not necessary then. Nonabelian gauge invariance is introduced by a linear operator condition in every order of perturbation theory separately:

$$[Q, T_n(x_1, ..., x_n)] = d_Q T_n(x_1, ..., x_n) = i \sum_{l=1}^{n} \partial_\mu^{x_l} T^\mu{}_{n/l}(x_1, ..., x_n) \equiv div. \tag{2}$$

where the charge Q is the generator of the linear (abelian!) BRS transformations of the free asymptotic field operators which defines an antiderivation d_Q in the algebra, generated by the fundamental field operators. The $T^{\nu}_{n/l}(x_1,\ldots,x_n)$ are n-point distributions of an extended theory which also can be inductively constructed in the causal formalism. They serve for an explicit representation of the commutator $[Q,T_n(x_1,...,x_n)]$ as a divergence in the sense of vector analysis.
Physical unitarity, i.e. decoupling of the unphysical degrees of freedom, is shown as a direct consequence of the linear operator gauge invariance condition (2) and of the nilpotency of the charge Q. Perturbatively, physical unitarity means

$$\tilde{T}_n^{P_\perp} = P_\perp T_n^+ P_\perp + div \quad \forall n \tag{3}$$

where div denotes distributions of divergence form as in the condition of gauge invariance (2), $P_\perp$ is the projection operator on the physical subspace, $+$ denotes the hermitean conjugation with regard to the Hilbert scalar product of the Fock space. The $\tilde{T}_n^{P_\perp}$ are the n-point distributions of the inverse $(P_\perp S(g) P_\perp)^{-1}$-matrix restricted to the physical subspace:

$$(P_\perp S(g) P_\perp)^{-1} = \sum_n \frac{1}{n!}\int d^4x_1\ldots\int d^4x_n \quad \tilde{T}_n^{P_\perp}(x_1,\ldots,x_n)g(x_1)\ldots g(x_n).$$

The n-point distributions $\tilde{T}_n^{P_\perp}$ are computed by formal inversion of (1). They are equal to the following sum over subsets of
$X=\{x_1,\ldots,x_n\}$

$$\tilde{T}_n^{P_\perp}(X) = \sum_{r=1}^{n}(-)^r \sum_{Pr} P_\perp T_{n_1}(X_1)P_\perp \ldots P_\perp T_{n_r}(X_r)P_\perp.$$

The perturbative statement (3) implies the following statement about a formal power series:

$$(S_\perp)^{-1}(g) = S_\perp^+(g) + div(g) \qquad S_\perp = P_\perp S P_\perp \tag{4}$$

Finally, normalizability of the theory means in the Epstein-Glaser approach that the number of the finite constants to be fixed by physical conditions stays the same in all orders of perturbation theory. This property is based on scaling properties of the theory only. The following conditions are shown to be sufficient for this property :
(a) The specific coupling $T_{n=1}$ of the theory has maximal mass dimension four and
(b) the singular order of the fundamental (anti-) commutator distributions of the free asymptotic fields are smaller than zero.
Note that in this context normalizability does not necessarily mean that the theory can be normalized in a gauge invariant way, a more far reaching quality generally referred as renormalizability.

Considering genuine massive nonabelian gauge theories, normalizability is established *per definitionem* in the theory by suitable choice of the defining equations. For example , we choose the following commutator relations of the asymptotic field operators and their corresponding equation of motion. The massive gauge potentials in a general linear ξ–gauge, transforming according to the adjoint representation of $SU(N)$, satisfy

$$(\Box + m^2)A_\mu^a(x) - (\frac{\xi - 1}{\xi})\partial_\mu(\partial^\nu A_\nu) = 0. \tag{5}$$

$$\left[A_\mu^a(x), A_\nu^b(y)\right]_- = i\delta_{ab}(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2})D_m(x-y) - i\delta_{ab}\frac{\partial_\mu\partial_\nu}{m^2}D_M(x-y) \tag{6}$$

D_m denotes the Pauli-Jordan commutation distribution with mass m.
The masses m and M are related:

$$M^2 = m^2\xi \tag{7}$$

The right side of (6) represents the general ansatz compatible with normalizability, Poincare invariance, field equation (5) and causality.
The ghost fields may fulfill (in a general ξ-gauge):

$$\{u_a(x), \tilde{u}_b(y)\}_+ = -i\delta_{ab}D_M(x-y) \tag{8}$$

$$(\Box + M^2)u_a(x), \quad (\Box + M^2)\tilde{u}_a(x) = 0. \tag{9}$$

Note that this relation between the masses of the gauge bosons and the ghosts is already suggested by the most general gauge invariant quadratic terms in the specific coupling in the massless theory (see Lemma 3.1 and 4.1 in [13]) This relation is uniquely fixed by gauge invariance.
It is well-known that the gauge boson field can be splitted into the Proca component (representing the three physical transverse components) and the unphysical part:

$$A_\mu^a = A_\mu^{a,phys} - \frac{1}{m^2\xi}\partial_\mu(\partial^\nu A_\nu^a) \tag{10}$$

$$\left[A_\mu^{a,phys}(x), A_\nu^{b,phys}(y)\right]_- = i\delta_{ab}(g_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2})D_m(x-y) \tag{11}$$

Having defined the Fock space of free asymptotic fields by the first two defining equations (5) and (6) we can further pursue the standard procedure in the causal formalism [13]: We choose a reasonable gauge invariance condition and then construct the most general gauge invariant specific coupling, the third defining equation of the theory in the causal formalism.

As has been firstly noticed by Curci and Ferrari [2, see also 7], a direct taking over of the formula of the generator Q_{CF} from the massless case in a general ξ-gauge [13]

$$Q_{CF} = \int \frac{\partial_\nu A^\nu}{\xi} \overleftrightarrow{\partial_0}\, u d^3\bar{x} \tag{12}$$

leads to a missing nilpotency of Q_{CF} in the massive case. One easily checks that Q_{CF}^2 is proportional to m^2, because of $\left[\partial^\mu A_\mu^a(x), \partial^\nu A_\nu^b(y)\right]_- \neq 0$.
But our analysis of the massless case shows that the nilpotency of Q_{CF} is the crucial input to determine unitarity in the physical subspace (3) as a direct consequence of the operator

gauge invariance condition (2). So this gauge invariance condition does not seem to be very useful. Nevertheless, at the end of this letter we will come back to these specific couplings, which are gauge invariant in respect to the charge Q_{CF} in (12).

Stückelberg's idea, generalized to the nonabelian case [4,5], is to introduce an additional scalar field $\zeta^a(x)$ transforming also according to the adjoint representation

$$[\zeta_a(x), \zeta_b(y)] = -i\delta_{ab} D_M(x-y) \qquad (\Box + M^2)\zeta_a(x) = 0 \tag{13}$$

Note that we have chosen the mass M of the unphysical component of the gauge bosons and the ghosts. Now we introduce also generalized BRS transformations of the free asymptotic fields which involve this new field $\zeta(x)$ [5].
The corresponding generator Q^s of these Stückelberg gauge transformations in the Fock space of asymptotic field is (see formula 3.31 in [5]; note that we can leave out the Z-factors because we directly work in the well-defined Fock space of free asymptotic fields and does not use the LSZ-formalism, moreover we have generalized the formula 3.31 in [5] to a general linear ξ-gauge):

$$Q_s = \int \eta^a(x) \overleftrightarrow{\partial_0}\, u_a(x) d^3\bar{x}$$
$$\text{with} \quad \eta^a(x) := \frac{\partial_\mu A^\mu_a(x)}{\xi} + m\zeta^a(x) \tag{14}$$

As one easily verify, we have $Q_s^2 = 0$ because of $[\eta(x), \eta(y)]_- = 0$ and arrive at a well-defined anti-derivation d_{Q^s} in the graded algebra of fields: The gradation is introduced by the ghost charge

$$Q_c := i \int d^3x : (\tilde{u} \overleftrightarrow{\partial_0}\, u) : .$$

The anti-derivation d_{Q_s} in the graded algebra is then given by

$$d_{Q^s}\hat{A} := Q_s\hat{A} - (e^{i\pi Q_c}\hat{A}e^{-i\pi Q_c})Q_s$$

with

$$d_{Q^s}A^\mu_a = i\partial^\mu u_a, \quad d_{Q^s}u_a = 0, \quad d_{Q^s}\tilde{u}_a = -i\eta_a,$$
$$d_{Q^s}\zeta_a = imu_a, \quad d_{Q^s}\partial^\mu A^a_\mu = -iM^2 u_a, \quad d_{Q^s}\eta_a = 0. \tag{15}$$

According to the standard procedure in the causal formalism [13] we construct the most general gauge invariant specific coupling with respect to this antiderivation:
Lemma: The most general gauge invariant coupling T^g_1, **(A)** $d_{Q^s}T^g_1 = div$, which is also invariant under the special Lorentz group $L^\uparrow_+$ **(B)** and under the structure group $G = SU(N)$ **(C)**, which has ghost number zero - $G(T^g_1) = 0$ - **(D)** and has maximal mass dimension 4 **(E)** and is invariant under the discrete symmetry transformations **(F)** can be written as

$$T^g_1 = -igf_{a'b'c'} : A^{a'}_\kappa A^{b'}_\lambda \partial_\kappa A^{c'}_\lambda : -\frac{1}{2}igf_{a'b'c'} : A^{a'}_\kappa u_{b'}\partial_\kappa \tilde{u}_{c'} : + \tag{16.a.b.}$$

$$+\frac{1}{2}igf_{a'b'c'} : A_\kappa^{a'}\partial^\kappa u_{b'}\tilde{u}_{c'} : +\frac{1}{2}igf_{a'b'c'} : A_\kappa^{a'}\zeta_{b'}\partial_\kappa\zeta_{c'} : + \qquad (16.c.d.)$$

$$+\alpha \quad \partial_\kappa \left[igf_{a'b'c'} : A_{a'}^\kappa u_{b'}\tilde{u}_{c'} :\right] + \beta \quad d_{Q^s}\left[gf_{a'b'c'} : u_{a'}\tilde{u}_{b'}\tilde{u}_{c'} :\right] \qquad (16.e.f.)$$

The explicit representation of $d_{Q^s}T_1^g$ as a divergence is given by

$$d_{Q^s}T_1^g = i\partial_\mu T_{1,g}^\mu + i\gamma \quad \partial_\mu B_{1,g}^\mu \qquad (17)$$

$$T_{1,g}^\mu = -igf_{abc} : u_a A_\nu^b(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) : -i\frac{1}{2}gf_{abc} : u_a u_b \partial^\mu \tilde{u}_c : + \qquad (18.a.b.c.)$$

$$+i\frac{1}{2}gf_{abc} : u_a\partial^\mu u_b\tilde{u}_c : -i\frac{1}{2}gf_{abc} : u_a A_b^\mu \partial_\nu A_c^\nu : + \qquad (18.d.e.)$$

$$+\frac{1}{2}igf_{abc} : u_a\zeta_b\partial^\mu\zeta_c : + \qquad (18.f.)$$

$$+i\alpha\left[gf_{abc} : \partial_\mu u_a u_b \tilde{u}_c : +\frac{1}{\xi}gf_{abc} : A_\mu^a u_b \partial_\kappa A_c^\kappa : +gf_{abc} : A_\mu^a u_b m\zeta_c :\right] \qquad (18.g.h.i.)$$

$$B_{1,g}^\mu = \partial_\nu\left\{gf_{abc}u_a A_b^\mu A_c^\nu :\right\} \qquad (18.j.)$$

α, β, γ are free constants.
The **proof** of this statement is straightforward and analogous to the one in the massless case (see Appendix A of [13]).
Note that all Lorentz invariant **(B)**, G-invariant **(C)** terms with ghost number zero **(D)** and with **four** normalordered operators which would be compatible with normalizability **(E)** and are invariant under the discrete symmetry transformations **(F)** are ruled out by the gauge invariance condition **(A)**.
Moreover we left out the quadratic terms compatible with the conditions **(A)**-**(F)** in formula (16) because in the causal formalism the information about such quadratic terms is already contained in the fundamental (anti-)commutation relations and the dynamical equations for the operators.
In addition, T_1^g in (16) is also anti-gauge invariant in respect to the anti-charge

$$Q_s = \int \eta^a(x)\overleftrightarrow{\partial_0}\,\tilde{u}_a(x)d^3\bar{x} \quad \text{with} \quad \bar{Q}_s^2 = 0) :$$

$$[\bar{Q}_s, T_1^g] = div \qquad (19)$$

We have thus defined a manifestly normalizable theory which is gauge invariant to first order of perturbation theory and respects certain further symmetry conditions. We now have to examine if one can prove a corresponding condition of gauge invariance to all orders of perturbation theory inductively

$$d_{Q^s}T_n = div \qquad (20)$$

Before studying this explicitly, we should emphasize that the unitarity of the S-matrix in the physical subspace would be a direct consequence of such a condition (20) - analogously to the massless case. The three physical components of the massive gauge boson would decouple from all other fields. The inductive proof of this statement is completely analogous to the one in the massless case (see chapter 5 of [12]): Again, the crucial point is the fact that the physical subspace Ker$N$ of the Fock space has the following representation (see formula 3.34 of [5])

$$\mathrm{Ker}N = \mathrm{Ker}Q_s/\mathrm{Range}Q_s \tag{21}$$

where N is the number operator of the unphysical particles only (excluding the three physical components of the massive gauge boson).
Knowing this fact (21), we could repeat the proof of unitarity in the physical subspace worked out in the massless case without any changes (see [12], Chapter 7), provided equation (20) holds! Another proof of this implication can be found in [14]. From the perspective of the causal formalism the operator gauge invariance condition (20) in the Stückelberg model is sufficient for the unitarity of the S-matrix in the physical subspace, that means that the perturbative condition (3) holds in every order of perturbation theory. However, we now show that the operator gauge invariance condition (20) is already violated in the tree contribution at second order of perturbation theory:
We prove that there is no normalization of $T_{n=2}|_{tree}$ which is gauge invariant, i.e. $d_{Q_s}T_{n=2}|_{tree}$ $= div$. The latter statement is equivalent to the insolvability of the corresponding anomaly equation (see [13])

$$d_{Q_s}N - 2A \stackrel{!}{=} div \tag{22}$$

where N represents free local normalization terms in $T_{n=2}|_{tree,4}$ and A represents the local anomaly terms which arise in the natural splitting in second order of perturbation theory in order to construct $T_{n=2}|_{tree,4}$ [15]:
According to Epstein-Glaser method one has to construct the causal commutator

$$D_{n=2}(x,y) = [T_1^g(x), T_1^g(y)], \tag{23}$$

in order to arrive at $T_{n=2}$. One verifies that gauge invariance of the causal commutator $D_{n=2}$ is a direct consequence of gauge invariance in first order (17):

$$d_{Q_s}D_{n=2}(x,y) = [Q_s,\, [T_1^g(x),\, T_1^g(y)]] =$$

$$= i\partial_\nu^x([T_{1,g}^\nu(x), T_1^g(y)]) + i\partial_\nu^y([T_1^g(x), T_{1/g}^\nu(y)]) \tag{24}$$

The question is whether the same (divergence form) is true for the commutator $[Q_s,\, R_2(x,y)]$ obtained by causal splitting of $[Q_s,\, D_2(x,y)]$ into a retarded and a advanced distribution. There is only one mechanism to spoil gauge invariance in the tree contribution [14]. The unique splitting solution of the Pauli-Jordan distribution

$$D(x-y) = D^{\mathrm{ret}}(x-y) - D^{\mathrm{av}}(x-y) \tag{25}$$

lead to a local term in the gauge invariance condition because instead of

$$(\Box + m^2)D_m(x-y) = 0 \tag{26}$$

we have after natural splitting

$$(\Box + m^2)D_m^{ret}(x-y) = \delta(x-y). \tag{27}$$

Consequently, the procedure is straightforward: We have to pick up all local terms A arising in the natural splitting of the causal distribution $d_{Q^s} D_{n=2}$ and compare them with the free normalization terms N in $T_{n=2}$ in order to find a solution of the anomaly equation (22). For this purpose, we focus on the local operator terms proportional to $: uA_\nu \partial_\nu \zeta\zeta :$. Because of the constraint of normalizability which implies that the maximal mass dimension of the anomaly must be 5, there are exactly two independent operator terms in this sector. Thus the most general anomaly term arising in the natural splitting can be written as

$$A|_{u\partial_\nu\zeta A^\nu\zeta} = \alpha_1 : u_a \zeta_b A_\nu^{a'} \partial^\nu \zeta_{b'} : f_{abc} f_{a'b'c} \delta +$$

$$\alpha_2 : u_a \partial^\nu \zeta_b A_\nu^{a'} \zeta_{b'} : f_{abc} f_{a'b'c} \delta. \tag{28}$$

Since the operator $\partial_\nu \zeta$ cannot be represented by a variation of any fundamental field (see 15), the term $d_Q N$ cannot contribute to the sector $: uA_\nu \partial_\nu \zeta\zeta :$. As a consequence, operator gauge invariance implies

$$A|_{u\partial_\nu\zeta A^\nu\zeta} \stackrel{!}{=} div|_{u\partial_\nu\zeta A^\nu\zeta} \tag{29}$$

instead of (22). The subscript on the right hand side of (29) of course means that one has to keep only these terms of the total derivative div which contributes to the specified sector. There is only one divergence term contributing to this sector, namely

$$A|_{u\partial_\nu\zeta A^\nu\zeta} = \partial_\nu \left[: u_a \zeta_b A^\nu_{a'} \zeta_{b'} : f_{abc} f_{a'b'c} \delta \right] |_{u\partial_\nu\zeta A^\nu\zeta} \tag{30}$$

Because of (30), equation (29) implies that $\alpha_1 = \alpha_2$. In the following, we explicitly calculate α_1 and α_2, and check this necessary condition of gauge invariance. Using formulae (16) and (18), we list all local terms in the specialized sector which arise in the natural splitting of the commutator $\partial^x_\nu([T^\nu_{1,g}(x), T^g_1(y)]$ according to the procedure described above.:

$$\begin{aligned} A = & (-i) g^2 \frac{1}{2} f_{abc} f_{a'b'c} : u_a A_\nu^b \zeta_{a'} \partial^\nu \zeta_{b'} : \delta(x-y) \\ & + (-i) g^2 \frac{1}{4} f_{abc} f_{c'a'c} : u_a \zeta_b A_\kappa^{a'} \partial^\kappa \zeta_{c'} : \delta(x-y) \\ & + (+i) g^2 \frac{1}{4} f_{abc} f_{a'b'c} : u_a \zeta_b A_\kappa^{a'} \partial^\kappa \zeta_{b'} : \delta(x-y) \end{aligned} \tag{31}$$

Using the Jacobi-identity $f_{abc} f_{a'b'c} = -f_{ab'c} f_{ba'c} - f_{aa'c} f_{b'bc}$ in the first term, one arrives at $\alpha_1 \neq \alpha_2$.

Thus, the Stückelberg gauge invariance condition (20) already breaks down in second order of perturbation theory in tree terms. Note that the constraint of normalizability is essential for this conclusion.

The corresponding breakdown of the perturbative unitarity condition (3) can also directly shown.

Therefore, perturbative normalizability and physical unitarity cannot be established simultaneously in this class of models. But the operator gauge invariance condition (2) would be sufficient for physical unitarity in perturbation theory (3).

At this point a short remark about the relation to the conventional Lagrange formalism is in order. If one starts with the Stueckelberg Lagrangean [4,5], the following question naturally arises: Are there any point transformation of the fields (which preserve the origin) so that this Lagrangean is power counting normalizable in a manifest way. It is well-known that two Lagrangean which are related by such a field transformation have the same S-matrix [16]. From the viewpoint of our analysis we immediately can answer this question with no - provided the propagators are not changed - because the asymptotic part of the BRS symmetry which is only relevant for our analysis is not changed by field transformations (which preserve the origin). So our argument is in this sense field-coordinate-independent .

We come back to the Curci-Ferrari model which is defined in the causal formalism as the most general gauge invariant specific coupling with respect to the charge Q_{CF} in (12). Because of the missing nilpotency of this charge Q_{CF} these models are not expected to be unitary to all orders in perturbation theory.

A causal analysis (until second order) of a specific coupling which is gauge invariant in respect to the charge Q_{CF} in (12) is given in [17]:

$$T_1 = \frac{i}{2} g f_{abc} : A^a_\mu A^b_\nu F^{\nu\mu}_c : -ig f_{abc} : A^a_\mu u_b \partial^\mu \tilde{u}_c : -\frac{i}{2} g f_{abc} : \partial_\mu A^\mu_a u_b \tilde{u}_c : . \tag{32}$$

In contrast to the Stückelberg model, operator gauge invariance can be preserved in second order of perturbation theory

$$[Q_{CF}, T_{n=2}] = div, \tag{33}$$

but using (10), one easily shows that the perturbative condition of physical unitarity (3) in second order

$$\frac{1}{2}(P_\perp T_2^+(x_1,x_2) + T_2(x_1,x_2)P_\perp) =$$
$$= P_\perp T_1(x_1) P_\perp T_1(x_2) P_\perp + P_\perp T_1(x_2) P_\perp T_1(x_1) P_\perp + div \tag{34}$$

breaks down for all $\xi \neq 0$. The case $\xi = 0$ needs further consideration. The generalizations of these findings are straightforward. The general specific coupling which is gauge invariant in respect to Q_{CF} in (12) can be written as

$$T_1 = \frac{i}{2} g f_{abc} : A^a_\mu A^b_\nu F^{\nu\mu}_c : -\frac{i}{2} g f_{abc} : A^a_\mu u_b \partial^\mu \tilde{u}_c : +$$

$$+\frac{i}{2} g f_{abc} : A^a_\mu \partial^\mu u_b \tilde{u}_c : + \quad \alpha \quad ig f_{abc} \partial_\mu (: A^\mu_a u_b \tilde{u}_c :), \quad \alpha \quad \text{free.} \tag{35}$$

We left out the possible two-operator terms in T_1 again. The four-operator terms compatible with conditions **(B)-(F)** again are ruled out by the gauge invariance condition, **(A)** $[Q_{CF}, T_{n=1}] = div$. But as in the massless case [13], the operator gauge invariance condition in second order, $[Q_{CF}, T_{n=2}|_{tree,4}] = div$, uniquely fixes the normalization of $T_{n=2}|_{tree,4}$ and naturally introduces a four gluon coupling and a four ghost coupling in $T_{n=2}$. In the perturbative analysis, the most general gauge invariant coupling in the general ξ-gauge , together with the local normalization terms in $T_{n=2}|_{tree,4}$, coincide with the interaction terms of the Curci-Ferrari Lagrangian - fixed in a linear ξ-gauge - which is invariant under the full BRS-transformations of the interacting fields and fulfills reasonable certain additional symmetry conditions. For example see formula (2.1) in [3a].

So in contrast to the Stückelberg models, the operator gauge invariance can be proven to all orders of perturbation theory, $[Q_{CF}, T_n] = div$ (2), along the same line as in the massless case, but this gauge invariance definition with respect to the charge Q_{CF} in (12) does not serve as a sufficient condition for physical unitarity because of its missing nilpotency. However, such models are useful because of the well-behaved $m \to 0$-limit. As it is proposed in [18], such models serve as a good infrared regularization of the massless theory. In fact, they also constitute a promising starting point in the causal approach for the investigation of the adiabatic limit $g \to 1$ in the massless theory. Such an investigation is crucial for the analysis of the physical infrared problem, which is naturally separated in the causal formalism by adiabatic switching of the n-point distributions T_n by a tempered testfunction g (see (1)).

Summing up, we have presented a short analysis of some genuine massive nonabelian gauge theories in the Epstein-Glaser approach in order to clarify the different reasons of the failure of unitarity in these models. Such an analysis in the well-defined Fock space of asymptotic fields is simplified because the asymptotic (linear) part of the BRS-symmetry has to be considered only.

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