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The Relation Between the Gravitational Stress Tensor and the Energy-Momentum Tensor

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Abstract We give a precise analysis of the usual statement "The theory of gravity is given by Einstein's equations where the lefthand side is the Einstein tensor and the righthand side of the energy-momentum tensor of matter."

1 Introduction

The statement "The theory of gravity is given by Einstein's equations where the lefthand side is the Einstein tensor and the righthand side is the energy-momentum tensor of matter" calls for the question "What is the energy-momentum tensor of matter?" This question is answered within the Lagrange formalism of classical field theory [1]. We take the position that the Lorentz group is the fundamental symmetry group of all of physics. A general Lorentz-invariant action then leads to an energy-tensor (related to translation invariance) and to an energy-momentum tensor (related to Lorentz-rotational invariance). All physical tensors are Lorentz-tensors. If the Lorentz-invariant action has additional symmetry groups, the physical relevant quantities should be covariant with respect to the additional symmetries. This can be expressed through the notion of "covariant derivatives." This point of view is used to describe electromagnetic interactions, where the additional symmetry group reflects the restmass zero, spin one nature of the electromagnetic field (EM-gauge group) and also for gravitational interactions, where the additional symmetry group reflects the restmass zero, spin two nature of the gravitational field (G-gauge group) [2]. The action for Einstein's Theory of Gravity is Lorentz-invariant and has the additional invariance under the G-gauge group. The G-gauge group is generated by smooth vector fields that vanish at infinity, i.e. by smooth coordinate transformations (general covariance group). The Lorentz group is thus not a subgroup of the G-gauge group. The meaning of general covariance is embedded in the restmass zero, spin two nature of the gravitational field.

The action for Einstein's Theory has two parts, namely the gravitational selfinteraction part and the gravitational-matter part. The Euler derivative of the latter part with respect to the gravitational field is called the gravitational stress tensor. The Euler derivative with respect to the graviational field of the first part is called the Einstein tensor. The gravitational stress tensor and the Einstein tensor are both Lorentz-tensors.

Thus, the Theory of Gravity is given by the vanishing of the sum of the Einstein tensor and the gravitational stress tensor.

On the other hand, Einstein's Theory of Gravity has a Lorentz-invariant action and thus an energy-momentum tensor.

In [2] we showed that the G-gauge group-covariant energy-momentum tensor belonging to the gravitational selfinteraction part, as well as the energy-momentum tensor belonging to the gravitational matter part, both vanish.

If we now evaluate the gravitational-matter part of Einstein's action, for the gravitational field being the Minkowski metric, we get an action involving the matter field only. This then leads to an energy-momentum tensor for the matter field. This tensor is related to the gravitational stress tensor evaluated for the Minkoswki metric, as this paper will show.

All attempts to modify Einstein's equations, calling on energy-momentum considerations, are not reasonable. Within the Lagrange formalism Einstein's Theory of Gravity is perfect; I like to be its advocate.

2 The Energy-Momentum Tensor for a Classical Lorentz-Invariant Action

Here we summarize the results in [1] and [2]. Let ϕ represent several multicomponent fields on Minkowski space. Summation over Lorentz-, spinor-, and internal indices is always implied.

The action is given by

$$A = \int dx\, L_0 + \int dx \partial_\mu B^\mu \tag{2.1}$$

where

$$L_0 = L_0(\phi, \partial_\mu \phi), \ B^\mu = B^\mu(\phi, \partial_\alpha \phi) \tag{2.2}$$

and the action is assumed to be Lorentz invariant. The variation of a field, induced by an infinitesimal coordinate variation

$$\delta x^\mu = \bar{x}^\mu - x^\mu, \tag{2.3}$$

is given by

$$(\delta\phi)(x) = \bar{\phi}(\bar{x}) - \phi(x) \tag{2.4}$$

With the following abbreviations

1)

$$H^\mu \equiv \frac{\partial L_0}{\partial \phi_\mu} \tag{2.5}$$

2)

$$G \equiv \varepsilon(\phi)L_0 \equiv \frac{\partial L_0}{\partial \phi} - \partial_\mu H^\mu \quad \text{Euler derivative} \tag{2.6}$$

3)

$$E^\mu{}_\sigma \equiv H^\mu \phi_\sigma - \delta^\mu{}_\sigma L_0 \quad \text{Energy tensor} \tag{2.7}$$

the variational principle becomes

$$\delta A = \int dx[G\delta\phi - G\phi_\sigma \delta x^\sigma] + \int dx \partial_\mu \left[-E^\mu{}_\sigma \delta x^\sigma + H^\mu \delta\phi + B^\mu(\partial_\alpha \delta x^\alpha) - B^\alpha(\partial_\alpha \delta x^\mu) + \delta B^\mu\right] \tag{2.8}$$

For coordinate variations, resulting in

$$\delta\phi = -S^\beta{}_\lambda(\phi)\partial_\beta \delta x^\lambda \tag{2.9}$$

and the abbreviations

4)

$$P^{\mu\beta}{}_\lambda \equiv \delta^\mu{}_\lambda B^\beta - \delta^\beta{}_\lambda B^\mu + \frac{\partial B^\mu}{\partial \phi} S^\beta{}_\lambda + \frac{\partial B^\mu}{\partial \phi_\beta}\phi_\lambda + \frac{\partial B^\mu}{\partial \phi_\alpha} S^\beta{}_\lambda \tag{2.10}$$

5)

$$K^{\mu\alpha}{}_\lambda \equiv H^\mu S^\alpha{}_\lambda + P^{\mu\alpha}{}_\lambda \tag{2.11}$$

the variation principle finally reads

$$\begin{aligned} \delta A = &\int dx \left[-G\phi_\sigma \delta x^\sigma - GS^\beta{}_\lambda \partial_\beta \delta x^\lambda\right] \\ &+ \int dx \partial_\mu \left[-E^\mu{}_\sigma \delta x^\sigma - K^{\mu\beta}{}_\lambda \partial_\beta \delta x^\lambda - \frac{\partial B^\mu}{\partial \phi_\alpha} S^\beta{}_\lambda \partial_\alpha \partial_\beta \delta x^\lambda\right] \end{aligned} \tag{2.12}$$

or

$$\begin{aligned} \delta A = &\int dx \left[\partial_\beta(GS^\beta{}_\lambda) - G\phi_\lambda\right] \delta x^\lambda \\ &+ \int dx \partial_\mu \left[-\left\{E^\mu{}_\sigma + GS^\mu{}_\sigma\right\} \delta x^\sigma - K^{\mu\beta}{}_\lambda \partial_\beta \delta x^\lambda - \frac{\partial B^\mu}{\partial \phi_\alpha} S^\beta{}_\lambda \partial_\alpha \partial_\beta \delta x^\lambda\right] \end{aligned} \tag{2.13}$$

Raising and lowering indices will be done with the Lorentz metric.

Translation invariances now implies

I.

$$\partial_\mu E^\mu_{\ \sigma} + G\phi_\sigma = 0 \tag{2.14}$$

We finally introduce the abbreviations

6)

$$Z^\mu_{\ \sigma} \equiv E^\mu_{\ \sigma} + GS^\mu_{\ \sigma} + \partial_\lambda K^{\lambda\mu}_{\ \ \ \sigma} \tag{2.15}$$

7)

$$W^{\lambda\mu\alpha} \equiv \frac{1}{2}(K^{\mu\alpha\lambda} + K^{\alpha\mu\lambda}) - \frac{1}{2}(K^{\lambda\mu\alpha} + K^{\mu\lambda\alpha}) - \frac{1}{2}(K^{\alpha\lambda\mu} + K^{\lambda\alpha\mu}) \tag{2.16}$$

Proper Lorentz invariance together with translation invariance now gives

II.

$$Z^{\mu\alpha} = Z^{\alpha\mu}$$

III.

$$\partial_\mu Z^\mu_{\ \sigma} = -G\phi_\sigma + \partial_\mu(GS^\mu_{\ \sigma}) + \partial_\mu\partial_\lambda K^{\lambda\mu}_{\ \ \ \sigma} \tag{2.17}$$

The energy-momentum tensor $T^{\mu\alpha}$ is given as follows:

Let

$$t^{\mu\alpha} \equiv \partial_\lambda W^{\lambda\mu\alpha} \tag{2.18}$$

Then

$$t^{\mu\alpha} = t^{\alpha\mu} \tag{2.19}$$

(i) If $\partial_\mu\partial_\lambda K^{\lambda\mu\alpha} = 0$, then

$$T^{\mu\alpha} = Z^{\mu\alpha} \tag{2.20}$$

(ii) If $\partial_\mu\partial_\lambda K^{\lambda\mu\alpha} \neq 0$, then

$$T^{\mu\alpha} = Z^{\mu\alpha} + t^{\mu\alpha} \tag{2.21}$$

The energy-momentum tensor is then symmetric and is conserved, provided the equations of motion $G = 0$ are satisfied.

<u>Example</u> Neutral, massive vector field $\phi = \{\phi_\alpha\}$

$$L(\phi_\alpha) = \eta^{\alpha\beta}\eta^{\mu\nu}(\partial_\mu\phi_\alpha)(\partial_\nu\phi_\beta) - m^2\eta^{\alpha\beta}\phi_\alpha\phi_\beta \tag{2.22}$$

$$L(\phi_\alpha) = (\partial_\mu\phi_\alpha)(\partial^\mu\phi^\alpha) - m^2\phi_\alpha\phi^\alpha \tag{2.23}$$

We first compute the auxiliary quantities

$$H^{\alpha,\mu} \equiv \frac{\partial L}{\partial\phi_{\alpha,\mu}}$$

$$H^{\alpha,\mu} = 2\partial^\mu\phi^\alpha \tag{2.24}$$

$$G^\alpha \equiv \varepsilon(\phi_\alpha)L \equiv \frac{\partial L}{\partial\phi_\alpha} - \partial_\mu H^{\alpha,\mu}$$

$$
\begin{aligned}
G^{\alpha} &= -2[\partial_\mu \partial^\mu \phi^\alpha + m^2 \phi^\alpha] && (2.25)\\
E^{\mu}{}_{\sigma} &\equiv H^{\alpha,\mu} \phi_{\alpha,\sigma} - \delta^{\mu}{}_{\sigma} L \\
E^{\mu}{}_{\sigma} &= 2(\partial^\mu \phi^\alpha)(\partial_\sigma \phi_\alpha) - \delta^{\mu}{}_{\sigma} L && (2.26)\\
\left\{S^{\beta}{}_{\lambda}(\phi)\right\}_{\alpha} &= \delta^{\beta}{}_{\alpha} \phi_\lambda \\
P^{\mu\beta}_{\ \ \lambda} &= 0 && (2.27)\\
K^{\mu\alpha}{}_{\lambda} &\equiv H^{\sigma\mu} (S^{\alpha}{}_{\lambda})_\sigma \\
K^{\mu\alpha}_{\ \ \lambda} &= 2(\partial^\mu \phi^\alpha) \phi_\lambda && (2.28)\\
Z^{\mu}{}_{\sigma} &\equiv E^{\mu}{}_{\sigma} + G^{\lambda} (S^{\mu}{}_{\sigma})_\lambda + \partial_\lambda K^{\lambda\mu}{}_{\sigma} \\
Z^{\mu}{}_{\sigma} &= 2(\partial^\mu \phi^\alpha)(\partial_\sigma \phi_\alpha) - 2m^2 \phi^\mu \phi_\sigma + 2(\partial_\lambda \phi_\sigma)(\partial^\lambda \phi^\mu) - \delta^{\mu}{}_{\sigma} L && (2.29)
\end{aligned}
$$

$$
\begin{aligned}
K^{\mu\alpha\lambda} + K^{\alpha\mu\lambda} &= 2\phi^\lambda \left[(\partial^\mu \phi^\alpha) + (\partial^\alpha \phi^\mu)\right] \\
W^{\lambda\mu\alpha} &\equiv \frac{1}{2}(K^{\mu\alpha\lambda} + K^{\alpha\mu\lambda}) - \frac{1}{2}(K^{\lambda\mu\alpha} + K^{\mu\lambda\alpha}) - \frac{1}{2}(K^{\alpha\lambda\mu} + K^{\lambda\alpha\mu}) \\
W^{\lambda\mu\alpha} &= \phi^\lambda[\partial^\mu \phi^\alpha + \partial^\alpha \phi^\mu] - \phi^\alpha[\partial^\lambda \phi^\mu + \partial^\mu \phi^\lambda] - \phi^\mu[\partial^\alpha \phi^\lambda + \partial^\lambda \phi^\alpha] && (2.30)
\end{aligned}
$$

Now the energy-momentum tensor becomes

$$
\begin{aligned}
T^{\mu\alpha} &= Z^{\mu\alpha} + \partial_\lambda W^{\lambda\mu\alpha} \\
T^{\mu\alpha} &= 2(\partial^\mu \phi^\lambda)(\partial^\alpha \phi_\lambda) - 2m^2 \phi^\mu \phi^\alpha + 2(\partial_\lambda \phi^\mu)(\partial^\lambda \phi^\alpha) - \eta^{\mu\alpha} L \\
&\quad + \partial_\lambda \left[\phi^\lambda(\partial^\mu \phi^\alpha + \partial^\alpha \phi^\mu) - \phi^\alpha(\partial^\lambda \phi^\mu + \partial^\mu \phi^\lambda) - \phi^\mu(\partial^\alpha \phi^\lambda + \partial^\lambda \phi^\alpha)\right] && (2.31)
\end{aligned}
$$

Observe that

$$Z^{\mu\alpha} = Z^{\alpha\mu} \qquad (2.32)$$

and that the energy tensor reads

$$E^{\mu\alpha} = 2(\partial^\mu \phi_\lambda)(\partial^\alpha \phi^\lambda) - \eta^{\mu\alpha} L \qquad (2.33)$$

$E^{\mu\alpha}$ is also symmetric.

3 The Gravitational Stress Tensor

Let ϕ be a general matterfield and $g = (g_{\alpha\beta})$ the gravitational field [2]. Einstein's Theory of Gravity is then described by the action

$$A = \int dx L_0(g) + \int dx L(g, \phi) \qquad (3.1)$$

This action is Lorentz invariant and in addition is also invariant under the G-gauge group. The action for the gravitational selfinteraction

$$A_0 = \int dx L_0(g) \qquad (3.2)$$

where

$$L_0 = \sqrt{g}R, \tag{3.3}$$

and R the scalar curvature formed from the gravitational field g, is itself Lorentz invariant. Thus it has an energy-momentum tensor. If we demand that this energy-momentum tensor is also G-gauge group covariant, then it is identically zero.

We now pay attention to the gravitational-matter part

$$A(g, \phi) = \int dx L(g, \phi) \tag{3.4}$$

of the total action A.

$L(g, \phi)$ is assumed to only depend on the field g and its first derivatives as well as the matterfield ϕ and its first derivatives. We also assume that there is no boundary term present. The gravitational stress tensor is now defined as the Euler derivative of the gravitational-matter part with respect to the gravitational field

$$L \equiv L(g, \phi) \tag{3.5}$$

$$\begin{aligned} M^{\alpha\beta} &= \varepsilon(g_{\alpha\beta})L \\ M^{\alpha\beta} &= \frac{\partial L}{\partial g_{\alpha\beta}} - \partial_\mu \frac{\partial L}{\partial g_{\alpha\beta}, \mu} \end{aligned} \tag{3.6}$$

The action $A(g, \phi)$ is Lorentz invariant and also invariant under the G-gauge group. The Lagrangian L is then of the form

$$L = \sqrt{g} L_M(g, \phi) \tag{3.7}$$

where $L_M(g, \phi)$ is invariant under the G-gauge group, and thus has the form

$$L_M(g, \phi) = L_M(g_{\alpha\beta}, \phi, D_\mu \phi), \tag{3.8}$$

where $D_\mu \phi$ is the covariant derivative of the matter field with respect to the $G-$ gauge group, and any internal group.

The G-gauge group covariant energy-momentum tensor belonging to the gravitational-matter part vanishes identically. This is expressed by (II.15) as

$$Z^\mu{}_\sigma = 0. \tag{3.9}$$

pertaining to the two fields g and ϕ. The auxiliary quantities relating to the matter field ϕ only will carry the symbol ˆ.

We then have the following relations and results

$$\hat{H}^\mu = \frac{\partial L}{\partial \phi_\mu} = \frac{\partial L}{\partial (D_\mu \phi)} \tag{3.10}$$

$$\hat{K}^{\mu\alpha}_{\ \ \lambda} \equiv \hat{H}^{\mu} S^{\alpha}_{\ \lambda}(\phi) \tag{3.11}$$

$$\hat{K}^{\mu\alpha\beta} \equiv \hat{K}^{\mu\alpha}_{\ \ \lambda} g^{\lambda\beta} \tag{3.12}$$

$$\hat{W}^{\lambda\mu\alpha} = \frac{1}{2}(\hat{K}^{\mu\alpha\lambda} + \hat{K}^{\alpha\mu\lambda}) - \frac{1}{2}(\hat{K}^{\lambda\mu\alpha} + \hat{K}^{\mu\lambda\alpha}) - \frac{1}{2}(\hat{K}^{\alpha\lambda\mu} + \hat{K}^{\lambda\alpha\mu}) \tag{3.13}$$

$$H^{\alpha\beta,\mu} = \frac{\partial L}{\partial g_{\alpha\beta,\mu}} \tag{3.14}$$

For the Lagrangian $L(g, \phi)$ we now compute the auxiliary quantities

$$\hat{H}^{\mu} = \frac{\partial L}{\partial(D_{\mu}\phi)} \tag{3.15}$$

$$H^{\alpha\beta,\mu} \equiv \frac{\partial L}{\partial g_{\alpha\beta,\mu}}$$

For tensor fields

$$H^{\alpha\beta\mu} = \frac{1}{2}\hat{W}^{\mu\alpha\beta} \tag{3.16}$$

$$E^{\mu}_{\ \sigma} = \hat{H}^{\mu}\phi_{\sigma} + H^{\alpha\beta,\mu} g_{\alpha\beta,\sigma} - \delta^{\mu}_{\ \sigma} L \tag{3.17}$$

$$E^{\mu}_{\ \sigma} = \hat{E}^{\mu}_{\ \sigma} + \frac{1}{2}\hat{W}^{\mu\alpha\beta} g_{\alpha\beta.\sigma} \tag{3.18}$$

$$K^{\mu\alpha}_{\ \ \ \lambda} = \hat{K}^{\mu\alpha}_{\ \ \ \lambda} + 2g_{\lambda\sigma} H^{\sigma\alpha,\mu} \tag{3.19}$$

$$K^{\mu\alpha}_{\ \ \ \lambda} = \hat{K}^{\mu\alpha}_{\ \ \ \lambda} + g_{\lambda\sigma}\hat{W}^{\mu\sigma\alpha} \tag{3.20}$$

$$Z^{\mu}_{\ \sigma} = E^{\mu}_{\ \sigma} + GS^{\mu}_{\ \sigma} + \partial_{\lambda} K^{\lambda\mu}_{\ \ \ \sigma} \tag{3.21}$$

$$Z^{\mu}_{\ \ \sigma} = \hat{E}^{\mu}_{\ \sigma} + \frac{1}{2}\hat{W}^{\mu\alpha\beta} g_{\alpha\beta,\sigma} + \hat{G}S^{\mu}_{\ \sigma}(\phi) + 2g_{\alpha\sigma} M^{\mu\alpha} + \partial_{\lambda}[\hat{K}^{\lambda\mu}_{\ \ \sigma} + g_{\sigma\varrho}\hat{W}^{\lambda\mu\varrho}] \tag{3.22}$$

The energy tensor $\hat{E}^{\mu}_{\ \sigma}$ and the tensor $\hat{Z}^{\mu}_{\ \sigma}$ belonging to the matter field ϕ are given by

$$\hat{E}^{\mu}_{\ \sigma} = \hat{H}^{\mu}\phi_{\sigma} - \delta^{\mu}_{\ \sigma} L \tag{3.23}$$

$$\hat{Z}^{\mu}_{\ \sigma} = \hat{E}^{\mu}_{\ \sigma} + \hat{G}S^{\mu}_{\ \sigma}(\phi) + \partial_{\lambda}\hat{K}^{\lambda\mu}_{\ \ \ \sigma} \tag{3.24}$$

Then with $Z^{\mu}_{\ \sigma} \equiv 0$, we get

$$\hat{Z}^{\mu}_{\ \sigma} + \partial_{\lambda}\left[g_{\sigma\varrho}\hat{W}^{\lambda\mu\varrho}\right] + \frac{1}{2}\hat{W}^{\mu\alpha\beta} g_{\alpha\beta,\sigma} + 2g_{\alpha\sigma} M^{\mu\alpha} \equiv 0 \tag{3.25}$$

or, raising indices with the inverse $g^{\alpha\beta}$,

$$M^{\mu\alpha} = -\frac{1}{2}\left[\hat{Z}^{\mu\alpha} + g^{\sigma\alpha}\partial_{\lambda}\left\{g_{\sigma\varrho}\hat{W}^{\lambda\mu\varrho}\right\} + \frac{1}{2}g_{\varrho\beta,\sigma}\hat{W}^{\mu\varrho\beta}\cdot g^{\sigma\alpha}\right] \tag{3.26}$$

The gravitational stress tensor thus has a matter contribution and also a gravitational contribution.

If we evaluate the right hand side bracket for the Minkowski metric, we get exactly (II.21), namely the energy-momentum tensor for the matterfied ϕ in Minkowski space.

Example. Neutral, massive vectorfield $\phi = \{\phi_\alpha\}$. The gravitational interaction term for this model is given by the Lagrangian

$$L = L(g,\phi) = \sqrt{g}\left[g^{\alpha\beta}g^{\mu\nu}(D_\mu\phi_\alpha)(D_\nu\phi_\beta) - m^2 g^{\alpha\beta}\phi_\alpha\phi_\beta\right] \tag{3.27}$$

with the covariant derivative

$$D_\mu\phi_\alpha = \partial_\mu\phi_\alpha - \Gamma^\sigma{}_{\alpha\mu}\phi_\sigma \tag{3.28}$$

Raising indices and lowering indices is performed with $g^{\alpha\beta}$ resp. $g_{\alpha\beta}$.

We find the following expressions

$$\begin{aligned}\frac{\partial L}{\partial g_{\alpha\beta}} =& \frac{1}{2}g^{\alpha\beta}L + \sqrt{g}\Big[\Gamma^{\alpha\mu\nu}\phi^\beta(D_\mu\phi_\nu) \\ &+ \Gamma^{\beta\mu\nu}\phi^\alpha(D_\mu\phi_\nu) - (D_\mu\phi^\alpha)(D^\mu\phi^\beta) \\ &- (D^\alpha\phi_\mu)(D^\beta\phi^\mu) + m^2\phi^\alpha\phi^\beta\Big]\end{aligned} \tag{3.29}$$

$$\begin{aligned}\frac{\partial L}{\partial g_{\alpha\beta,\mu}} =& \frac{1}{2}\sqrt{g}\Big[\phi^\mu(D^\alpha\phi^\beta + D^\beta\phi^\alpha) \\ &- \phi^\alpha(D^\mu\phi^\beta + D^\beta\phi^\mu) \\ &- \phi^\beta(D^\mu\phi^\alpha + D^\alpha\phi^\mu)\Big]\end{aligned} \tag{3.30}$$

The gravitational stress tensor is then given by

$$\begin{aligned}M^{\alpha\beta} =& \frac{1}{2}g^{\alpha\beta}L - \sqrt{g}\Big[(D^\mu\phi^\alpha)(D_\mu\phi^\beta) + (D^\alpha\phi_\mu)(D^\beta\phi_\mu) - m^2\phi^\alpha\phi^\beta\Big] \\ &+ \frac{1}{2}\sqrt{g}D_\mu\Big[\phi^\alpha(D^\mu\phi^\beta + D^\beta\phi^\mu) + \phi^\beta(D^\mu\phi^\alpha + D^\alpha\phi^\mu) - \phi^\mu(D^\alpha\phi^\beta + D^\beta\phi^\alpha)\Big]\end{aligned} \tag{3.31}$$

If we evaluate $M^{\alpha\beta}$ for the Minkowskimetric we get $(-\frac{1}{2})$ times the energy-momentum tensor of the neutral, massive vectorfield in Minkowski space as given by (II.31).

The energy tensor for the matterfield alone is given by

$$\hat{E}^\mu{}_\sigma = 2\sqrt{g}(D^\mu\phi^\alpha)\phi_{\alpha,\sigma} - \delta^\mu{}_\sigma L \tag{3.32}$$

4 Conclusions

Einstein's Theory of Gravity is investigated within the Lagrange Formalism. The action is the sum of the graviational selfinteraction and the gravitational-matter part. The action is Lorentz-invariant and in addition is also invariant with respect to the G-gauge group. The G-gauge group has its roots in the fact that the gravitational field belongs to the spin 2, restmass zero representation of the Lorentz group. The G-gauge group can be identified with the group of smooth coordinate transformations which vanish at infinity: the Lorentz group is not a subgroup of the G-gauge group.

We consider the Lorentz group as the symmetry group of all of physics. This leads to the concepts of the energy-tensor and a symmetric energy-momentum tensor.

For Einstein's Theory of Gravity the G-gauge covariant energy-momentum tensor vanishes identically. The gravitational stress tensor is now defined as the Euler derivative of the gravitational-matter part with respect to the gravitational field. We have shown that the gravitational stress tensor, evaluated for the gravitational field being the Minkowski metric, is proportional to the energy momentum tensor of the gravitational-matter part, also evaluated at the Minkowski metric.

The right hand side of Einstein's equation is well defined through the gravitational stress tensor, which has a matter contribution and also a contribution due to gravity. Any modification of the right hand side of Einstein's equation, based on energy considerations, is inappropriate.

References

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