THE EPR BINGO

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Dedicated to Walter Hunziker and Klaus Hepp on the occasion of their 60th birthdays.

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Abstract. It is argued by means of an example that the Einstein-Podolsky-Rosen effect can transport valuable information at superluminal speed.

The question of whether we ought to worry or not about the EPR paradox is as unsettled at present as ever it was. This note is written by a worrier who sees a serious problem in the paradox. It is not its purpose, however, to convince the non-worriers that they are wrong. I am merely concerned with highlighting a particular detail of the problem, namely its relativistic aspect: the question whether the EPR effect can transport information with superluminal velocities. My answer, arrived at by means of an example, will be “yes”. I will argue throughout within the formal framework of quantum mechanics, disregarding the more general settings in which e.g. the Bell inequalities are derived and discussed. My purpose is to illuminate the paradoxical aspects of quantum mechanics, not to propose possible ways out of the conundrum.

I assume that the reader is acquainted with the paradox. Let me nevertheless describe it briefly, from the point of view of a worrier, using D. Bohm’s standard example of the decay of a particle of spin 0 into two particles of spin $\frac{1}{2}$. This admirable example exhibits the problems in a maximally succinct way. Consider a particle of spin 0 at rest, which decays at time $t = 0$ into two particles $A$ and $B$ with spin $\frac{1}{2}$. They are emitted back-to-back so that their orbital angular momentum is zero to an excellent approximation. We are only

$^1$For a recent account see ref. [1]
interested in the spin part of the 2-particle state which is present at \( t > 0 \). Let \( |\pm\rangle^A \) be the eigenstate of the component \( \sigma_z \) of the particle \( A \) in a prescribed direction, called the z-direction, with eigenvalue \( \pm \frac{1}{2} \). \( |\pm\rangle^B \) is defined analogously. Because of the conservation of angular momentum the spin state of the \( A-B \)-system is given by

\[
|\iota\rangle = 2^{-\frac{1}{2}} (|+\rangle^A |-=\rangle^B - |-=\rangle^A |+\rangle^B).
\]

The z-direction is the same for both particles, but this common direction can be chosen freely. As is well known, this freedom of choice is essential for the emergence of a paradox. But it is not going to be an important feature of my example.

Assume that at time \( T > 0 \) the z-component \( s^A \) of the \( A \)-spin is measured. Then a measurement of the z-component of the \( B \)-spin at time \( t = T + \epsilon \), \( \epsilon \) arbitrarily small, yields \textit{with certainty} the value \( s^B = -s^A \). This is an \textit{objective} statement about the state of \( B \) after time \( T \).

I am aware that this claim is far from being universally accepted. Let me therefore briefly defend my point of view, again without expecting to convince anybody who is not already convinced. First I note that the state of a system may be considered a representation of its history, in other words a description of how the system has been prepared. More exactly, the state represents those parts of the history that are relevant for making predictions on the future development of the system. With this interpretation it is clearly meaningful to talk about the state of an individual system. Second, what about the objectivity of the above statement? It is often claimed that a prediction of the outcome of a possible measurement is essentially meaningless as long as such a measurement is not actually performed or at least planned, since in that case if refers to a purely hypothetical, unreal event. But this defeatistic attitude is contrary to the physical practice. Most physicists agree that physics is concerned with studying a real world existing independently of our mind, and that only under such an assumption is research in physics a meaningful and worthwhile enterprise. Our statements about the world must have some possibly low but non-vanishing degree of realism to them. And my objectivity claim is not very strong, it is quite close to that required minimum of realism. In fact, I maintain that it is a tacit assumption which every physicist constantly uses in his daily work as a matter of course, usually without even being conscious of it, except in the rare moments when he is thinking about the problem of interpreting quantum mechanics. Hence I will stick to the notion that my statement about the properties of \( B \) after time \( T \) is objective, whatever that may exactly mean. In the language of quantum mechanics it means that after \( T \) the \( A-B \)-system is in the product state

\[
|f\rangle = |+\rangle^A |-=\rangle^B,
\]

and thus the particle \( B \) is in the state \( |-=\rangle^B \), if e.g. the \( A \)-measurement has yielded the result \( +\frac{1}{2} \). But before the time \( T \) the system was definitely not in the state \( |f\rangle \), because this state is not a state of total spin 0, it contains an admixture of spin 1. This argument applies also if the state is considered to describe only our knowledge of the system, not the system itself. Even in this case it cannot be maintained that the particle \( B \) did “have” a fixed value of \( s_z \) before \( T \), and we only did not know which one. This would mean that the \( A-B \)-system was
represented before $T$ by the density matrix

$$\rho = \frac{1}{2}(|++)\langle+| + |--\rangle\langle-+|)$$

(3)

in obvious notation, which density matrix also contains an inadmissible spin-1 part. But this means that the measurement of $s^4$ has caused an objective, instantaneous change of the state of $B$.

This is a highly disturbing finding even in the non-relativistic case, because the effect does not depend on the distance between $A$ and $B$, and because quantum mechanics gives us no inkling of a mechanism that could produce such a puzzling effect. The situation becomes even more worrying in relativistic quantum mechanics, and this brings me at last to the real subject of this note. The problem is that the change of the state of $B$ happens simultaneously with the measurement of $s^4$. But simultaneity of distant events is no relativistic invariant. Hence: in which frame are these two events simultaneous? The usual way out of the quandary is the assertion that the question is physically irrelevant, since the change in the state of $B$ is not a directly observable physical event. What counts is only that no observable acausalities can occur, in particular that the EPR effect does not permit the transmission of information with superluminal speed. However, the second prohibition (of superluminal transmission) is not necessarily a particular instance of the first prohibition (of acausalities). The following is true: the observer in $A$ has no possibility of influencing the outcome of his measurement of $s^4$ or the outcomes of repeated such measurements effected on a sequence of identically prepared $A-B$-systems. Therefore he cannot use such measurements to codify messages which then would be transmitted instantaneously to $B$. As a result of this, the EPR effect cannot be used for sending messages into one's own past. E.g. a student immediately after an examination cannot send a list of the questions he has been asked to himself before the examination. In this sense the effect does not lead to acausalities. But this does not preclude the possibility of transmitting valuable information with superluminal velocities. And this claim I wish to substantiate with an admittedly somewhat fanciful example.

Imagine a time in the more or less distant future, after the solar system has been colonized. At a certain date $T_1$ the following advertisement is distributed throughout the solar system, issued by a professional betting company with headquarters on earth but operating system-wide:

"We have in our possession at our headquarters a certain particle of spin $\frac{1}{2}$. Two weeks from now, at the date $T_2$, at 12 o'clock universal standard time the previously unknown spin component of this particle in the direction pointing towards the north star will be measured. All our branch offices accept bets as to the outcome of this measurement until shortly before the result of the measurement becomes known".

Consider such a branch office located on Titan, the largest moon of Saturn. Since Saturn is at all times more than a light-hour away from earth, the outcome of the spin measurement cannot possibly become known on Titan before 13.00 o'clock on day $T_2$. Therefore the office in question accepts bets until that time. Assume now this: in order to make sure that the spin state of the crucial particle, called $A$, is completely unknown, a freshly minted particle is used, one that has been created in the decay of an unstable particle immediately before its
capture and safeguarding. Assume further that this decay was of the EPR type and that the particle $A$ has been captured by a method which did in no way interfere with its spin. Assume finally that an inhabitant of Titan has succeeded in a devious way, unbeknown to everyone, to get hold of the EPR twin $B$ of $A$, again in a way which leaves its spin state strictly alone, and to take it with him to his home. Then, shortly after 12 noon on day $T_2$ he measures the relevant spin component $s^B$ of his particle, learns from this the outcome $s^A = -s^B$ of the rigorously supervised spin measurement effected on the strongly guarded particle $A$, places at once a substantial bet at his neighbourhood betting shop, reappears there at 14 o’clock and collects his winnings. Hence in this case the EPR effect has transported very practical information instantaneously, for what could be more practical than winning money!

And thus we are confronted with the relativistic lack of meaning of the notion of simultaneity of distant events. At what time $t_m$ has particle $B$ “learned” which result it will have to exhibit at the measurement of $s^B$? No matter where we choose this mysterious time $t_m$ before the $B$-measurement, we can find a frame of reference in which it lies earlier than both measurements, i.e. before the system has come into contact with either of the two measuring devices or has indulged in any other interactions involving its spin variables. But, as has been remarked before, after the time $t_m$ the $A$-$B$-system is in a product state as far as spin is concerned, or possibly in a mixture of product states, and these states are not pure spin 0 but contain an admixture of spin 1, in contradiction with the conservation of angular momentum. This fact also precludes the excuse that possibly the mere announcement of the intention of measuring a specified spin component has already effected a reduction of the wave packet plunging the system into the impure state (3). The idea that such a mere declaration of intent could change the total spin of the $A$-$B$-system is clearly absurd.

References