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Time of Events in Quantum Theory[1]

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Abstract. We enhance elementary quantum mechanics with three simple postulates that enable us to define time observable. We discuss shortly justification of the new postulates and illustrate the concept with the detailed analysis of a delta function counter.

Zeit ist nur dadurch, daß etwas geschieht und nur dort wo etwas geschieht.
(E.Bloch)

1 Introduction

Time plays a peculiar role in Quantum Mechanics. It differs from other physical quantities like position or momentum. When discussing position a dialogue may look like this: [4]

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[1]This paper is dedicated to Klaus Hepp and to Walter Hunziker on the occasion of their sixtieth anniversary

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[4]We use the method chosen by Galileo in his great book "Dialogues Concerning Two New Sciences"[1]. Galileo is often refered to as the founder of modern physics. The most far–reaching of his achievements was his counsel's speech for mathematical rationalism against Aristotle's logico–verbal approach, and his insistence for combining mathematical analysis with experimentation.

SP: What is the position?
SG: Position of what?
SP: Of the particle.
SG: When?
SP: At t=t1.
SG: The answer depends on how you are going to measure this position. Are you sure you have detectors put everywhere that interact with the particle only during the time interval (t-dt,t+dt) and not before?

When talking about time we will have something like this:

SP: What is time?
SG: Time of what?
SP: Time of a particle.
SG: Time of your particle doing what?
SP: Time of my particle leaving the box where it was trapped. Or time at which my particle enters the box.
SG: Well, it depends on the box and it depends on the method you want to apply to ascertain that the event has happened.
SP: Why can't we simply put clocks everywhere, as it is common in discussions of special relativity? And let these clocks note the time at which the particle passes them?
SG: Putting clocks disturbs the system. The more clocks you put - the more you disturb. If you put them everywhere - you force wave packet reductions a'la GRW. If you increase their time resolution more and more - you increase the frequency of reductions. When the clocks have infinite resolutions - then the particle stops moving - this is the Quantum Zeno effect [2].
SP: I do not believe these wave packet reductions. Zeh published a convincing paper whose title tells its content: "There are no quantum jumps nor there are particles" [3], and Ballentine [4, 5], proved that the projection postulate is wrong.
SG I remember these papers. They had provocative titles...
SV. First of all Ballentine did not claim that the projection postulate is wrong. He said that if incorrectly applied - then it leads to incorrect results. And indeed he showed how incorrect application of the projection postulate to the particle tracks in a cloud chamber leads to inconsistency. What he said is this: "According to the projection postulate, a position measurement should "collapse" the state to a position eigenstate, which is spherically symmetric and would spread in all directions, and so there would be no tendency to subsequently ionize only atoms that lie in the direction of the incident momentum. An approximate position measurement would similarly yield a spherically symmetric wave packet, so the explanation fails." This is exactly what he said. And this is correct. This shows how careful one has to be with the projection postulate. If the projection postulate is understood as operating with an operator on a state vector: $\psi \longrightarrow R\psi/||R\psi||$, then the argument does not apply. Thus a correct application would be to *multiply* the moving Gaussian of the

In the dialog SG=Sagredo, SV=Salviati, SP=Simplicio particle, something like:

$$\psi(x,t) = \exp(ip(x - a(t))\exp(-(x - a(t))^2/\sigma(t))$$

which is spherically symmetric, but only up to the phase, by a static Gaussian modelling a detector localized around a:

$$f(x) = \exp(-\alpha(x-a)^2)$$

The result is again a moving Gaussian. And in fact, such a projection postulate is not a postulate at all. It can be derived from the simplest possible Liouville equation.

SP: Has this "correct", as you claim, cloud chamber model been published? Have its predictions been experimentally verified?

SV: A general theory of coupling between quantum system and a classical one is now rather well understood [6]. The cloud chamber model has been published quite recently, you can take a look at [7, 8]. Belavkin and Melsheimer [9] tried to derive somewhat similar result from a pure unitary evolution, but I am not able to say what assumptions they used, what approximations they made and what exactly are their results.

SP: Hasn't the problem been solved long ago in the classical paper by Mott [10]?

SV Mott did not even attempt to derive the timing of the tracks. In the cloud chamber model of Refs. [7, 8], that I understand rather well, because I participated in its construction, it is interesting that the detectors – even if they do not "click" – influence the continuous evolution of the wave packet between reductions. They leave a kind of a "shadow". This is another case of a "interaction-free" experiment discussed by Dicke [11, 12], and then by Elitzur and Vaidman [13] in their "bomb–test" allegory, and also by Kwiat, Weinfurter, Herzog and Zeilinger [14]. The shadowing effect predicted by EEQT [5] may be tested experimentally. I believe it will find many applications in the future, and I hope these will be not only the military ones! Yet we must now not digress upon this particular topic since you are waiting to hear what I think about the problem of time in quantum theory. We already know that "time" must be "time of something". Time of something that happens. Time of some event.[6] But in quantum theory events are not simply space-time events as it is in relativity. Quantum theory is specific in the sense that there are no events unless there is something external to the quantum system that "causes" these events. And this something external must not be just another quantum system. If it is just another quantum system - then nothing happens, only the state vector continuously evolves in parameter time.

SP But is it not so that there are no sharp events? Nothing is sharp, nothing really sudden. All is continuous. All is approximate.

SG How nothing is sharp, do we not register "clicks" when detecting particles?

SP I do not know what clicks are you talking about ...

SG How you don't know? Ask the experimentalist.

SP I *am* an experimentalist!

SV The problem you are discussing is not an easy one to answer. I pondered on it many times, but did not arrive at a clear conclusion. Nevertheless something can be said with

[5]Salviatti refers here to "Event Enhanced Quantum Theory" of reference [6] - paper apparently well know to the participants of the dialog.

[6]Heisenberg proposed the word "event" to replace the word "measurement", the latter word carrying a suggestion of human involvement.

certainty. First of all you both agree that in physics we always have to deal with idealizations. For instance one can argue that there are no real numbers, that the only, so to say, experimental numbers are the natural numbers. Or at most rational numbers. But real numbers proved to be useful and today we are open to both possibilities: of a completely discrete world, and of a continuous one. Perhaps there is also a third possibility: of a fuzzy world. Similarly there are different options for modeling the events. One can rightly argue that they are never sharp. But do they happen or not? Do we need counting them? Do we need a theory that describes these counts? We do. So, what to do? We have no other choice but to try different mathematical models and to see which of them better fit the experiment, better fit the rest of our knowledge, better explain what makes Nature tick. In the cloud chamber model that we were talking about just a while ago the events are unsharp in space but they are sharp in time. And the model works quite well. However, if you try to work out a relativistic cloud chamber model, then you see that the events must be also smeared out in the coordinate time.[7] Nevertheless they can still be sharp in a different "time", called "proper time" after Fock and Schwinger. If time allows I will tell you more about this relativistic theory, but now let us agree that in a nonrelativistic theory sharp localization of events in time does not contradict any known principles. We will remember at the same time that we are dealing here with yet another idealization that is good as long as it works for us. And we must not hesitate to abandon it the moment it starts to lead us astray. The principal idea of EEQT is the same as that expressed in a recent paper by Haag [17]. Let me quote only this: "... we come almost unavoidably to an evolutionary picture of physics. There is an evolving pattern of events with causal links connecting them. At any stage the 'past' consists of the part which has been realized, the 'future' is open and allows possibilities of new events ..."

SG Let me interrupt you. Perhaps we should remember what Bohr was telling us. Bohr insisted that the apparatus has to be described in terms of classical physics; this point of view is a common-place for experimental physicists. Indeed any experimental article observes this rule. This principle of Bohr is not in any way a contradiction but simply the recognition of the fact that any physical theory is always the expression of an approximation and an idealization. Physics is always a little bit false. Epistemology must also play role in the labs. Physics is a system of analogies and metaphors. But these metaphors are helping us to understand how Nature does what it does.

SP I agree with this. So what is your proposal? How to describe time of events in a nonrelativistic quantum theory? Does one first have to learn EEQT - your "Event Enhanced Quantum Theory" that you are so proud of? I know many theoretical physicists dislike your explicit introduction of a classical system. They prefer to keep everything classical in the background. Never put it into the equations.

SV Here we have a particularly lucky situation. For this particular purpose of discussing time of events it is not necessary to learn EEQT. It is possible to describe time observation with simple rules. This is normal in standard quantum mechanics. You are told the rules, and you are told that they work. So you believe them and you are happy that you were told them. In EEQT Schrödinger's evolution and reduction of the wave function appear as special cases of a single law of motion which governs all systems equally. EEQT is one

[7] Cf. an illuminating discussion of this point in [15, 16].

of the few approaches that allow you to *derive* quantum mechanical postulates and to see that these postulates reflect only part of the truth. Here, when discussing time of events we do not need the full predictive power of EEQT. This is so because after an event has been registered the experiment is over. We are not interested here in what happens to our system after that. Therefore we need not to speak about jumps and wave packet reductions. It is only if you want to **derive** the postulates for time measurements, only then you will have to look at EEQT. But instead of deriving the rules, it is even better to see if they give experimentally valid predictions. We know too many cases where good formulas were produced by doubtful methods and bad formulas with seemingly good ones. Using the right tool makes the job easier.

SP I become impatient to see your postulates, and to see if I can accept them as reasonable even before any experimental testing. Only if I see that they are reasonable, only then I will have any motivation to see whether they really be derived from EEQT, or perhaps in some other way.

2 Time of Events

We start our discussion on quite a general and somewhat abstract level. Only later on, in examples, we will specialize both: our system and the monitoring device. We consider quantum system described using a Hilbert space $\mathcal{H}$.[8] To answer the question "time of what?", we must select a *property* of the system that we are going to monitor. It must give only "yes-no", or *one-zero* answers. We denote this binary variable with the letter α. In our case, starting at $t = 0$, when the monitoring begins, we will get continuously $\alpha = 0$ reading on the scale, until at a certain time, say $t = t_1$, the reading will change into "yes". Our aim is to get the statistics of these "first hitting times", and to find out its dependence of the initial state of the system and on its dynamics.
Speaking of the "time of events" one can also think that "events" are transitions which occur; sometimes the system is changing its state randomly - and these changes are registered. There are two kinds of probabilities in Quantum Mechanics the transition probabilities and other probabilities - those that tell us **when** the transitions occur. It is this second kind of probabilities that we will discuss now.

2.1 First Postulate - the Coupling

Our first postulate reads: the coupling to a "yes-no" monitoring device is described by an operator $\Lambda \geq 0$ in the Hilbert space $\mathcal{H}$. In general Λ may explicitly depend on time but here, for simplicity, we will assume that this is not the case. That means: to any real experimental device there corresponds a Λ. In practice it may be difficult to produce the Λ that describes exactly a given device. As it is difficult to find the Hamiltonian that takes

[8]More generally we would need two Hilbert spaces: $\mathcal{H}_{no}$ and $\mathcal{H}_{yes}$ that can be different, but for the present discussion we need not be pedantic, so we will assume them to be identified.

into account *exactly* all the forces that act in a system. Nevertheless we believe that an exact Hamiltonian exists, even if it is hard to find or impractical to apply. Similarly our postulate states that an exact Λ exists, although it may be hard to find or impractical to apply. Then we use an approximate one, a model one.

It should be noticed that we do not assume that Λ is an orthogonal projection. This reflects the fact that our device - although giving definite "yes-no" answers, gives them acting upon possibly *fuzzy* criteria. In the limit when the criteria become sharp one should think of Λ as $\Lambda \longrightarrow \lambda E$, where λ is a coupling constant of physical dimension t^{-1} and E a projection operator. In the general case it is usually convenient to write $\Lambda = \lambda\Lambda_0$, where Λ_0 is dimensionless.

It is also important to notice that the property that is being monitored by the device need not be an *elementary* one. Using the concepts of quantum logic (cf. [18, 19]) the property need not be *atomic* - it can be a composite property. In such a case, when thinking about physical implementation of the procedure determining whether the property holds or no, there are two extreme cases. Roughly speaking the situation here is similar to that occurring in discussion of superpositions of state preparation procedures. Some procedures lead to a coherent superpositions, some other lead to mixtures. Similarly with composite detectors: one possibility is that we have a distributed array of detectors that can act independently of each other, and our event consists on activating *one of them*. Another possibility is that we have a *coherent* distributed detector like a solid state lattice that acts as *one* detector. In the first case (called *incoherent*) Λ will be of the form

$$\Lambda = \sum_\alpha g_\alpha^\star g_\alpha,$$

while in the second *coherent* case:

$$\Lambda = (\sum_\alpha g_\alpha)^\star (\sum_\alpha g_\alpha),$$

where g_α are operators associated to individual constituents of the detector's array. More can be said about this important topic, but we will not need to analyze it in more details for the present purpose.

2.2 Second Postulate - the Probability

We assume that, apart of the monitoring device, our system evolves under time evolution described by the Schrödinger equation with a self–adjoint Hamiltonian $H_0 = H_0^\star$. We denote by $K_0(t) = \exp(-iH_0 t)$ the corresponding unitary propagator. Again, for simplicity, we will assume that H_0 does not depend explicitly on time.

Our second postulate reads: assuming that the monitoring started at time $t = 0$, when the system was described by a Hilbert space vector ψ_0, $\|\psi_0\| = 1$, and when the monitoring device was recording the logical value "no", the probability $P(t)$ that the change *no* $\rightarrow$ *yes* will happen before time t is given by the formula:

$$P(t) = 1 - \|K(t)\psi_0\|^2, \tag{2.1}$$

where

$$K(t) = \exp(-iH_0t - \frac{\Lambda t}{2}). \tag{2.2}$$

Remark: The factor $\frac{1}{2}$ in the formula above is put here for consistency with the notation used in our previous papers.

It follows from the formula (2.1) that the probability that the counter will be triggered out in the time interval (t,t+dt), provided it was not triggered yet, is $p(t)dt$, where $p(t)$ is given by

$$p(t) = \frac{d}{dt}P(t) =< K(t)\psi_0, \Lambda K(t)\psi_0 > . \tag{2.3}$$

We remark that $\int_0^\infty p(t)dt = P(\infty)$ is the probability that the detector will notice the particle at all. In general this number representing the total efficiency of the detector (for a given initial state) will be smaller than 1.

2.3 Third Postulate - the Shadowing Effect

As noticed above in general we expect $P(\infty) < 1$. That means that if the experiment is repeated many times, then there will be particles that were not registered while close to the counter; they moved away, and they will never be registered in the future. The natural question then arises: is the very presence of the counter reflected in the dynamics of the particles that pass the detector without being observed? Or we can put it as a "quantum espionage" question: **can a particle detect a detector without being detected?** And if so - which are the precise equations that tell us **how**?.

To answer this question it is not enough to use the two postulates above. One needs to make use of the Event Engine of EEQT once more.

Our third postulate reads: prior to any event, and independently of whether any event will happen or not, the state of the system is described by the vector $\hat{\psi}_t$ undergoing the non-linear evolution given by:

$$\hat{\psi}_t = \frac{\psi_t}{\|\psi_t\|}, , \tag{2.4}$$

where

$$\psi_t = K(t)\psi_0. \tag{2.5}$$

It is not too difficult to think of an experiment that will test this prediction.
Fig. 1 shows four shots from time evolution of a gaussian wavepacket monitored by a gaussian detector placed at the center of the plane. The efficiency of the detector is in this case ca. $P(\infty) \simeq 0.55$. There is almost no reflection. The shadow of the detector that is seen on the fourth shot can be easily interpreted in terms of ensemble interpretation: once we count only

those particles that were *not* registered by the detector, then it is clear that there is nothing or almost nothing behind the detector. However a careful observer will notice that there is a local maximum exactly behind the counter. This is a quantum effect, that of "interference of alternatives". It has consequences for the rate of future events for an individual particle.

2.4 Justification of the postulates

The above postulates are more or less "natural". They are in agreement with the existing ideas of non-unitary[9] evolution. So, for instance, in [20] the authors considered the ionization model. They wrote: 'According to the usual procedure the ionization probability $P(t)$ should be given by $P(t) = 1 - |\Psi|^2$'.
Even if our postulates are natural, it is worthwhile to notice that EEQT allows us to interpret them, to understand them and to derive them, in terms of *classical* Markov processes. First of all let us see that the above formula for $P(t)$ can be understood in terms of an inhomogeneous Poisson decision process as follows.[10] Assume the evolution starts with some quantum state ψ_0, of norm one, as above. Define the positive function $\lambda(t)$ as

$$\lambda(t) = (\hat{\psi}_t, \Lambda\hat{\psi}_t), \tag{2.6}$$

where Then $P(t)$ above happens to be nothing but the first-jump probability of the inhomogeneous Poisson process with intensity $\lambda(t)$. It is instructive to see that this is indeed the case. To this end let us divide the interval $(0, t)$ into n subintervals of length $\Delta t = t/n$. Denote $t_k = (k-1)\Delta t$, $k = 1, \ldots, n$. The inhomogeneous Poisson process of intensity $\lambda(t)$ consists then of taking independent decisions 'jump–or–not–jump' in each time subinterval. The probability for jumping in the k-th subinterval is assumed to be $p_k = \lambda(t_{k-1})\Delta t$ (that is why λ is called the *intensity* of the process). Thus the probability $P_{not}(t)$ of not jumping up to time t is

$$P_{not}(t) = \lim_{n\to\infty} \prod_{k=1}^{n} (1 - p_k) = \exp(-\int_0^t \lambda(s)ds). \tag{2.7}$$

Let us show that $1 - P_{not}(t)$ can be identified with $P(t)$ given by Eq. (2.7). To this end notice that

$$\begin{aligned} \frac{d}{dt}(1 - P(t)) &= - < \psi_t, \Lambda\psi_t >= -\lambda(t)\|\psi_t\|^2 \\ &= -\lambda(t)(1 - P(t)). \end{aligned}$$

Thus $1 - P(t)$, given by Eq. (2.1) satisfies the same differential equation as $P_{not}(t)$ given by Eq. (2.7). Because $1 - P(0) = P_{not}(0) = 1$, it follows that $1 - P(t) = P_{not}(t)$, and so $P(t) = 1 - P_{not}(t)$ indeed is the first jump probability of the inhomogeneous Poisson process with intensity $\lambda(t)$.

This observation is useful but rather trivial. It can not yet stand for a justification of the formula (2.1) - this for the simple reason that the jump process above, based upon a

[9]Known in the literature also under the name of "non-hermitian"

[10]A mathematical theory of a counter that leads to an inhomogeneous Poisson process, starting from formal postulates that are different than ours was given almost fifty years ago by Res Jost [21] .

continuous observation of the variable α and registering the time instant of its jump, is not a Markovian process. It would become Markovian if we know $\lambda(t)$, but to know $\lambda(t)$ we must know $\hat{\psi}_t$. This leads us to consider pairs $x_t = (\hat{\psi}_t, \alpha_t)$, where $\hat{\psi}_t$ is the Hilbert space vector describing quantum state, and α_t is the yes-no state of the counter. Then $\hat{\psi}_t$ evolves deterministically according to the formula (2.4), the intensity function $\lambda(t)$ is computed on the spot, and the Poisson decision process described above is responsible for the jump of value of α - in our case it corresponds to a "click" of the counter. The time of the click is a random variable T_1, well defined and computable by the above prescription.

This prescription sheds some light onto the meaning of the quantum state vector ψ. We see that ψ codes in itself information that is being used by a decision mechanism working in an entirely classical way - the only randomness we need is that of a biased (by $\lambda(t)$) classical roulette. Until we ask *why* the bias is determined by this particular functional of the quantum state, until then we do not have to invoke more esoteric concepts of quantum probability – whatever they mean. But, in fact, it is possible to understand somewhat more, still in pure classical terms. We will not need this extra knowledge in the rest of this paper, but we think it is worthwhile to sketch here at least the idea.

In the reasoning above we were interested only in what governs the time of the first jump, when the counter clicks. But in reality nothing ends with this click. A photon, for instance, when detected, is transformed into another form of energy. So, if we want to continue our process in time, after T_1, we must feed it with an extra information: how is the quantum state transformed as the result of the jump. So, in general, we have a classical variable α that can take finitely many, denumerably many, a continuum, or more, possible values, and to each ordered pair $(\alpha \to \beta)$ there corresponds an operator $g_{\alpha\beta}$. The transition $(\alpha \to \beta)$ is called an *event*, and to each event there corresponds a transformation of the quantum state $\psi \to \frac{g_{\beta\alpha}\psi}{\|g_{\beta\alpha}\psi\|}$. In the case of a counter there is only one β. In general, when there are several β-s, we need to tell not only when to jump, but also where to jump. One obtains in this way a piecewise deterministic Markov process on pure states of the total system: (quantum object, classical monitoring device). It can be then shown [6, 22] that this process, including the jump rate formula (2.6) follows uniquely from the simplest possible Liouville equation that couples the two systems.

3 The Time of Arrival

As the most natural application of the above concept of "time of event" we consider the notion of "time of arrival" of a particle to a certain state. There are several methods available for computing the "time of arrival" distribution given our postulates. We shall take the approach that seems to us to be the simplest one. One by one we shall specialize our assumptions about Λ.

3.1 One elementary detector

Let $K(t)$ be given by Eq. (2.2), and let[11]

$$K_0(t) = \exp(-iH_0 t). \tag{3.1}$$

Then $K(t)$ satisfies the Schrödinger equation

$$\dot{K} = -iH_0 K(t) - \frac{\Lambda}{2} K(t). \tag{3.2}$$

This differential equation, together with initial data $K(0) = I$, is easily seen to be equivalent to the following integral equation:

$$K(t) = K_0(t) - \frac{1}{2}\int_0^t K_0(t-s)\Lambda K(s)ds. \tag{3.3}$$

By taking the Laplace transform and by the convolution theorem we get the Laplace transformed equation:

$$\tilde{K} = \tilde{K}_0 - \frac{1}{2}\tilde{K}_0 \Lambda \tilde{K}. \tag{3.4}$$

Let us consider the case of a maximally sharp measurement. In this case we would take $\Lambda = |a><a|$, where $|a>$ is some Hilbert space vector. It is not assumed to be normalized; in fact its norm stands for the strength of the coupling (notice that $<a|a>$ must have physical dimension t^{-1}). Taking look at the formula (2.3) we see that now $p(t) = |<a|\psi_t>|^2$ and so we need to know $<a|K(t)\psi_0>$ rather than the full propagator $K(t)$. Multiplying Eq. (3.4) from the left by $<a|$ and from the right by $|\psi_0>$ we obtain:

$$<a|\tilde{\psi}> = \frac{2<a|\tilde{K}_0|\psi_0>}{2+<a|\tilde{K}_0|a>} \tag{3.5}$$

where $\tilde{\psi}$ is the Laplace transform of $t \rightarrow \psi(t)$:

$$\tilde{\psi}(z) = \int_0^\infty e^{-tz}\psi(t)dt = \tilde{K}(z)\psi_0, \qquad \Re(z) \geq 0. \tag{3.6}$$

3.2 Composite detector

We consider now the simplest case of a composite detector. It will be an incoherent composition of two simple ones. Thus we will take:

$$\Lambda = |a_1><a_1| + |a_2><a_2|. \tag{3.7}$$

[11]From this time on the subscript $_0$ may refer to either the initial state, as in ψ_0 or to free evolution, as in H_0, or to initial state evolving under free evolution. In case of confusion the actual meaning should be derived from the context.

Remark Notice that if $< a_1|a_2 >= 0$, then coherent and incoherent compositions are indistinguishable, as in this case, with $g_i = |a_i >< a_i|$, we have that $\sum_i g_i{}^\star g_i = (\sum_i g_i)^\star(\sum_i g_i)$. For $p(t)$ we have now the formula:

$$p(t) = \sum_i | < a_i|\psi_t > |^2, \tag{3.8}$$

and to compute the complex amplitudes $< a_i|\psi_t >$ we will use the Laplace transform method as in the case of one detector. To this end one applies $< a_i|$ from the left and $|\psi_0 >$ from the right to Eq. (3.4) and solves the resulting system of two linear equations to obtain:

$$\left.\begin{aligned} < a_1|\tilde{\psi} > &= \tfrac{2}{\Delta}\left((2+(22)) < a_1|\tilde{\psi_0} > -(12) < a_2|\tilde{\psi_0} > \right) \\ < a_2|\tilde{\psi} > &= \tfrac{2}{\Delta}\left((2+(11)) < a_2|\tilde{\psi_0} > -(21) < a_1|\tilde{\psi_0} > \right) \end{aligned}\right\} \tag{3.9}$$

where we used the notation

$$(ij) =< a_i|\tilde{K_0}|a_j >, \tag{3.10}$$

$$|\tilde{\psi_0} >= \tilde{K_0}|\psi_0 >, \tag{3.11}$$

and where Δ stands for

$$\Delta = 4 + 2((11) + (22)) + ((11)(22) - (12)(21)). \tag{3.12}$$

The probability density $p(t)$ is then given by

$$p(t) = \sum_i |\phi_i(t)|^2, \tag{3.13}$$

where ϕ_i is the inverse Fourier transform

$$\phi_i(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ity}\tilde{\phi}_i(iy)dy \tag{3.14}$$

of

$$\tilde{\phi}_i(iy) = \lim_{x=0+} < a_i|\tilde{\psi}(x+iy) > \ . \tag{3.15}$$

By the Parseval formula we have that $P(\infty)$ is given by:

$$P(\infty) = \frac{1}{2\pi}\sum_i \int_{-\infty}^{\infty} |\tilde{\phi}_i(iy)|^2 dy\ . \tag{3.16}$$

3.3 Example: Dirac's δ counter for ultra-relativistic particle

Let us now specialize the model by assuming that we consider a particle in R^1 and that the Hilbert space vector $|a >$ approaches the improper position eigenvector $\sqrt{\kappa}\delta(x-a)$ localized at the point a. This corresponds to a point-like detector of strength κ placed at a.[12] We see from the equation (2.3) that $p(t)$ is in this case given by:

$$p(t) = |\phi(t)|^2, \tag{3.17}$$

where the complex amplitude $\phi(t)$ of the particle arriving at a is:

$$\phi(t) =< a|\psi(t) >, \tag{3.18}$$

or, from Eq. (3.5)

$$\tilde{\phi} = \frac{2\sqrt{\kappa}}{2+\kappa\tilde{K}_0(a,a)}\tilde{\psi}_0(a) \tag{3.19}$$

where $\tilde{\psi}_0$ stands for the Laplace transform of $K_0(t)\psi_0$.
Let us now consider the simplest explicitly solvable example - that of an ultra-relativistic particle on a line. For H_0 we take $H_0 = -ic\frac{d}{dx}$, then the propagator K_0 is given by $K_0(x,x';t) = \delta(x'-x+ct)$, and its Laplace transform reads $\tilde{K}_0(x,x';z) = \frac{1}{c}e^{(x-x')z/c}$. In particular $\tilde{K}_0(a,a;z) = \frac{1}{c}$ and from Eq. (3.19) we see that the amplitude for arriving at the point a is given by the "almost evident" formula:

$$\phi(t) = const(\kappa) \times \psi(a-ct), \tag{3.20}$$

where $const(\kappa) = \sqrt{\kappa}/(1+\frac{\kappa}{2c})$. It follows that probability that the particle will be registered is equal to

$$P(\infty) = \frac{\kappa/c}{(1+\frac{\kappa}{2c})^2}\int_{-\infty}^{a} dx|\psi_0(x)|^2 \tag{3.21}$$

which has a maximum $P(\infty) = 1/2$ for $\kappa = 2c$ if the support of ψ_0 is left to the counter position a. We notice that in this example the shape of the arrival time probability distribution $p(t)$ does not depend on the value of the coupling constant - only the effectiveness of the detector depends on it. For a counter corresponding to a superposition $\sum_i \sqrt{\kappa_i}\delta(x-a_i)$ we obtain for $P(\infty)$ exactly the same expression as for one counter but with κ replaced with $\sum_i \kappa_i$.

3.4 Example: Dirac's δ counter for Schrödinger's particle

We consider now another example corresponding to a free Schrödinger's particle on a line. We will study response of a Dirac's delta counter $|a >= \sqrt{\kappa}\delta(x-a)$, placed at $x = a$, to a Gaussian wave packet whose initial shape at $t = 0$ is given by:

$$\psi_0(x) = \frac{1}{(2\pi)^{1/4}\eta^{1/2}}\exp\left(\frac{-(x-x_0)^2}{4\eta^2} + 2ik(x-x_0)\right). \tag{3.22}$$

[12]The case of Hermitian singular *delta*-function perturbation was discussed by many authors - see [23, 24, 25, 26, 27, 28, 29] and references therein

In the following it will be convenient to use dimensionless variables for measuring space, time and the strength of the coupling:

$$\xi = \frac{x}{2\eta}, \qquad \tau = \frac{\hbar t}{2m\eta^2}, \qquad \alpha = \frac{m\eta\kappa}{\hbar} \tag{3.23}$$

We denote

$$\xi_0 = x_0/2\eta,\ \xi_a = a/2\eta,\ v = 2\eta k \tag{3.24}$$

In these new variables we have:

$$\psi_0(\xi) = \left(\tfrac{2}{\pi}\right)^{1/4} e^{-(\xi-\xi_0)^2+2iv(\xi-\xi_0)} \tag{3.25}$$

$$K_0(\xi',\xi;\tau) = \left(\tfrac{1}{\pi i\tau}\right)^{1/2} \exp\left(\tfrac{i(\xi'-\xi)^2}{\tau}\right) \tag{3.26}$$

$$\tilde{K}_0(\xi',\xi;z) = (iz)^{-\frac{1}{2}} \exp\left(-2\sqrt{-iz}\,|\xi'-\xi|\right) \tag{3.27}$$

We can compute now explicitly $\tilde{\psi}(z)$ of Eq. (3.6):

$$\tilde{\psi}_0(a;z) = \frac{1}{2}(2\pi)^{1/4}(iz)^{-1/2}e^{-d^2-2ivd}\left[\mathrm{w}(u_+)+\mathrm{w}(u_-)\right] \tag{3.28}$$

where

$$u_\pm = i\sqrt{-iz} \pm (v-id), \qquad d = \xi_0 - \xi_a, \tag{3.29}$$

and the amplitude $\tilde{\phi}$ of Eq. (3.19), when rendered dimensionless,[13] reads

$$\tilde{\phi}(z) = (2\pi)^{1/4}\alpha^{1/2}e^{-d^2-2ivd}\,\frac{\mathrm{w}(u_+)+\mathrm{w}(u_-)}{2\sqrt{iz}+\alpha} \tag{3.30}$$

with the function $w(z)$ defined by

$$\mathrm{w}(u) = e^{-u^2}\,\mathrm{erfc}(-iu) \tag{3.31}$$

(see Ref. [30], Ch. 7.1.1 – 7.1.2). We have also used the formula

$$\int_0^\infty e^{-(ax^2+2bx+c)}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2-ac}{a}\right)\mathrm{erfc}\left(\frac{b}{\sqrt{a}}\right) \tag{3.32}$$

valid for $\Re(a) > 0$ (see [30], Ch. 7.4.2).
To compute $p(t)$ from Eqs. (3.13,3.14) the correct boundary values of the complex square root (with the cut on the negative real half-axis) must be taken. Thus for $z = x+iy$, $x = 0+$ we should take

$$\sqrt{iz} = \begin{cases} i\sqrt{y} & y \geq 0 \\ \sqrt{-y} & y \leq 0 \end{cases} \tag{3.33}$$

$$\sqrt{-iz} = \begin{cases} \sqrt{y} & y \geq 0 \\ -i\sqrt{-y} & y \leq 0 \end{cases} \tag{3.34}$$

---

[13]We should have $|\phi(t)|^2dt = |\phi(\tau)|^2d\tau$

The time of arrival probability curves of the counter for several values of the coupling constant are shown in Fig.2. The incoming wave packet starts at $t = 0$, $x = -4$, with velocity $v = 4$. It is seen from the plot that the average time at which the counter, placed at $x = 0$, is triggered is about one time unit, independently of the value of the coupling constant. This numerical example shows that our model of a counter serves can be used for measurements of time of arrival. It is to be noticed that the shape of the response curve is almost insensitive to the value of the coupling constant. Fig.3 shows the curves of Fig.2, but rescaled in such a way that the probability $P(\infty) = 1$. The only effect of the increase of the coupling constant in the interval $0.01 - 100$ is a slight shift of the response time to the left - which is intuitively clear. Notice that the shape of the curve in time corresponds well to the shape of the initial wave packet in space.
For a given velocity of the packet there is an optimal value of the coupling constant. In our dimensionless units it is $\alpha_{opt} \asymp 2v$. Figure 4 shows this asymptotically linear dependence. At the optimal coupling the total response probability $P(\infty)$ approaches the value 0.5, - the same as in the ultra–relativistic case.
By numerical calculations we have found that the maximal value of $P(\infty)$ that can be obtained for a single Dirac's delta counter and Schrödinger's particle is slightly higher than 0.7, that corresponds to the value $\alpha = 1.3$ of the coupling constant. The dependence of $P(\infty)$ on the coupling constant for a static wave packet (that is $v = 0$) centered exactly over the detector is shown in Fig.5. Fig.6 shows the dependence of $P(\infty)$ on both variables: v and α.

The value 0.7 for the maximal response probability $P(\infty)$ of a detector may appear to be rather strange. It is however connected with the point–like structure of the detector in our simple model. For a composite detector, for instance already for a two–point detector, this restriction does not apply and $P(\infty)$ arbitrarily close to 1.0 can be obtained. Our method applies as well to detectors continuously distributed in space. In this case the efficiency of the detector (for a given initial wavepacket) will depend on the shape of the function $\Lambda(x)$. The absorptive complex potentials studied in [31, 32] are natural candidates for providing maximal efficiency as measured by $P(\infty)$ defined at the end of Sec. 2.2.

4 Concluding Remarks

Our approach to the quantum mechanical measurement problem was originally shaped to a large extent by the important paper by Klaus Hepp [33]. He wrote there, in the concluding section: 'The solution of the problem of measurement is closely connected with the yet unknown correct description of irreversibility in quantum mechanics...'Our approach does not pretend to give an ultimate solution. But it attempts to show that this "correct description" is, perhaps, not too far away.
In the present paper we have only been able to scratch the surface of some of the new mathematical techniques and physical ideas that are enhancing quantum theory in the framework of EEQT, that free the quantum theory from the limitations of the standard formulation. For a long time it was considered that quantum theory is only about averages. Its numerical

predictions were supposed to come *only* from expectation values of linear operators. On the other hand in his 1973 paper [34] Wigner wrote: 'It seems unlikely, therefore, that the superposition principle applies in full force to beings with consciousness. If it does not, or if the linearity of the equations of motion should be invalid for systems in which life plays a significant role, the determinants of such systems may play the role which proponents of the hidden variable theories attribute to such variables. All proofs of the unreasonable nature of hidden variable theories are based on the linearity of the equations ...'. Weinberg [35] attempted to revive and to implement Wigner's idea of non-linear quantum mechanics. He proposed a nonlinear framework and also methods of testing for linearity. Warnings against potential dangers of nonlinearity are well known, they were summarized in a recent paper by Gisin and Rigo [36]. The scheme of EEQT avoids these pitfalls and presents a consistent and coherent theory. It introduces necessary nonlinearity in the algorithm for generating sample histories of individual systems, but preserves linearity on the ensemble level. It is not only about averages but also about individual events (cf. the event generating PDP algorithm of ref. [6]). Thus it explains more, it predicts more and it opens a new gateway leading beyond today's framework and towards new applications of Quantum Theory. These new applications may involve the problems of consciousness. But in our opinion (supported in the all quoted papers on EEQT, and also in the present one) quantum theory does not need neither consciousness nor human observers - at least not more than any other probabilistic theory. On the other hand, to understand mind and consciousness we may need Event Enhanced Quantum Theory. And more.

In the abstract to the present paper we stated that we "enhance elementary quantum mechanics with three simple postulates". In fact the PDP algorithm *replaces* the standard measurement postulates and enables us to *derive* them in a refined form. This is because EEQT *defines* precisely what *measurement* and *experiment* is - without any involvement of consciousness or of human observers. It is only for the purpose of the present paper - to introduce time observable into elementary quantum mechanics as simply as possible - that we have chosen to present our three postulates as postulates rather than theorems. The time observable that we introduced and investigated in the present paper is just one (but important) trace of this nonlinearity.[14] Time of arrival, time of detector response, is an "observable", is a random variable whose probability distribution function can be computed according to the prescription that we gave in the previous section. But its probability distribution is not a bilinear functional of the state and as a result "time of arrival" can not be represented by a linear operator, be it Hermitian or not. Nevertheless our "time" of arrival is a "safe" nonlinear observable. Its safety follows from the fact that what we called "postulates" in the present paper are in fact "theorems" of the general scheme EEQT. And EEQT is the *minimal* extension of quantum mechanics that accounts for events: no extra unnecessary hidden variables, and linear Liouville equation for ensembles.

Our definition of time of arrival bears some similarity to the one proposed long ago by Allcock [40]. Although we disagree in several important points with the premises and conclusions of this paper, nevertheless the detailed analysis of some aspects of the problem given by Allcock was prompting us to formulate and to solve it using the new perspective and the

[14] This is why our time observable does not contradict the well known objections by Pauli [37]. Cf. also the discussion in a recent book by Bush, Grabowski and Lahti [38]). For the same reason it does not fall into the family analysed axiomatically by Kijowski [39].

new tools that EEQT endowed us with. Our approach to the problem of time of arrival goes in a similar direction as the one discussed in an (already quoted) interesting recent paper by Muga and co–workers [32]. We share many of his views. The difference being that what the authors of [32] call "operational model" we promote to the role of a fundamental new postulate of quantum theory. We justify it and point out that it is a theorem of a more fundamental theory - EEQT. Moreover we take the non–unitary evolution before the detection event seriously and point out that the new theory is experimentally falsifiable.
Once the time of arrival observable has been defined, it is rather straightforward to apply it. In particular our time observable solves Mielnik's "waiting screen problem" [41]. But not only that; with our **precise** definition at hand, one can approach again the old puzzle of time–energy uncertainty relation in the spirit of Wigner's analysis [42] (cf. also [43, 44]. One can also approach afresh the other old problem: that of decay times (see [45] and references therein) and of tunneling times ([46, 47, 48] and references therein). This last problem needs however more than just one detector. We need to analyse the joint distribution probability for two separated detector. We must also know how to describe the unavoidable disturbance of the wave function when the first detector is being triggered. For this the simple postulates of this paper do not suffice. But the answer is in fact quite easy if using the event generating algorithm of EEQT.
More investigations needs also our "shadowing effect" of section 2.3. Every "real" detector acts not only as an information exchange channel, but also as an energy–momentum exchange channel. Every real detector has not only its "information temperature" described by our coupling constant λ (cf. Sec. 2.1), but also ordinary temperature. Experiments to test the effect must take care in separating these different contributions to the overall phenomenon. This is not easy. But the theory is falsifiable in the laboratory and critical experiments might be feasible within the next couple of years.
In the introductory chapter the problem of extension of the present framework to the relativistic case has been shortly mentioned. Work in this direction is well advanced and we hope to be able to report its result soon. But this will not be end of the story. At the very least we have much to learn about the nature and the mechanism of the coupling between Q and C.[15]

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[15] More comments in this direction can be found in [6] and also under WWW address of the Quantum Future Project: http://www.ift.uni.wroc. pl/~ajad/qf.htm

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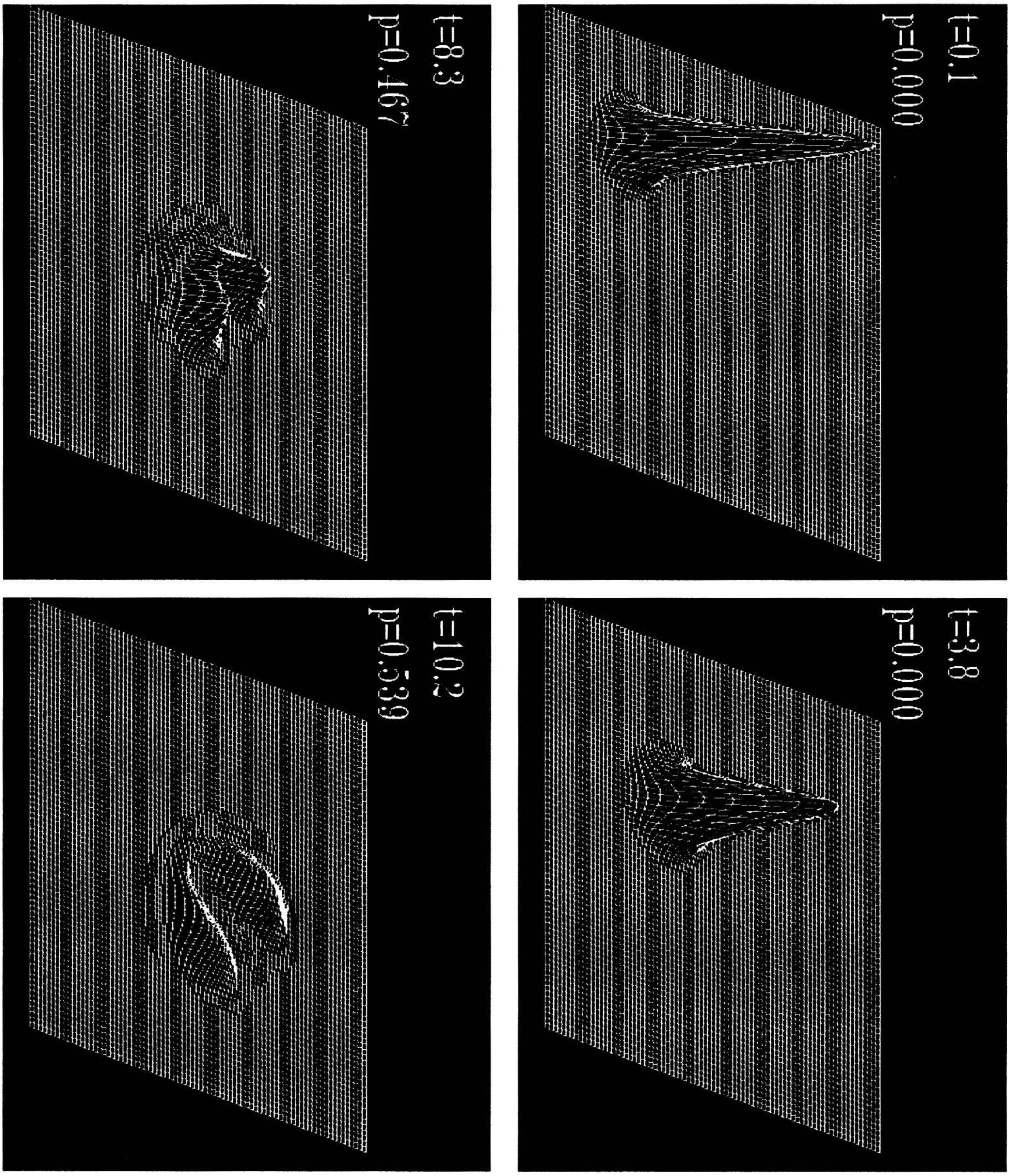

Figure 1: Four shots from the time evolution of a gaussian wavepacket monitored by a gaussian detector placed at the center of the plane. The efficiency of the detector is in this case ca. $P(\infty) \simeq 0.55$.

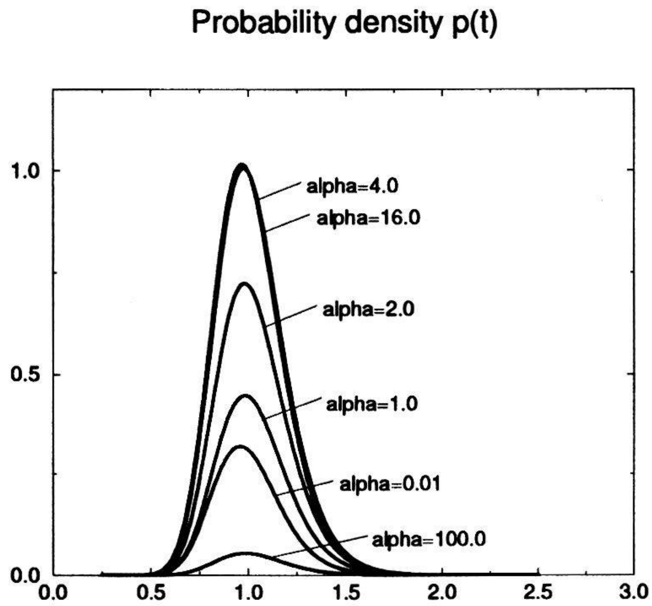

Figure 2: Probability density of time of arrival for a Dirac's delta counter placed at $x = 0$, coupling constant alpha. The incoming wave packet starts at $t = 0$, $x = -4$, with velocity $v = 4$

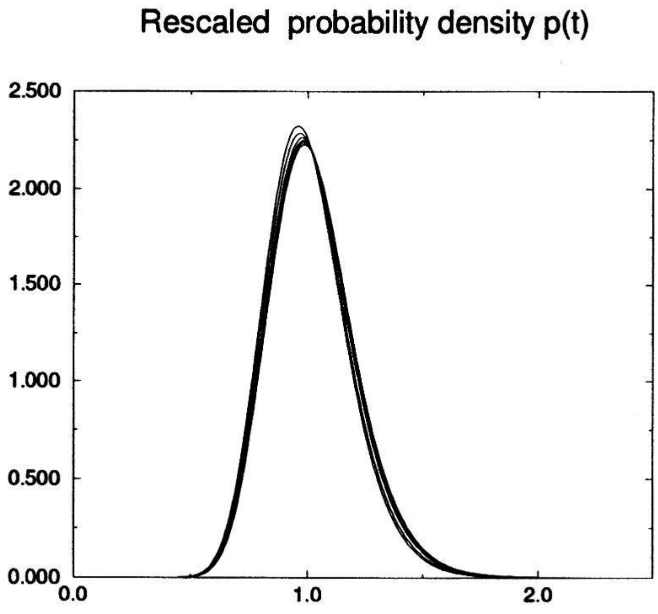

Figure 3: Rescaled probability densities of Fig.1

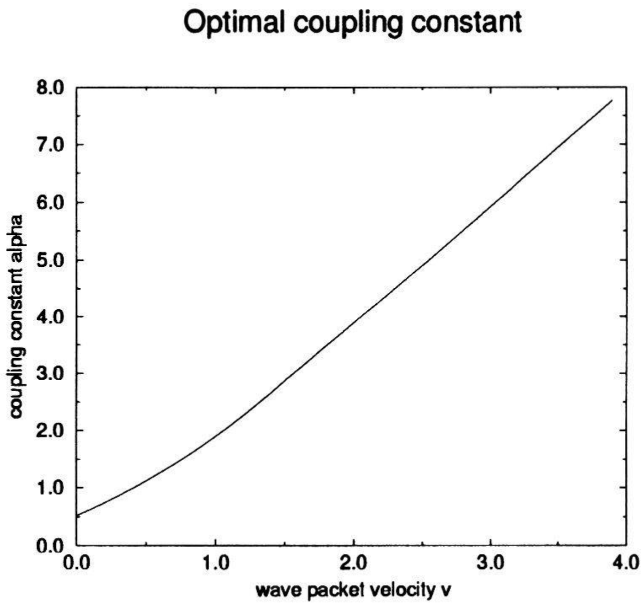

Figure 4: Optimal coupling constant as a function of velocity of the incoming wave packet. The dependence pretty soon saturates to a linear one. At the saturation value $P(\infty) \asymp 0.5$.

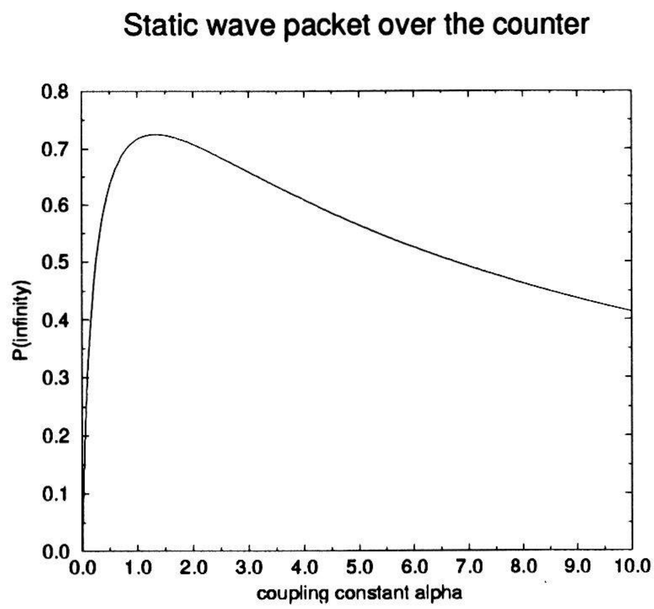

Figure 5: $P(\infty)$ as a function of α for a static wave packet centered over the counter. The maximum, of $P(\infty) = 0.725448$ is reached for $\alpha = 1.3216$.

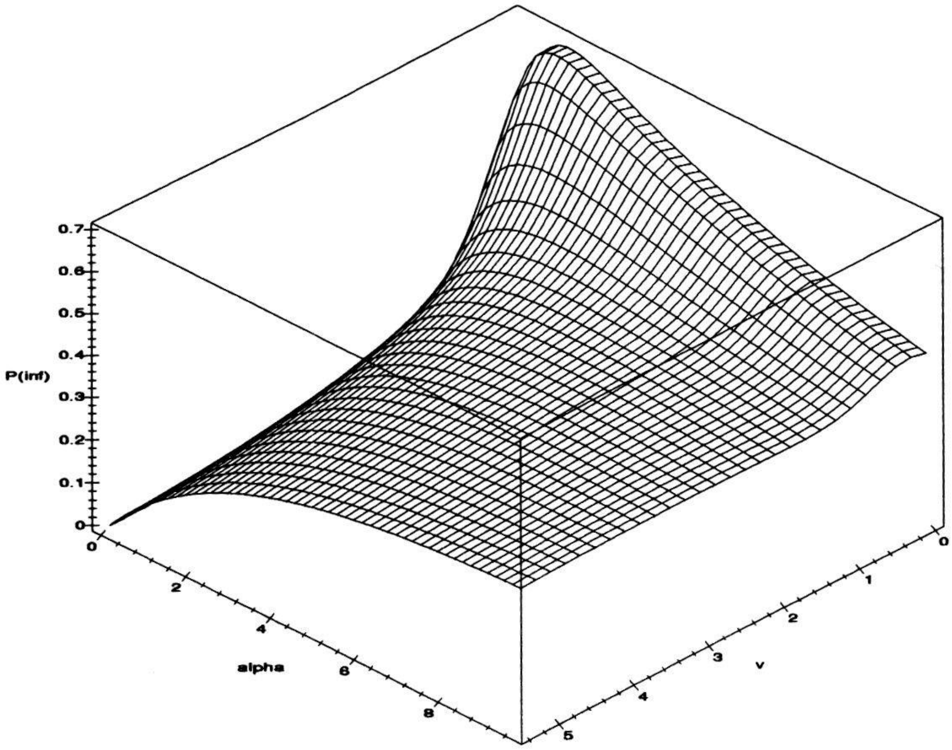

Figure 6: $P(\infty)$ as a function of velocity v and coupling constant α for a static wave packet centered over the detector.