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# String Cosmology: Concepts and Consequences

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*Abstract.* I discuss the main ideas/assumptions underlying string cosmology and show how they lead to a two-parameter family of "minimal" models. I then outline how the spectrum of scalar, tensor and electromagnetic perturbations are related to those parameters, and mention their most relevant physical consequences. Finally, I briefly report on recent progress on the exit problem in string cosmology.

## 1 Basic facts in quantum string theory (QST)

I am listing below a few basic properties of strings, emphasizing those that are most relevant to our subsequent discussion:

**1.** Unlike its classical counterpart, quantum string theory contains a fundamental length scale $\lambda_s$ representing [1] the ultraviolet, short-distance cut-off (equivalently, a high-momentum cut-off at $E = M_s \equiv \lambda_s^{-1}$).

**2.** Tree-level masses are either zero or of $O(M_s)$. Quantum mechanics allows massless strings with non-zero angular momentum [2] while, classically, $M^2 >$ const. $\times J$. The existence of such states is obviously a crucial property of QST, without which it could not pretend to be a candidate theory of all known interactions.

**3.** The effective interaction of the massless fields at $E \ll M_s$ takes the form of a classical, gauge-plus-gravity field theory with specified parameters. It is described by an effective action [3],[4] of the (schematic) type:

$$\Gamma_{eff} = \frac{1}{2}\int d^4x\sqrt{-g}\, e^{-\phi}\left[\lambda_s^{-2}(\mathcal{R} + \partial_\mu\phi\partial^\mu\phi) + F_{\mu\nu}^2 + \bar{\psi}\not{D}\psi + R^2 + \ldots\right] + \left[\text{higher orders in } e^{\phi}\right] . \quad (1.1)$$

Equation (1.1) contains two dimensionless expansion parameters. One of them, $g^2 \equiv e^{\phi}$, controls the analogue of QFT's loop corrections, while the other, $\lambda^2 \equiv \lambda_s^2 \cdot \partial^2$, controls string-size effects, which do not, of course, exist in QFT.

**4.** As indicated in (1.1), QST has (actually needs!) a new particle/field, the so-called dilaton $\phi$, a scalar massless particle (at the perturbative level). It appears in $\Gamma_{eff}$ as a Jordan–Brans–Dicke [5] scalar with a "small" negative $\omega_{BD}$ parameter, $\omega_{BD} = -1$.

**5.** The dilaton's VEV provides [6] a unified value for:

a) The gauge coupling(s) at $E = O(M_s)$.

b) The gravitational coupling in string units.

c) Yukawa couplings, etc., at the string scale.

In formulae:

$$\ell_p^2 \equiv 8\pi G_N \hbar = e^{\phi}\lambda_s^2 ,$$
$$\alpha_{GUT}(\lambda_s^{-1}) \simeq \frac{e^{\phi}}{4\pi} , \quad (1.2)$$

implying (from $\alpha_{GUT} \approx 1/20$) that the string-length parameter $\lambda_s$ is about $10^{-32}$ cm. Note, however, that, in a cosmological context in which $\phi$ evolves in time, the above formulae can only be taken to give the *present* values of $\alpha$ and $\ell_p/\lambda_s$. In the scenario we will advocate, both quantities were much smaller in the very early Universe!

**6.** Dilaton couplings at large distance are such [7] that a massless dilaton is most likely ruled out [7],[8] by precision tests [9] of the equivalence principle, i.e.

$$M_\phi > 10^{-4} \text{ eV} . \quad (1.3)$$

**7.** Details about the dilaton potential are unknown, yet:

a) On theoretical grounds, in critical superstring theory, the dilaton potential has to go to zero as a double exponential as $\phi \to -\infty$ (weak coupling):

$$V(\phi) \sim \exp\left(-c\exp(-\phi)\right) = \exp\left(-\frac{c}{4\pi\alpha_{GUT}}\right) , \quad (1.4)$$

with $c$ a positive (but model-dependent) constant.

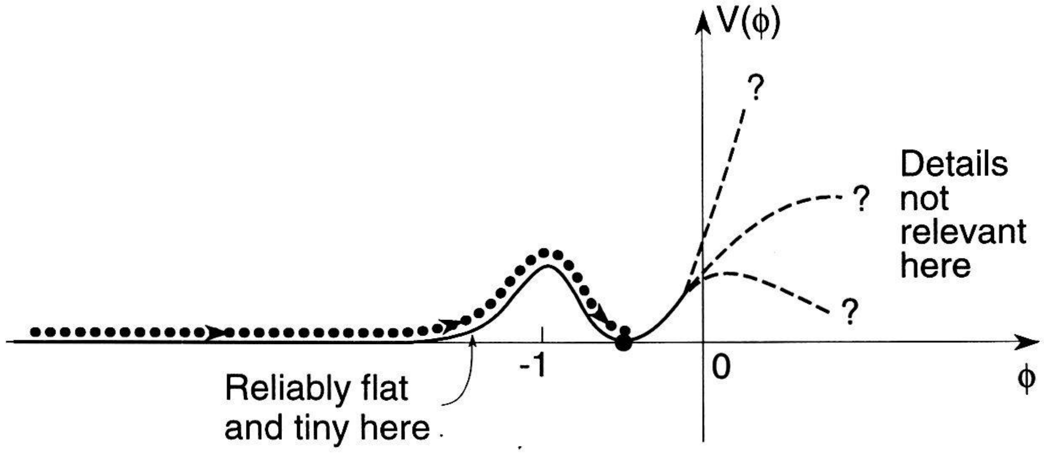


Figure 1: A possible dilaton potential with illustration of an inflation-driving rolling dilaton (large dots).

b) On physical grounds it should have a non-trivial minimum at its present value ($\langle\phi\rangle = \phi_0 \sim 0$) with a vanishing cosmological constant, $V(\phi_0) = 0$.

A typical potential satisfying a) and b) is shown in Fig. 1. The dotted lines at $\phi > 0$ represent our ignorance about strongly coupled string theory. Fortunately, the details of what happens in that region will not be very relevant for our subsequent discussion.

**8.** There is an exact (all-order) vacuum solution for (critical) superstring theory. Unfortunately, it corresponds to a free theory ($g = 0$ or $\phi = -\infty$) in flat, ten-dimensional, Minkowski space-time, nothing like the world we seem to be living in!

Before closing this section I would like to comment briefly on a point that appears to be the source of much confusion, even among experts: it is the debate between working in the (so-called) string and Einstein "frames" (not to be confused with different coordinate systems). Since the two frames are related by a local field redefinition (a conformal, dilaton-dependent rescaling of the metric) all physical quantities are independent of the frame. The question is: What should we call the metric? Although, to a large extent, this is a question of taste, one's intuition may work better with one definition than with another. Note also that, since the dilaton is time-independent today, the two frames now coincide.

Let us compare advantages and drawbacks in each frame.

A) **String Frame.** This is the metric appearing in the fundamental (Polyakov) action for the string. Classical, weakly coupled strings sweep geodesic surfaces with respect to this metric. Also, the dilaton dependence of the low energy effective action takes the simple form indicated in (1.1) only in the string frame. The advantage of this frame is that the string cut-off is fixed and the same is true for the value of the curvature at which higher orders in the $\sigma$-model coupling $\lambda$ become relevant. The main disadvantage is that the gravitational action is not so easy to work with.

B) **Einstein Frame.** In this frame the pure gravitational action takes the standard Einstein–Hilbert form. Consequently, this is the most convenient frame for studying the cosmological evolution of metric perturbations. The Planck length is fixed in this frame while the string length is dilaton- (hence generally time-) dependent. In the Einstein frame $\Gamma_{eff}$ takes the form:

$$\begin{aligned}\Gamma_{eff} &= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left[R+\partial_\mu\phi\partial^\mu\phi+e^{-\phi}F_{\mu\nu}^2+\partial_\mu A\partial^\mu A+e^{\phi}m^2A^2\right]\\ &\quad+\left[G_N e^{-\phi}R^2+\ldots\right], \qquad (1.5)\end{aligned}$$

showing that, in this frame, masses are dilaton-dependent (even at tree level) and so is the value of $R$ at which higher-order stringy corrections become important. It is for the above reasons that I will choose to base my discussion (although not always the calculations) in the string frame.

# 2 Main ideas/assumptions of string cosmology

The very basic postulate of (our own version of) string cosmology [10], [11] is that the Universe did indeed start near its trivial vacuum mentioned at the end of the previous section.

Fortunately, if one looks at the space of homogeneous (and for simplicity spatially-flat) perturbative vacuum solutions, one finds that the trivial vacuum is a very special, *unstable* solution. This is depicted in Fig. 2a for the simplest case of a ten-dimensional cosmology in which three spatial dimensions evolve isotropically while six "internal" dimensions are static (it is easy to generalize the discussion to the case of dynamical internal dimensions, but then the picture becomes multidimensional).

The straight lines in the $H, \dot{\bar{\phi}}$ plane (where $\dot{\bar{\phi}} \equiv \dot{\phi} - 3H$) represent the evolution of the scale factor and of the coupling constant as a function of the cosmic time parameter (arrows along the lines show the direction of the time evolution). As a consequence of a stringy symmetry [10], [12], known as "Scale Factor Duality (SFD)", there are two branches (two straight lines). Furthermore, each branch is divided by the origin in two time-reversal-related parts. In the general case of a generic Bianchi I cosmology including an antisymmetric tensor field $B_{\mu\nu}$, SFD is part of a much larger non-compact continuous $O(d,d)$ symmetry of the field equations [13].

It is straightforward to check that the low-energy effective action implies

$$\ddot{\bar{\phi}} = 3H^2 \geq 0 \;, \qquad (2.1)$$

showing that the flow in Fig. 2a can only be upward (at low curvature). The origin (the trivial vacuum) is an "unstable" fixed point: a small perturbation in the direction of positive $\dot{\bar{\phi}}$ makes the system evolve further and further from the origin, meaning larger and larger

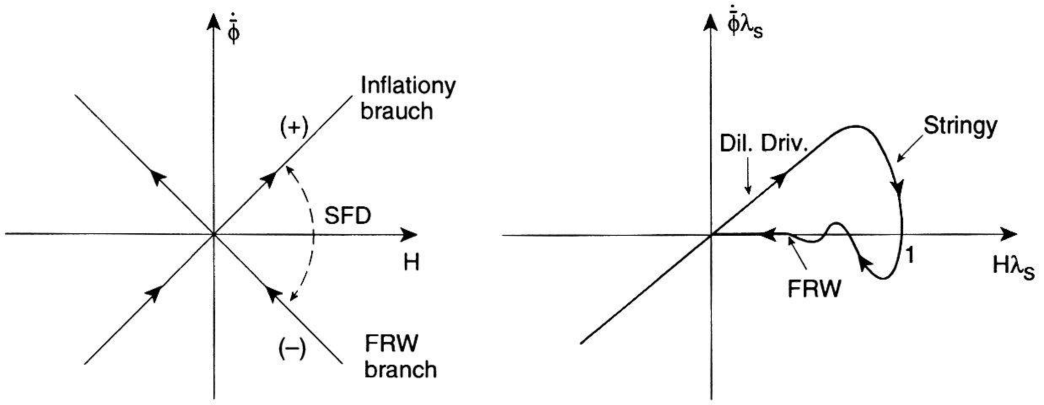


Figure 2: Phase diagrams for the perturbative (a) regime and a conjectured non-perturbative solution (b) to the branch-change problem.

coupling and absolute value of the Hubble parameter. This means an accelerated expansion or an accelerated contraction, i.e. in the latter case, inflation. It is tempting to assume that those patches of the original Universe that had the right kind of fluctuation have grown up to become (by far) the largest fraction of the Universe today.

In order to arrive at a physically interesting scenario, however, we have to somehow connect the top-right inflationary branch to the bottom-right branch, since the latter is nothing but the standard FRW cosmology, which has presumably prevailed for the last few billion years or so. Here the so-called "exit problem" arises. At lowest order in $\lambda^2$ (small curvatures in string units) the two branches do not talk to each other. The inflationary (also called +) branch has a singularity in the future (it takes a finite cosmic time to reach $\infty$ in our graph, if one starts from anywhere but the origin), while the FRW $(-)$ branch has a singularity in the past (the usual big-bang singularity).

It is widely believed that QST has a way to avoid the usual singularities of Classical General Relativity or at least a way to reinterpret them [14],[15]. It thus looks reasonable to assume that the inflationary branch, instead of leading to a non-sensical singularity, will evolve into the FRW branch at values of $\lambda^2$ of order unity. This is schematically shown in Fig. 2b, where we have gone back from $\dot{\bar{\phi}}$ to $\dot{\phi}$ and we have implicitly taken into account the effects of a non-vanishing dilaton potential at small $\phi$ in order to freeze the dilaton at its present value. The need for the branch change to occur at large $\lambda^2$, first argued for in [16], has recently been proved in [17]. Some recent progress on this crucial issue will be described in Section 5.

There is a rather simple way to parametrize a class of scenarios of the kind defined above. They contain (roughly) three phases and two parameters. Indeed:

In phase I the Universe evolves at $g^2, \lambda^2 \ll 1$ and is thus close to the trivial vacuum. This phase can be studied using the tree-level low-energy effective action (1.1) and is characterized by a long period of dilaton-driven inflation. The accelerated expansion of the Universe, instead of originating from the potential energy of an inflaton field, is driven by the growth of the coupling constant (i.e. by the dilaton's kinetic energy, see ref. [18] for a similar kind of inflationary scenario) with $\dot{\phi} = 2\dot{g}/g \sim H$ during the whole phase.

Phase I supposedly ends when the coupling $\lambda^2$ reaches values of $O(1)$, so that higher-derivative terms in the effective action become relevant. Assuming that this happens while $g^2$ is still small (and thus the potential is still negligible), the value $g_s$ of $g$ at the end of phase I (the beginning of phase II) is an arbitrary parameter (a modulus of the solution)..

During phase II, the stringy version of the big bang, the curvature as well as $\dot{\phi}$ are assumed to remain fixed at their maximal value given by the string scale (i.e. we expect $\lambda \sim 1$). The coupling $g$ will instead continue to grow from the value $g_s$ until it is its own turn to reach values of $O(1)$. At that point, assuming a branch change to have occurred as a result of large curvatures and/or coupling, the dilaton will be attracted to the true non-perturbative minimum of its potential; the standard FRW cosmology can then start, provided the Universe was heated-up and filled with radiation (this is not a problem, see end of Sect. 3). The second important parameter of this scenario is the duration of phase II or better the total red-shift, $z_s \equiv a_{end}/a_{beg}$, which has occurred from the beginning to the end of the stringy phase.

Our present ignorance about this most crucial phase (and in particular about the way the exit can be implemented) prevents us from having a better description of this phase which, in principle, should not introduce new arbitrary parameters ($z_s$ should be eventually determined in terms of $g_s$).

During Phase III, the Universe evolves towards smaller and smaller curvatures, but stays at moderate-to-strong coupling. This is the regime in which usual QFT methods are applicable. The details of the particular gauge theory emerging from the string's non-perturbative vacuum will be very important in determining the subsequent evolution, and in particular the problem of structure formation, dark matter and the like.

Our scenario contains implicitly an arrow of time, which points in the direction of increasing entropy, inhomogeneity and structure. As a result of the amplification of primordial vacuum fluctuations, the Universe is *not* coming back to its initial simple (and unique) state (the origin in Fig. 2), but to the much more structured (and interesting) state in which we are living today. Actually, the arrow of time itself should be determined by the direction in which entropy is growing. This will force us to identify (*by definition*) the perturbative vacuum with the initial state of the Universe!

# 3 Observable consequences

All the observable consequences I will discuss below have something to do with the well-known phenomenon [19] of amplification of vacuum quantum fluctuations in cosmological backgrounds. Any conformally flat cosmological background is known:

a) to amplify tensor perturbations, i.e. to produce a stochastic background of gravitational waves;

b) to induce scalar-metric perturbations from the coupling of the metric either to a fluid or to scalar particles (in our context to the dilaton).

By contrast, because of the scale-invariant coupling of gauge fields in four dimensions, electromagnetic (EM) perturbations are *not* amplified in a conformally flat cosmological background (even if inflationary). In string cosmology, the presence of a time-dependent dilaton in front of the gauge-field kinetic term yields, on top of the two previously mentioned effects,

c) an amplification of EM perturbations corresponding to the creation of macroscopic magnetic (and electric) fields.

Various physically interesting questions arise in connection with the three effects I have just mentioned. These include the following:

1. Does the Universe remain quasi-homogeneous during the whole string-cosmology history?
2. Does one generate a phenomenologically interesting (i.e. measurable) background of GW?
3. Can one produce large enough seeds for generating the observed galactic (and extragalactic) magnetic fields?
4. Can scalar, tensor (and possibly EM) perturbations explain the large-scale anisotropy of the CMB observed by COBE?
5. Do these perturbations have anything to do with the CMB itself?

In the following I will give—without derivation—some partial answers to each one of the above questions, referring to the literature [20] for the all-important missing details.

1. **Does the Universe remain quasi-homogeneous throughout the whole string-cosmology history?**

   The answer to this question turns out to be yes! This is not a priori evident since, in commonly used gauges [21] for scalar perturbations of the metric (e.g. the so-called longitudinal gauge in which the metric remains diagonal), such perturbations appear to grow very large during the inflationary phase and to destroy homogeneity or, at least, to prevent the use of linear perturbation theory.

   In ref. [22] it was shown that, by a suitable choice of gauge (an "off-diagonal" gauge), the growing mode of the perturbation can be tamed. The bottom line is that scalar perturbations in string cosmology behave no worse than tensor perturbations, to which we now turn our attention.

2. **Does one generate a phenomenologically interesting (i.e. measurable) background of GW?**

The canonical variable $\psi$ for tensor perturbations (i.e. for GW) is defined by:

$$g_{\mu\nu} = a^2[\eta_{\mu\nu} + h_{\mu\nu}]$$
$$\psi = (a/g)\ h = a\ e^{-\phi/2} h\ , \tag{3.1}$$

where $h$ stands for either of the two transverse-traceless polarizations of the gravitational wave. As long as the perturbation is inside the horizon, $\psi$ remains constant while $h$ is adiabatically damped. By contrast, outside the horizon, $\psi$ is amplified according to

$$\psi_k \sim (a/g) \left[ C_k + D_k \int_{\eta_{ex}}^{\eta} d\eta'\ g^2(\eta')\ a^{-2}(\eta') \right] \tag{3.2}$$

where, for each Fourier mode of (comoving) wave number $k$, $\eta_{ex} = k^{-1}$.

The first term in (3.2) clearly corresponds to the freezing of $h$ itself, while the second term represents the freezing of its associated canonical momentum. In standard (non-dilatonic) inflationary models, the first term dominates since $a$ grows very fast. In our case, the second term dominates since the growth of $a$ is over-compensated by the growth of $g$ (i.e. of $\phi$). This is equivalent to saying that, in the Einstein frame, our background describes a contracting Universe.

After matching the result (3.2) with the usual oscillatory, damped behaviour of the radiation-dominated epoch, one arrives at the final result [23] for the predicted stochastic background of GW today. The energy density per logarithmic interval of frequency is given, in critical units by:

$$\frac{d\Omega_{GW}}{d\ln\omega} = z_{eq}^{-1} (g_s)^2 \left(\frac{\omega}{\omega_s}\right)^3 \left[ \ln\left(\frac{\omega_s}{\omega}\right) + (z_s)^{-3} \left(\frac{g_s}{g_1}\right)^{-2} \right]^2\ ,\ \omega < \omega_s\ . \tag{3.3}$$

where $\omega = k/a$ is the proper frequency, $z_{eq} \sim 10^4, \omega_s \sim z_s^{-1}(g_1)^{1/2} \times 10^{11}$ Hz $\equiv z_s^{-1}\omega_1$ and $g_1$ is the present value of the string coupling.

In Fig. 3 we show the spectrum of stochastic gravitational waves expected from our two-parameter model. For a given pair $g_s, z_s$ one identifies a point in the $\omega, \delta h_\omega$ plane as illustrated explicitly in the case of $g_s = 10^{-3}, z_s = 10^6$. The resulting point (indicated by a large dot) represents the end-point $\omega_s, \delta h_{\omega_s}$ of the $\omega^{1/2}$ spectrum corresponding to scales crossing the horizon during the dilatonic era.

Although the rest of the spectrum is more uncertain, one can argue that it has to join smoothly the point $\omega_s, \delta h_{\omega_s}$ to the true end-point $\delta h \sim 10^{-30}, \omega \sim 10^{11}$Hz. The latter corresponds to a few gravitons produced at the maximal amplified frequency $\omega_1$, the last scale to go outside the horizon during the stringy phase. The full spectrum is also shown in the figure for the case $g_s = 10^{-3}, z_s = 10^6$, with the wiggly line representing the less well known high frequency part.

Curves of constant $\Omega_{GW}$ are also shown. If $g_s < 1$, as we have assumed, spectra will always lie below the $\Omega_{GW} = 10^{-4}$ line corresponding to a production of as many

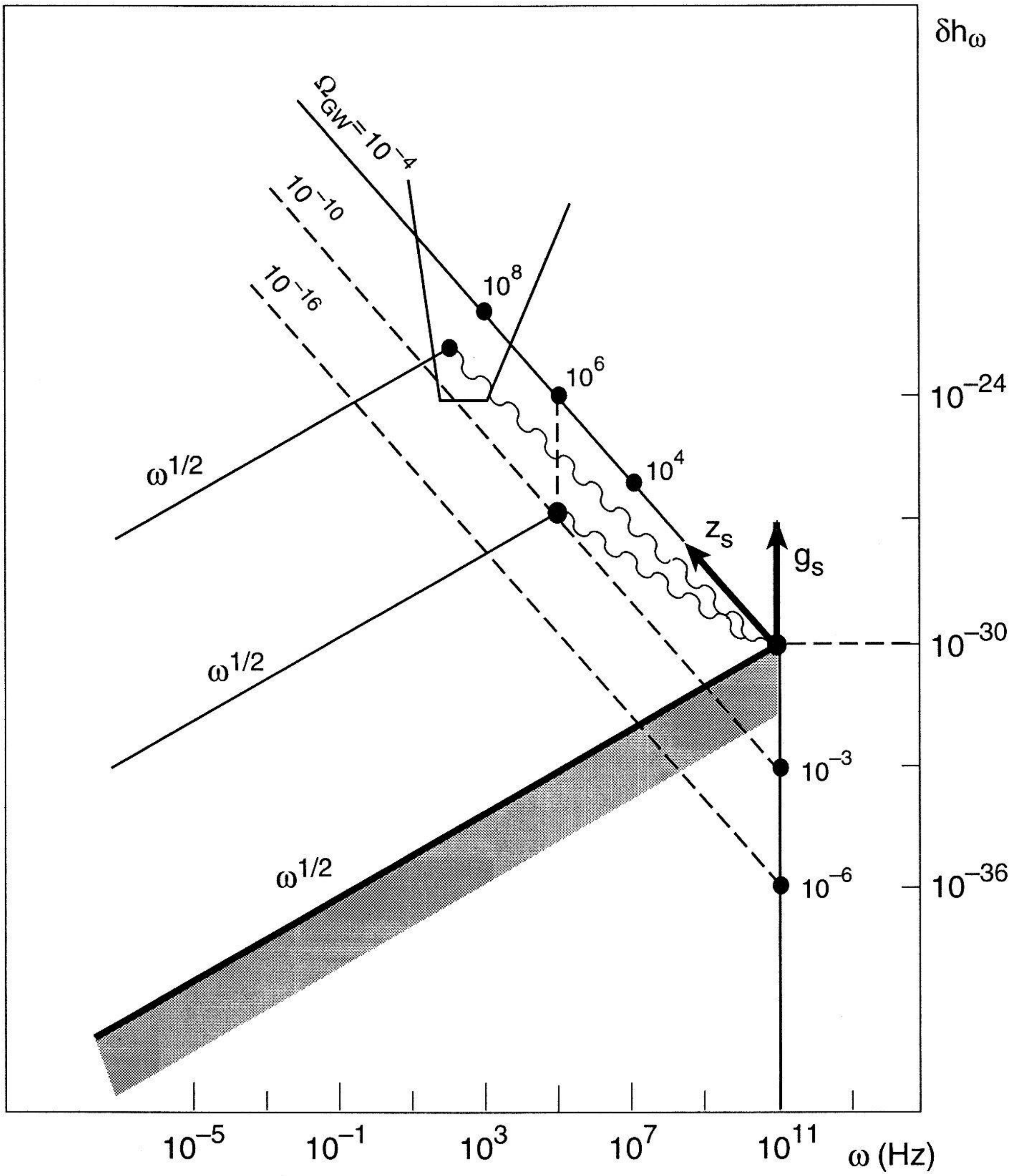


Figure 3: Gravitational-wave spectrum against the expected sensitivity of advanced LIGO-VIRGO.

photons as gravitons. On the other hand, since the actual spectrum (3.3) contains two duality-related contributions, it will never lie below the self-dual spectrum ending at $\delta h \sim 10^{-30}, \omega \sim 10^{11}$ Hz (the thick line bordering the shaded region). In conclusion all possible spectra sweep the angular wedge inside the two above-mentioned lines.

The odd-shaped region in Fig. 3 shows the expected sensitivity of the so-called "Advanced LIGO" (and possibly also VIRGO) project [24]. While there is no hope to detect our spectrum at LIGO if $g_s = 10^{-3}, z_s = 10^6$, perspectives would be better for, say, $g_s = 10^{-1}, z_s = 10^8$ (see Ref. [25] for a more complete analysis of LIGO's sensitivity).

Resonant bars might also be able to reach comparable sensitivity in the kHz region [26], while microwave cavities, if conveniently developed, could be used in the region $10^6$–$10^9$ Hz [27].

3. **Can large enough seeds be produced for the generation of observed galactic (and extragalactic) magnetic fields?**

As already mentioned, seeds for generating the galactic magnetic fields through the so-called cosmic dynamo mechanism [28] can be generated in our scenario by the amplification of the quantum fluctuations of the EM field. In this case the canonical variable is just the (Fourier transform of the) usual $A_\mu$ potential. In analogy with (3.2), its amplification, while outside the horizon, is described by the asymptotic solution:

$$A_k \sim g^{-1}\left[C_k + D_k \int_{\eta_{ex}}^{\eta} d\eta' g^2(\eta')\right] \ , \tag{3.4}$$

which leads [29],[30] to an overall amplification of the electromagnetic field by a factor $|c_k|^2 \sim (g_{re}/g_{ex})^2 + (g_{ex}/g_{re})^2$. Note that the spectrum is invariant under $g \to g^{-1}$, i.e. under ordinary electric-magnetic duality. In our cosmological scenario we have excluded the possibility of a decreasing coupling constant and, therefore, the main contribution to the amplification comes from the second term on the r.h.s. of eq. (3.4), which gives $|c_k|^2 \sim (g_{re}/g_{ex})^2$.

One can express this result in terms of the fraction of electromagnetic energy stored in a unit of logarithmic interval of $\omega$ normalized to the one in the CMB, $\rho_\gamma$. One finds:

$$r(\omega) = \frac{\omega}{\rho_\gamma}\frac{d\rho_B}{d\omega} \simeq \frac{\omega^4}{\rho_\gamma}|c_-(\omega)|^2 \equiv \frac{\omega^4}{\rho_\gamma}(g_{re}/g_{ex})^2 \ . \tag{3.5}$$

The ratio $r(\omega)$ stays constant during the phase of matter-dominated as well as radiation-dominated evolution, in which the Universe behaves like a good electromagnetic conductor [31]. In terms of $r(\omega)$ the condition for seeding the galactic magnetic field through ordinary mechanisms of plasma physics is [31]

$$r(\omega_G) \geq 10^{-34} \tag{3.6}$$

where $\omega_G \simeq (1 \text{ Mpc})^{-1} \simeq 10^{-14}$ Hz is the galactic scale. Using the known value of $\rho_\gamma$, we thus find, from (3.5, 3.6):

$$g_{ex} < 10^{-33} \ , \tag{3.7}$$

i.e. a very tiny coupling at the time of exit of the galactic scale.

The conclusion is that string cosmology stands a unique chance in explaining the origin of the galactic magnetic fields. Indeed, if the seeds of the magnetic fields are to be attributed to the amplification of vacuum fluctuations, their present magnitude can be interpreted as prime evidence that the fine structure constant has evolved to its present value from a tiny one during inflation. The fact that the needed variation of the coupling constant ($\sim 10^{30}$) is of the same order as the variation of the scale factor needed to solve the standard cosmological problems, can be seen as further evidence for scenarios in which coupling and scale factor grow roughly at the same rate during inflation.

4. **Can scalar, tensor (and possibly EM) perturbations explain the large-scale anisotropy of the CMB observed by COBE?**

The answer here is certainly negative as far as scalar and tensor perturbations are concerned. The reason is simple: for spectra that are normalized to $O(1)$ (at most) at the maximal amplified frequency $\omega_1 \sim 10^{11}$ Hz, and that grow like $\omega^{1/2}$, one cannot have any substantial power at the scales $O(10^{-18}$ Hz) to which COBE is sensitive. The origin of $\Delta T/T$ at large scale would have to be attributed to other effects (e.g. to the electromagnetic perturbations themselves [32] or to topological defects).

5. **Do all these perturbations have anything to do with the CMB itself?**

Stated differently, this is the question of how to arrive at the hot big bang of the SCM starting from our "cold" initial conditions. The reason why a hot Universe can emerge at the end of our inflationary epochs (phases I and II) goes back to an idea of L. Parker [33], according to which amplified quantum fluctuations can give origin to the CMB itself if Planckian scales are reached.

Rephrasing Parker's idea in our context amounts to solving the following bootstrap-like condition: at which moment, if any, will the energy stored in the perturbations reach the critical density?

The total energy density $\rho_{qf}$ stored in the amplified vacuum quantum fluctuations is given by:

$$\rho_{qf} \sim N_{eff}\, \frac{M_s^4}{4\pi^2}\, (a_1/a)^4 \ , \tag{3.8}$$

where $N_{eff}$ is the number of effective (relativistic) species, which get produced (whose energy density decreases like $a^{-4}$) and $a_1$ is the scale factor at the (supposed) moment of branch-change. The critical density (in the same units) is given by:

$$\rho_{cr} = e^{-\phi} M_s^2 H^2 \ . \tag{3.9}$$

At the beginning, with $e^{\phi} \ll 1$, $\rho_{qf} \ll \rho_{cr}$ but, in the $(-)$ branch solution, $\rho_{cr}$ decreases faster than $\rho_{qf}$ so that, at some moment, $\rho_{qf}$ will become the dominant sort of energy while the dilaton kinetic term will become negligible. It would be interesting to find out what sort of initial temperatures for the radiation era will come out of this assumption.

# 4 Recent progress on the exit problem

Three lines have recently been followed in order to tackle this most important issue. They can be characterized by the mechanism invoked for the exit as follows:

a) Higher derivatives

b) Loop corrections

c) Quantum cosmology à la Wheeler–DeWitt

Because of lack of space-time I will talk mostly about the last (and probably also least) mechanism, also because this is where most of the activity has been in recent months. I will first say, however, a few words about the first two mechanisms which are probably the most relevant but also the hardest to analyze.

a) **Exit via higher derivatives:** The idea is to justify the strong curvature transition from the dilatonic to the string phase by proving the existence of an exact De Sitter-like solution to the field equation, which acts as a late time attractor for the perturbative super-inflationary branch. Some old negative results in this direction [34] were shown [35] to be evaded at next-to-leading order if one allows for a dynamical dilaton. Preliminary results [36] also look very encouraging vis-à-vis the all-order problem and the evolution from pre-big-bang initial conditions.

b) **Exit via loop corrections:** The idea here is to invoke the back reaction from particle production as the relevant mechanism. Since the back reaction is an $O(e^{\phi}\alpha' H^2)$ correction, its effect is contained in one-loop $O(R^2)$ contributions to the effective action. A class of such contributions were analysed by Antoniadis et al. [37] in the case of a spatially flat ($k = 0$) cosmology and more recently by Easther and Maeda [38] in the case of a closed Universe ($k = 1$). Both groups find non-singular solutions to the loop-corrected field equations. However, neither group is actually able to obtain solutions that start in the weak-coupling dilaton-driven superinflationary regime and later evolve through a branch change.

   In order to study a fully analytic model, Rey [39] has recently considered the same problem in the context of two-dimensional dilaton gravity models à la CGHS [40], models previously considered as toy models for the information paradox in black hole physics. His work was later clarified and extended in ref. [41]. Starting with pre-big-bang-type initial conditions one finds that the lowest-order singularity is avoided while the dilaton keeps growing indefinitely. No branch change actually occurs. This is why, even after the introduction of a dilaton potential, the dilaton is not attracted by its minima and the exit problem remains unsolved.

   My feeling is that no single mechanism will be enough to bring about a successful exit. However, it is quite possible that by combining the two effects a successful scenario can emerge whereby the higher derivative effects induce the first transition from phase

I to phase II in the weak-coupling regime, while the loop effects will induce the final transition from phase II to phase III as soon as the coupling becomes of $O(1)$.

c) **Exit via quantum cosmology:** Recently several groups [42] have attempted to describe the transition from the pre- to the post-big-bang without modifying the low-energy tree-level effective action by exploiting the quantum cosmology approach based on the Wheeler–DeWitt (WDW) equation. In Refs. [43] an $O(d,d)$-invariant WDW equation was derived in the $d^2+1$-dimensional mini-superspace consisting of an homogeneous Bianchi I metric, antisymmetric tensor and dilaton. The $O(d,d)$ symmetry helps avoiding the ordering ambiguities which usually plague the WDW equation. For the time being, only the mathematically simpler case of an $O(d,d)$-invariant potential $V(\bar{\phi})$ has been analysed since, in that case, $d^2$ conserved charges can be defined and the "radial" part of the WDW equation reduces to a one-dimensional Schrödinger equation for a scattering problem.

It is amusing that, from such a point of view, the initial state of the Universe is described by a right-moving plane wave, which later encounters a potential giving rise to both a transmitted and a reflected (i.e. left-moving) wave. The transmission coefficient gives the probability that the Universe ends up in the pre-big-bang singularity, while the reflection coefficient gives the probability of a successful exit into the post-big-bang decelerating expansion.

For certain forms of $V(\bar{\phi})$ the wave is classically reflected and the WDW approach just confirms this expectation by giving a 100% probability for the exit. However, even when there is no classical exit, the probability of wave-reflection is non-zero because of quantum tunnelling. The quantum probability of a classically forbidden exit turns out to be exponentially suppressed in the coupling constant $e^{\phi}$, which is just fine. Unfortunately it is also exponentially suppressed in the total volume of 3-space (in string units) after the pre-big-bang.

Work is in progress to establish whether such a huge suppression is still there when realistic potentials for the dilaton are used.

# 5 Conclusions

Let us summarize the main properties of string cosmology:

- Inflation comes naturally, without ad-hoc fields and/or fine-tuned potentials: simply, the accelerated growth of the coupling constant entails an inflationary expansion of the Universe. There is an underlying symmetry yielding, for any non-inflationary cosmology, an inflationary one.
- Initial conditions could hardly be simpler or more natural; yet, a simple Universe would evolve into a rich and complex one.
- The kinematical problems of the SCM are solved.

- Perturbations do not grow too large to spoil homogeneity.
- An interesting characteristic spectrum of GW is generated.
- Larger-than-usual electromagnetic perturbations are easily generated and could explain the galactic magnetic fields.
- The usual hot-big-bang cosmology can be the natural outcome of our inflationary scenario
- Unfortunately, a scale-invariant spectrum is all but automatic (unlike what happens in normal vacuum-energy-driven inflation).
- Our understanding of the high curvature (stringy) phase and of the mechanism inducing a branch change is still poor. However, the recent progress reported in Section 4 justifies some cautious optimism on these most important theoretical issues.

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