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DARK MATTER: PROBLEMS AND SOLUTIONS

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Abstract. The evidence for dark matter from dynamical studies, X-ray observations and gravitational lensing effects is first reviewed. It is argued that one needs a combination of baryonic and non-baryonic solutions. The dark matter in galactic disks (if real) is almost certainly baryonic, in which case white dwarfs or brown dwarfs would be possible candidates. The dark matter in galactic halos could also be partly baryonic and, in this case, it is likely to be contained in the remnants of a first generation of pregalactic or protogalactic stars. We discuss the various constraints on the nature of such remnants - with particular emphasis on lensing and dynamical limits - and conclude that brown dwarfs are the most plausible candidates. Microlensing searches should either confirm or exclude this possibility very soon. Unless one gives up the standard cosmological nucleosynthesis scenario, the dark matter in clusters or intergalactic space must be non-baryonic. This is probably in the form of cold elementary particle relics or primordial black holes, in which case it should also settle into galactic halos.

1 Evidence for Dark Matter

A gravitationally bound system of mass M, radius R and density ρ has a characteristic potential Φ which may be probed by studying the velocity of its components V, the temperature of the gas T or the deflection of light δ:

$$\Phi \sim GM/R \sim G\rho R^2, \quad V \sim \Phi^{1/2}, \quad kT \sim m\Phi, \quad \delta \sim \Phi/c^2 \tag{1.1}$$

A dark matter problem arises whenever the values of M or ρ inferred from measurements of Φ and R exceeds the mass or density in visible form. Evidence for dark matter has been claimed in four different contexts - the Galactic disk, galactic halos, clusters of galaxies and the background Universe - and the different methods of probing the potential are summarized in Table (1). The form of the dark matter need not be the same in all these contexts and this should be born in mind when discussing the evidence.

1.1 Local Dark Matter

There may be local dark matter in the Galactic *disk*; in this case, R is associated with the thickness of the disk $\sim$ 300 pc and measurements of the stellar velocity and density distribution perpendicular to the disk provide an estimate of the total disk density. Although it has long been suspected that this exceeds the density in visible stars, the evidence is very controversial. Bahcall (1984) used counts of F dwarfs and K giants to conclude that the density of unseen material must be at least 50% that of the visible material. He also concluded that the disk dark matter must have an exponential scale height of less than 700 pc, so that it must itself be confined to a disk. However, Bahcall assumed a particular model and doubt was cast in a series of papers by Kuijken & Gilmore (1989,1991), who used the full distribution function for the velocities and distances of K dwarfs rather than assuming a particular model. From more recent analyses of K giants, Bahcall et al. (1992) claimed a dark fraction of 60%, whereas Flynn & Fuchs (1995) got an upper limit of 10%. As discussed in Section 4, disk dark matter in the mass range $10^{-6} - 1 M_\odot$ can also be sought through its microlensing effects on the luminosity of stars in the Galactic Bulge. Such microlensing has been observed but may be attributable to ordinary disk stars.

1.2 Galactic Halos

There may be dark matter in the *halos* of galaxies with a mass $M_{dark} \sim 10 M_{vis}(R_h/100kpc)$ which depends upon the (uncertain) halo radius R_h. The best evidence for dark halos in spiral galaxies comes from rotation curve measurements, the dependence of the rotation speed V upon galactocentric distance R being a measure of the density profile $\rho(R)$. An important feature of the typical spiral rotation curve is that it is approximately constant at large R (Rubin et al. 1980). This implies that the mass within radius R increases like R, which is faster than the increase of visible mass. Indeed neutral hydrogen observations suggest that V continues to remain constant well beyond the visible stars (Sancisi & van Albada 1987). There seems to be a correlation between the form of the rotation curve and the galaxy luminosity: as R increases, the velocity always rises to its asymptotic value for low luminosities but it rises and then falls for high luminosities. Indeed Persic et al. (1995) claim that there is a "universal" rotation curve, characterized by a single parameter. The mass distribution in ellipticals is best studied using X-ray observations of the hot gas (since the stellar velocity dispersion does not determine the density profile uniquely). This again indicates the presence of dark matter, in many cases with the same $M \sim R$ law which characterizes spirals (Sarazin 1986). There is also evidence for dark matter in dwarf galaxies. Indeed the rotation curves in many dwarf irregulars (Lake et al. 1990) and the velocity dispersions in nearby dwarf spheroidals (Lin & Faber 1983, Aaronson 1983) indicate that they have even higher dark mass fractions than bright spirals. As discussed in Section 4, microlensing effects on the luminosity of stars in the Large Magellanic Cloud provide a direct probe of dark objects in our own halo with mass in the range $10^{-7} - 10M_{\odot}$ and may have already met with success.

1.3 Clusters of Galaxies

There may be dark matter associated with *clusters* of galaxies; in this case, R characterizes the size of the cluster $\sim$ 10 Mpc and the velocity dispersion indicates that the dynamical mass exceeds the visible mass by at least a factor of 10. This is confirmed by X-ray data on the gas emission, with the gas itself containing more mass than the galaxies: a typical rich cluster has 2-7% of its mass in galaxies, 10-30% in gas and 60-85% in dark matter (Bohringer & Neumann 1995). The gas is roughly isothermal beyond the central regions, indicating that the dark matter density scales as R^{-2}. The gas density itself falls off more slowly than this and it appears to be more extended than the dark matter (David et al. 1995). The galaxies are the most centrally concentrated component. Further evidence for dark matter in clusters comes from lensing: distant galaxies behind the cluster may be distorted into arclets by the cluster potential and the properties of these arclets can be used to infer the dark matter distribution (Tyson et al. 1990, Smail et al. 1995). The lensing estimates of the dark mass are roughly consistent with the dynamical estimates, as discussed by Schneider (1996).

1.4 Background Dark Matter

None of the forms of matter discussed above can have the critical density required for the Universe to recollapse: $\rho_{crit} = 3H_o^2/8\pi G$. However, according to the currently popular inflation theory (Guth 1981), in which the Universe undergoes an exponential expansion phase at some early time, the total density should have almost exactly the critical value ($\Omega \equiv \rho/\rho_{crit} = 1$), corresponding to $M_{dark} \sim 100M_{vis}$. This would have two possible implications: either there is another dark component which is distinct from the clustered dark matter or galaxy formation is biased (Kaiser 1984, Dekel & Rees 1987), in the sense that galaxies form preferentially in a small fraction of the volume of the Universe. In either case, one would expect the mass-to-light ratio to increase as one goes to larger scales and there is some indication of this from dynamical studies. One can probe the density on scales above 10 Mpc, for example, by analysing large-scale streaming motions of clusters of galaxies (Dressler et al. 1987, Bertschinger & Dekel 1989) or by determining the dipole moment of the IRAS sources (Rowan-Robinson et al. 1990). In all these analyses, the inferred density depends on the bias parameter b and more sophisticated analyses are required to determine Ω and b separately (Nusser & Dekel 1993, Peacock & Dodds 1994). A variety of other methods can be used to probe the background density. Measurements of the deceleration parameter using supernovae as distance indicators (Perlmutter et al. 1995) suggest that Ω is close to 1 and the fact that the fraction of clusters with substructure is high implies that Ω is at least 0.5 (Forman et al. 1995). As discussed in Section 4, gravitational lensing effects may provide evidence for intergalactic dark matter in the form of compact objects and upper limits on the fraction of lensed quasars also puts a limit $\Omega_\Lambda < 0.7$ on the density contribution from a cosmological constant (Kochanek 1995). As discussed by Lasenby (1996), a powerful probe of both the baryonic and total density parameter is the spectrum of the microwave background anistropies since these parameters determine the amplitude and angular scale, respectively, of the 1st Doppler peak.

Table (1): *evidence for dark matter*

	Dynamics	X-rays	Lensing
Local	✓	✗	?
Spirals	✓	✗	✓
Ellipticals	?	✓	?
Dwarfs	✓	✗	✗
Clusters	✓	✓	✓
Background	✓	✗	✓

2 Baryonic Versus Non-Baryonic Dark Matter

The main argument for both baryonic and non-baryonic dark matter comes from Big Bang nucleosynthesis. This is because the success of the standard picture in explaining the primordial light element abundances only applies if the baryon density parameter Ω_b lies in the range $0.010h^{-2} < \Omega_b < 0.015h^{-2}$ (Walker etal. 1991) where $h = H_o/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. The upper and lower limits come from the upper bounds on the helium abundance and the sum of the deuterium and helium-3 abundances, respectively. Although more recent estimates by Copi et al. (1995) allow a wider range of values, $0.007h^{-2} < \Omega_b < 0.022h^{-2}$, the upper limit implies that Ω_b is well below 1, which suggests that no baryonic candidate could provide the critical density required in the inflationary scenario. This conclusion also applies if one invokes inhomogeneous nucleosynthesis since one requires $\Omega_b < 0.09h^{-2}$ even in this case (Mathews et al. 1993). The standard scenario therefore assumes that the total density parameter is 1, with only 1-10% being baryonic.

On the other hand, the value of Ω_b allowed almost certainly exceeds the density of visible baryons Ω_v. A careful inventory by Persic & Salucci (1992) shows that the contributions to Ω_v are 0.0007 from spirals, 0.0015 from ellipticals and spheroidals, $0.00035h^{-1.5}$ from hot gas within an Abell radius for rich clusters, and $0.00026h^{-1.5}$ from hot gas out to a virialization radius in groups and poor clusters. This gives a total of $(2.2 + 0.6h^{-1.5}) \times 10^{-3}$, although neutral hydrogen in galaxies gives another contribution of $0.2 \times 10^{-3}h^{-1}$ (Rao & Briggs 1993). For reasonable values of h, they infer $\Omega_V = 0.003$, which is well below the baryon density inferred from nucleosynthesis. Dar (1995) gets a somewhat larger value of $(4.5 + 0.9h^{-1.5}) \times 10^{-3}$, corresponding to $\Omega_V = 0.006$, but one still needs some baryonic dark matter.

Which of the dark matter problems mentioned in Section 1 could be baryonic? Baryons would certainly suffice to explain the dark matter in galactic disks: even if all disks have the 60% dark component envisaged for our Galaxy by Bahcall et al. (1992), this only corresponds to $\Omega_d = 0.001$ - well below the nucleosynthesis value for Ω_b. On the other hand, the cluster dark matter has a density $\Omega_c = 0.1 - 0.2$ and this cannot be baryonic unless one invokes inhomogeneous nucleosynthesis. Whether dark baryons could explain galactic halos depends on the typical halo radius: if our own halo is typical, the density associated with halos would be $\Omega_h \simeq 0.02h^{-1}(R_h/70kpc)$ and, for reasonable values of R_h, this could be either less or more than Ω_b. The various values of Ω required by these arguments are summarized in Figure (1). This emphasizes that there are four distinct issues:

$\Omega_b = \Omega_{vis}$? Do we really need dark baryons? Jedamzik et al. (1995) have argued that inhomogenous Big Bang nucleosynthesis could reduce the lower limit on Ω_b enough to eliminate the need for baryonic dark matter. This may apply even in the homogeneous scenario since recent measurements of the deuterium abundance in quasar absorption systems gives $\chi(D) \simeq 2 \times 10^{-4}$, which is an order of magnitude larger than the standard interstellar abundance (Songaila et al. 1994, Carswell et al. 1994, Rugers & Hogan 1996). In this case, the upper limit on Ω_b is reduced to $0.005h^{-2}$, which is only marginally larger than the Persic-Salucci estimate of Ω_V. However, other groups find the standard deuterium abundance in quasar absorption systems (Tytler er al. 1996), so the evidence for such a high value of $\chi(D)$ is inconclusive.

$\Omega_b = \Omega_h$? Could dark baryons suffice to explain galactic halos? The estimate $\Omega_h \simeq 0.02h^{-1}(R_h/70\text{kpc})$ is compatible with the Copi et al. (1995) limit of $\Omega_b < 0.022h^{-2}$ only for $R_h < 70h^{-1}$ kpc. For the Milky Way, the minimum halo radius consistent with rotation curve measurements, the local escape speed, the kinematics of globular clusters and the dynamics of the Local Group is about 70 kpc (Fich & Tremaine 1991), which would just be compatible with this. However, Zaritsky et al. (1993) argue from observations of the satellite systems of other galaxies that spirals typically have 200 kpc halos, which would not be.

$\Omega_c = \Omega_h$? Could all the dark matter in clusters derive from halos? Although the cluster dark mass cannot all be associated with individual galaxies *now* - else dynamical friction would result in the most massive galaxies being dragged into the cluster centre (White 1976) - it may still have derived from the galaxies *originally*. Indeed in the hierarchical clustering picture one would expect the galaxies inside a cluster to be stripped of much of their individual halos, thereby forming a collective halo (White & Rees 1978). Bahcall et al. (1995a) advocate this possibility: in their scenario both elliptical and spiral galaxies have a halo radius of $200h^{-1}$ kpc but ellipticals have a higher mass-to-light ratio. Since ellipticals are preferentially contained in clusters, they claim this explains why clusters have a larger mass-to-light ratio than spirals. However, unless one invokes a rather exotic cosmological nucleosynthesis scenario (cf. Gnedin & Ostriker 1992), this scenario could only account for all the cluster dark matter if the original galactic halos were non-baryonic.

$\Omega = 1$? We have seen that the background dark matter can be identified with the cluster dark matter (even though $\Omega_c < 1$) providing one invokes biased galaxy formation. However, several people (eg. Coles & Ellis 1994) have emphasized that the direct evidence for $\Omega = 1$ is poor. Indeed Bahcall et al. (1995a) argue that there is no need for Ω to exceed Ω_c. One problem with the standard scenario with $\Omega_b \sim 0.01 - 0.1$ and $\Omega = 1$ is that X-ray data suggest that the ratio of visible baryon mass (in stars and hot gas) to total mass in clusters is anomalously high compared to the mean cosmic ratio. For example, ROSAT observations suggest that the baryon fraction within the central 3 Mpc of the Coma cluster is about 25% (White et al. 1993) and there is now evidence that this is a fairly widespread phenomena (White & Fabian 1995). It is hard to understand how the extra baryon concentration would come about, since dissipation should be unimportant on these scales (Gunn & Thomas 1995), so this has been referred to as the "baryon catastrophe". Unless one invokes a cosmological constant, it suggests that either Ω is well below 1 or Ω_b is higher than allowed by the homogeneous nucleosynthesis scenario.

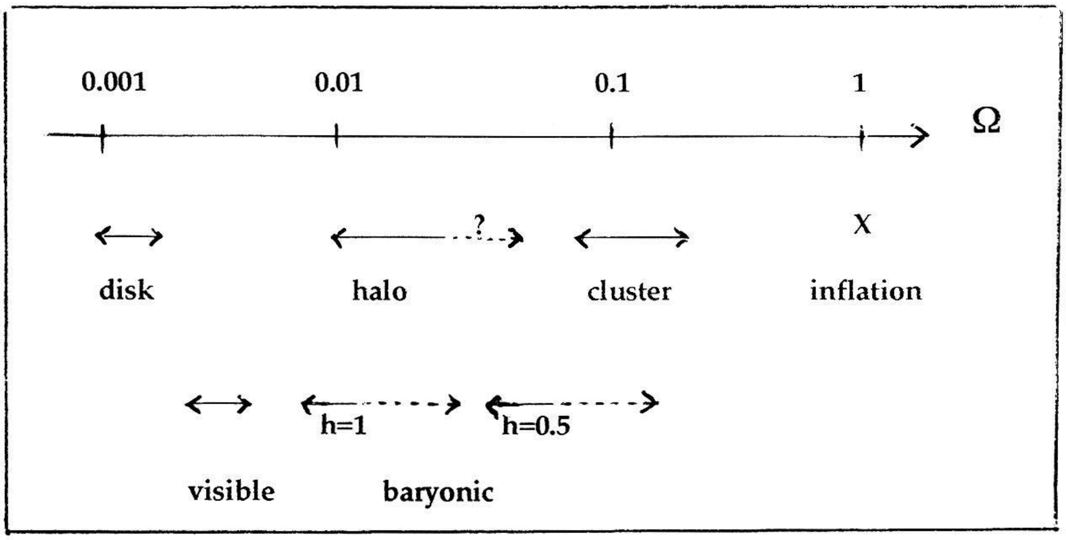


Figure (1). *This compares the density associated with the various dark matter problems to the density in visible form and the baryonic density required by cosmological nucleosynthesis (with the inhomogeneous case shown dotted).*

3 Possible Sites for Dark Baryons

Henceforth most of the emphasis will be on baryonic dark matter, so we must address thequestion of where the dark baryons are located and what form they may take. Apart from the dark baryons in galactic disks, which anyway have a negligible cosmological density ($\Omega_d \simeq 0.001$), there are four possibilities and these are summarized in Table (2)]:

* **Hot Intergalactic Medium.** The discrepancy between Ω_b and Ω_V could be resolved if the missing baryons were in a hot intergalactic medium but, in this case, the temperature

T would need to be finely tuned. Since the Gunn-Peterson test requires the neutral hydrogen density to be $\Omega(HI) < 10^{-8}h^{-1}$ (Steidel & Sargent 1988) out to a redshift of 3 and since the COBE limit on the Compton distortion of the microwave background ($y < 3 \times 10^{-5}$) requires an ionized hydrogen density

$$\Omega(HII) < 0.03(y/10^{-5})(T/10^{8}K)^{-1}h^{-1}[(1+z)^{3/2}-1]^{-1} \sim 0.1(T/10^{7}K)^{-1}h^{-1} \tag{3.1}$$

at that redshift (Mather et al. 1994), the temperature must lie in the range $10^4 - 10^6 K$ if $\Omega_{IGM} \sim \Omega_b$. Positive evidence for an intergalactic may come from the recent detection of helium absorption (Jacobsen et al. 1993, Davidsen et al. 1996).

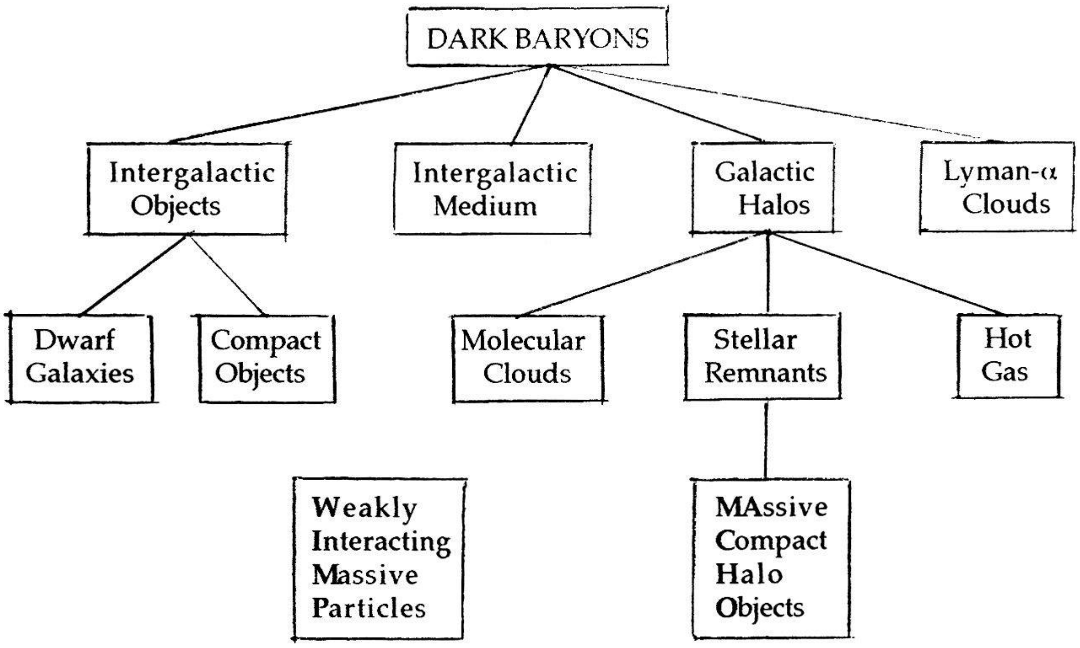
* **Lyman-a clouds.** Although the density parameter associated with "damped" clouds is probably around $0.003h^{-2}$ (Lanzetta et al. 1991), comparable to the density in galaxies and therefore consistent with the idea that these are protogalactic discs, the density associated with undamped systems is unknown and - depending on their ionized fraction and geometry - could be much larger (Rees 1986). However, by the present epoch the undamped clouds could have fragment into stars (indeed they could be the birth sites of Population III objects), so this does not exclude the other possibilities discussed below.

* **Dark Intergalactic Objects.** The usual estimate of Ω_V does not include the contribution from an intergalactic population of dark objects, such as dwarf galaxies (Bristow & Phillipps 1993) or low surface brightness galaxies (McCaugh 1994). Indeed it has recently been claimed that dwarf galaxies may provide all of the missing baryons (Impey 1996). There could also be intergalactic dark matter in the form of compact objects (Carr & Sakellariadou 1996): either the remnants of a first generation of pregalactic stars or primordial black holes which formed before cosmological nucleosynthesis. Only the latter could have the critical density required by inflation. Otherwise most of the intergalactic dark matter would have to be in the form of "Weakly Interacting Massive Particles" or "WIMPs".

* **Galactic halos.** We have seen that galactic halos could contain all the dark baryons if the typical halo radius R_h is less than $70h^{-1}$ kpc. In this case, one might consider three possible forms for the dark baryons: hot gas, cold molecular clouds or "Population III" remnants. The first possibility would appear to be inconsistent with X-ray observations since the gas would need to have the virial temperature of $10^6 K$. The second possibility has been discussed by Pfenniger et al. (1994) and De Paolis (1995) but will not be considered further here. The third possibility corresponds to the "MAssive Compact Halo Object" or "MACHO" scenario. This is motivated by the fact that the existence of galaxies implies that there must have been density fluctuations in the early Universe and, in many scenarios, these fluctuations would also give rise to a first generation of "Population III" stars (Carr 1994). Note that, if the background non-baryonic dark matter is "cold", it will necessarily fall into halo potentials, so halos would most naturally consist of a *mixture* of WIMPs and MACHOs. In this case, the epoch of Population III formation will be very important for their relative distribution. If the Population III stars form *before* galaxies, one would expect their remnants to be distributed throughout the Universe, with the ratio of the WIMP and MACHO densities being the same everywhere and of order 10. If they form in the first phase of protogalactic collapse, one would expect the remnants to be confined to halos and clusters. In this case, their contribution to the halo density could be larger since the baryons

would dissipate and become more concentrated.

Table (2): *Summary of possible locations for dark baryons*



4 Gravitational Lensing Effects

One of the most useful signatures of compact objects is their gravitational lensing effects. Indeed it is remarkable that lensing could permit their detection over the entire mass range $10^{-7}M_\odot$ to $10^{12}M_\odot$. There are two distinct lensing effects and these probe different but nearly overlapping mass ranges: *macrolensing* (the multiple-imaging of a source) can be used to search for objects larger than $10^5 M_\odot$, while *microlensing* (modifications to the intensity of a source) can be used for objects smaller than $10^3 M_\odot$. The current constraints on the density parameter of compact objects in various mass ranges are brought together in Figure (2).

4.1 Macrolensing by Compact Objects

If one has a population of compact objects with mass M and density parameter Ω_C, then the probability of one of them multiply-imaging a source at redshift z $\sim$ 1 and the separation between the images are given by

$$P \simeq (0.1 - 0.2)\Omega_C, \qquad \Theta \simeq 6 \times 10^{-6}(M/M_\odot)^{1/2}h^{1/2} arcsec \tag{4.1}$$

(Press & Gunn 1973). One can therefore use upper limits on the frequency of macrolensing for different image separations to constrain Ω_C as a function of M (Nemiroff 1989). In particular, in the context of quasars, VLA observations imply $\Omega_C(10^{11} - 10^{13}M_\odot) < 0.4$ (Hewitt 1986) and Hubble Space Telescope data imply $\Omega_C(10^{10} - 10^{12}M_\odot) < 0.02$ (Surdej et al. (1993). To probe smaller scales, ,one must use high resolution radio sources: Kassiola et al. (1991) have invoked lack of lensing in 40 VLBI objects to infer $\Omega_C(10^7 - 10^9 M_\odot) < 0.4$, while a study of VLBA sources leads to a limit $\Omega_C(10^5 - 10^7 M_\odot) < 0.1$ (Henstock 1996). Future observations could strengthen these constraints, as indicated by the broken lines in Figure (2).

4.2 Microlensing in Macrolensed Quasars

If a galaxy is suitably positioned to image-double a quasar, then there is also a high probability that an individual halo object will traverse the line of sight of one of the images (Gott 1981) and this will give intensity fluctuations in one but not both images. The effect would be observable for objects bigger than $10^{-4}M_\odot$ but the timescale of the fluctuations, being of order $40(M/M_\odot)^{1/2}$y, would only show up over a reasonable time for $M < 0.1M_\odot$. There is already evidence of this effect for the quasar 2237+0305 (Irwin et al. 1989, Corrigan et al. 1991); the observed timescale for the variation in the luminosity of one of the images is taken to indicate a mass below $0.1M_\odot$ (Webster et al. 1991), although Wambsganss et al. (1990) argue that it might be as high as $0.5M_\odot$. Further microlensing in this quasar has been reported by Pen et al. (1993).

4.3 The Effect of Microlensing on Quasar Luminosity

More dramatic but more controversial evidence for the effect of microlensing on quasar luminosity comes from Hawkins (1993, 1996), who has been monitoring 300 quasars in the redshift range 1-3 over the last 17 years using a wide field Schmidt camera. He finds quasi-sinusoidal variations of amplitude 0.5m on a timescale 5 y and attributes this to lenses with mass $\sim 10^{-3}M_\odot$. The crucial point is that the timescale decreases with increasing z, which is the opposite to what one would expect for intrinsic variations [see Alexander (1995) and Baganoff & Malkan (1995) for a contrary view.] The timescale also increases with the luminosity of the quasar. He explains this by noting that the luminosity should increase with the size of the accretion disk. A rather worrying feature of Hawkins' claim is that it requires the density of the lenses to be close to critical so that the sources are being transited continuously (Schneider 1993). In this case, the lenses have to form before Big Bang nucleosynthesis, so he invokes primordial black holes which form at the quark-hadron phase transition at 10^{-5}s. However, this requires fine-tuning since the fraction of the Universe going into black holes at that time can only be 10^{-9}.

4.4 Line-to-continuum Effects for Quasars

In some circumstances, the continuum part of the quasar emission will be microlensed but not the line part. This is because only the continuum region may be small enough to act as a point-like source. (For a lens at a cosmological distance the Einstein radius is $0.05(M/M_\odot)^{1/2}h$ pc, whereas the size of the optical continuum and line regions are of order 10^{-4} pc and 0.1-1 pc respectively.) This would decrease the equivalent width of the emission lines, so in *statistical* studies of many quasars, one would expect the typical equivalent width to decrease as one goes to higher redshift because there would be an increasing probability of having an intervening lens. Recently Dalcanton et al. (1994) have compared the equivalent widths for a high and low redshift sample of quasars and find no difference. They infer the following limits:

$$\Omega_C(0.001 - 60M_\odot) < 0.2, \quad \Omega_C(60 - 300M_\odot) < 1, \quad \Omega_C(0.01 - 20M_\odot) < 0.1 \tag{4.2}$$

The mass limits come from the fact that the amplification of even the continuum region would be unimportant for $M < 0.001M_\odot$, while the amplification of the broad-line regions would be important (cancelling the effect) for $M > 20M_\odot$ if $\Omega_C = 0.1$ or $M > 60M_\odot$ if $\Omega_C = 0.2$ or $M > 300M_\odot$ if $\Omega_C = 1$. These limits are marginally incompatible with Hawkins' claim that $\Omega_C(10^{-3}M_\odot) \sim 1$.

4.5 Microlensing of Stars by Halo Objects in our own Galaxy

Attempts to detect microlensing by objects in our own halo by looking for intensity variations in stars in the Magellanic Clouds and the Galactic Bulge have now been underway for several years. This method is sensitive to lens masses in the range $10^{-8} - 10^2 M_\odot$ but the probability of an individual star being lensed is only $\tau \sim 10^{-6}$, so one has to look at many stars for a long time (Paczynski 1986). The duration of the variation and the likely event rate are

$$P \sim 0.2(M/M_\odot)^{1/2}y, \qquad \Gamma \sim N\tau P^{-1} \sim (M/M_\odot)^{-1/2}yr^{-1} \tag{4.3}$$

where $N \sim 10^6$ is the number of stars. Thus small masses give frequent short-duration events, while large masses give rare long-duration events. The key feature of microlensing events is that the light-curve is time-symmetric and achromatic and this may allow them to be distinguished from intrinsic stellar variations (Griest 1991). Three groups are involved in these searches and each now claims to have detected lensing events. The American/Australian group (MACHO) uses a dedicated telescope at Mount Stromlo to study 10^7 stars in red and blue light in the LMC, the SMC and the Galactic bulge. They currently have 7 LMC events and around 45 bulge events (Alcock et al. 1996). The timescale and frequency of the LMC events suggests that the halo objects have a most likely mass of $0.5M_\odot$ and a halo fraction of about 0.5. However, these precise figures should not be taken too seriously since they depend on the halo model and the number of events is small. (For comparison, the first year data gave a mass of $0.1M_\odot$ and a halo fraction of 0.2.) The French group (EROS) has been studying stars in the LMC: they are seeking 1-100 day events with digitized red and blue Schmidt plates obtained with the ESO telescope in Chile and 1 hour to 3 day events with

CCDs taken at the Observatoire de Haute Provence (Auborg et al. 1993). The CCD searches have given no results, which excludes objects in the range 10^{-7} to $10^{-3}M_\odot$ from having the halo density (Auborg et al. 1995), but analysis of 3×10^6 stars on the Schmidt plates yields one event with a duration of about two months. The Polish collaboration (OGLE) are using the Las Campanas telescope in Chile to look at 7×10^5 stars in the Galactic bulge and have claimed 11 events (Udalski et al. 1993). Even if halo objects are not responsible for the EROS and MACHO events, the upper limits on their frequency still give interesting constraints on Ω_C and these are shown in Figure (2) on the assumption that the halo density is $\Omega_h = 0.1$ (Renault 1996).

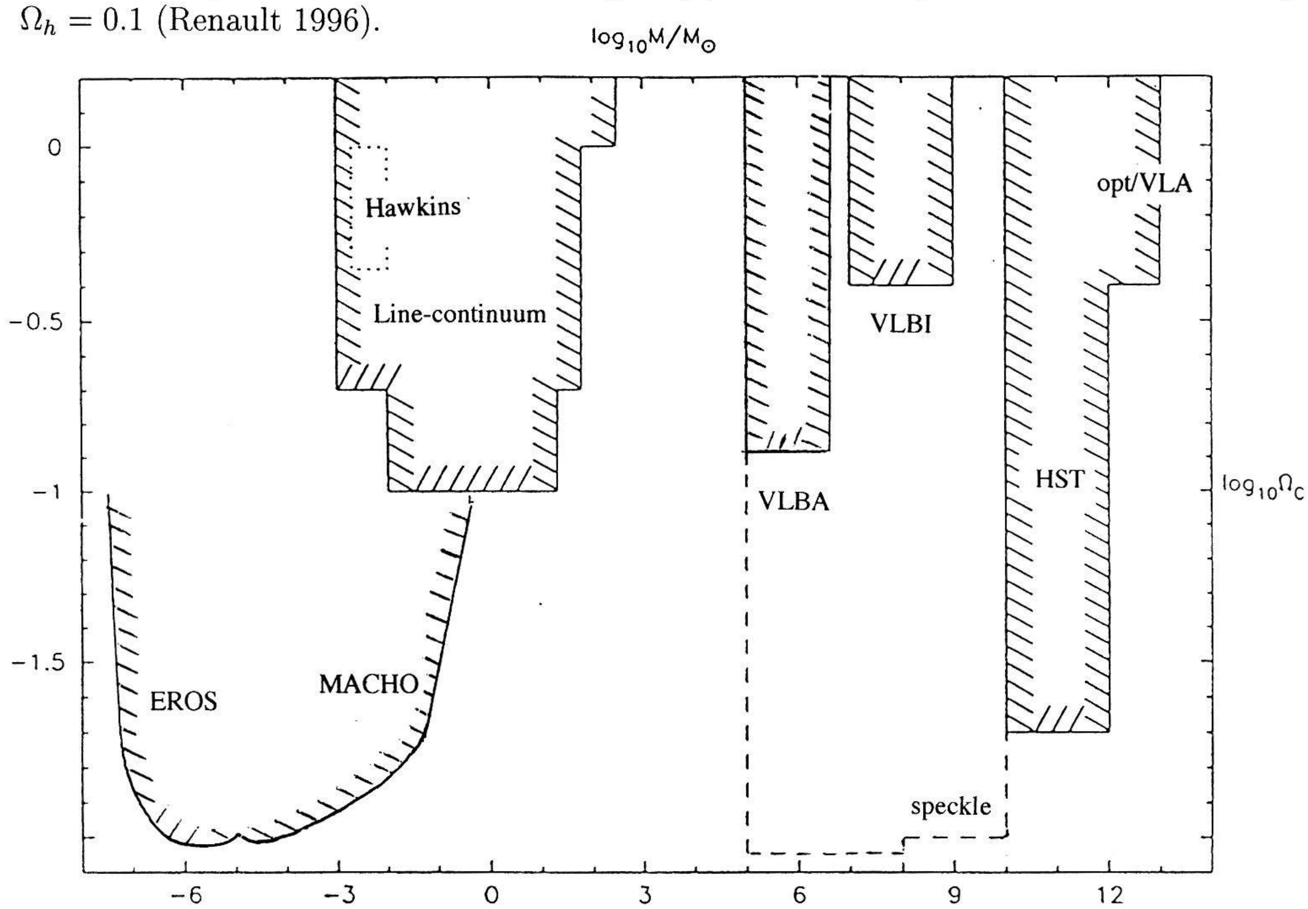


Figure (2). *Lensing constraints on the density parameter for compact objects.*

5 Dynamical Constraints

A variety of constraints can be placed on compact objects by considering their dynamical effects (Carr & Sakellariadou 1996). The constraints are usually calculated on the assumption that the objects are black holes but most of them also apply for dark clusters of smaller objects. This is important since several people have argued that one would expect compact objects to form in clusters (eg. Carr & Lacey 1986, Salpeter & Wasserman 1993, Kerins & Carr 1994, De Paolis et al. 1995). The limits are summarized as upper limits on the density parameter Ω_B for black holes of mass M in Figure (3), where the disk, halo and cluster density parameters are assumed to be 0.001, 0.1 and 0.2, respectively.

5.1 Disk Heating by Halo Holes

As halo objects traverse the Galactic disk, they will impart energy to the stars there. This will lead to a gradual puffing up of the disk, with older stars being heated more than younger ones. Indeed Lacey & Ostriker (1985) have argued that black holes of around $10^6 M_\odot$ could generate the observed puffing. More recent data is probably inconsistent with this picture and heating by spiral density waves or giant molecular clouds is now usually invoked (Lacey 1991). Nevertheless, one can still use the Lacey-Ostriker argument to place an upper limit on the density in halo objects of mass M (Carr et al. 1984):

$$\Omega_B < \Omega_h \ min\ [1, (M/M_{heat})^{-1}], \quad M_{heat} = 3 \times 10^6 (t_g/10^{10} y)^{-1} M_\odot \tag{5.1}$$

where t_g is the age of the Galaxy. Otherwise the disk would be more puffed up than observed.

5.2 The Disruption of Stellar Clusters by Halo Objects

Another type of dynamical effect associated with halo objects would be their influence on bound groups of stars (eg. globular clusters). Every time a halo object passes near a star cluster, the object's tidal field heats up the cluster and thereby reduces its binding energy. If the object is sufficiently large it will disrupt the cluster in a single fly-by. For smaller objects the disruption will be a gradual, requiring the cumulative effect of many traversals. By comparing the expected disruption time for clusters of mass m_c and radius r_c with the cluster lifetime t_L, one finds that the local density of halo holes of mass M must satisfy (Sakellariadou 1984, Wielen 1985, Carr & Sakellariadou 1996)

$$\rho_B < \begin{cases} m_c V/(G M t_L r_c) & \text{for} \quad M < M_c(V/V_c) \\ (m_c/G t_L^2 r_c^3)^{1/2} & \text{for } M_c(V/V_c) < M < M_c(V/V_c)^3 \end{cases} \tag{5.2}$$

where $V \simeq 300$ km s^{-1} is the characteristic speed of the holes and $V_c \sim (Gm_c/r_c)^{1/2}$ is the internal velocity of the cluster. Any lower limit on t_L therefore places an upper limit on ρ_B. The crucial point is that the limit is independent of M in the single-encounter regime, so it bottoms out at density of order $(\rho_c/G t_L^2)^{1/2}$. For globular clusters, we may take $m_c = 10^5 M_\odot$, $r_c = 10$pc and $t_L > 10^{10}$ y (Moore 1993) and eqn (5.2) then gives the limit shown by the broken line in Figure (3).

5.3 The Effect of Dynamical Friction on Halo Objects

Sufficiently massive halo objects will tend to lose energy to lighter objects and consequently drift towards the Galactic nucleus. In particular, one can show that the objects will be dragged into the nucleus by the dynamical friction of the Spheroid stars from within a Galactocentric radius (Carr & Lacey 1986)

$$R_{df} = (M/10^6 M_\odot)^{2/3} (t_g/10^{10} y)^{2/3} kpc \tag{5.3}$$

The total mass dragged into the Galactic nucleus therefore exceeds the observational upper limit of $3 \times 10^6 M_\odot$ unless

$$\Omega_B < \Omega_h \; min \; [1, (M/M_{df})^{-2}], \quad M_{df} = 3 \times 10^4 (a/2kpc)(t_g/10^{10}y)^{-1} M_\odot \qquad (5.4)$$

where a is the halo core radius. This is stronger than the disk-heating limit but there is an important caveat here since,once more than two black holes have accumulated in the centre of the Galaxy, they may be ejected via the "slingshot" mechanism (Hut & Rees 1992). Because limit (5.4) is not completely firm, it is shown broken in Figure (3).

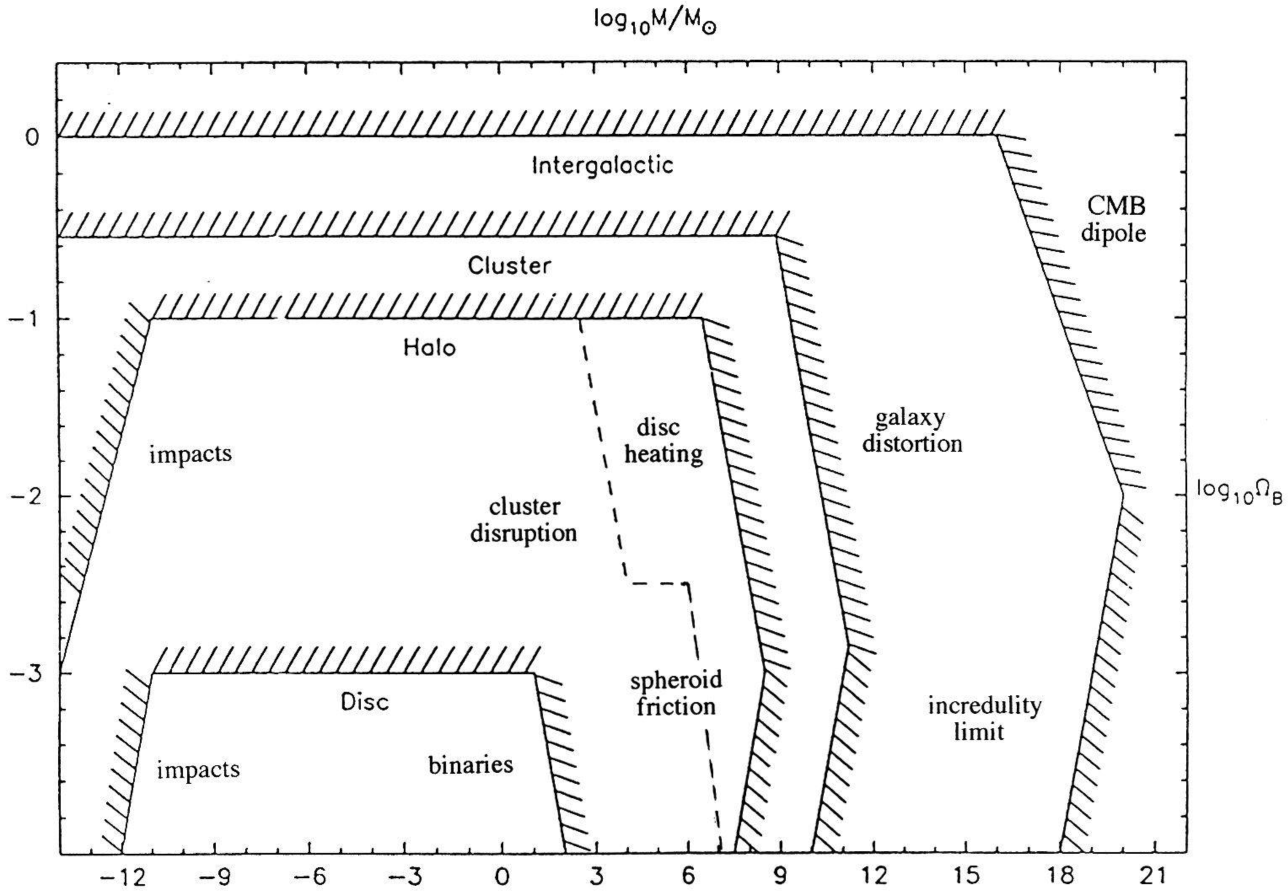


Figure (3). *Dynamical constraints on the density parameter for black holes in the Galactic disk, the Galactic halo, clusters of galaxies and the intergalactic medium.*

5.4 Constraints on Dark Objects Outside Halos

Bahcall et al. (1985) have argued that the disk dark matter could not comprise objects larger than $2M_\odot$ else they would disrupt the observed wide binaries, although this limit has been disputed by Wasserman & Weinberg (1987). The most interesting dynamical constraints on dark objects in clusters of galaxies comes from upper limits on the fraction of galaxies with unexplained tidal distortions. Van den Bergh (1969) applied this argument to the Virgo cluster and concluded that black holes binding the cluster could not be bigger than $10^9 M_\odot$.

The form of both this and the wide binary limit can be inferred from eqn (5.2). The most interesting dynamical constraint on intergalactic black holes comes from the fact that each galaxy would have a peculiar velocity due to its gravitational interaction with the nearest one. Since the CMB dipole anisotropy shows that the peculiar velocity of our own Galaxy is only 600 km s^{-1}, one infers a limit (Carr 1978)

$$\Omega_B < (M/10^{16} M_\odot)^{-1/2} \tag{5.5}$$

The limits on the bottom right of Figure (3) corresponds to the requirement that there be at least one object of mass M within each halo, each cluster and the Universe itself (the incredulity limit). Those on the left come from upper limits on the frequency of encounters with interstellar snowballs (Hills 1986).

6 Assessment of Baryonic Candidates

From a theoretical perspective, it is hard to predict the mass range of Population III stars (some people argue that they would have been smaller than at present, others that they would have been larger) but one can still use a wide variety of constraints - including the lensing and dynamical effects discussed above - to exclude certain candidates.

SNOWBALLS. This term is used to describe condensations of cold hydrogen which are smaller than $0.001 M_\odot$ and have atomic density; larger objects are degeneracy-supported and have more than atomic density. In fact, snowballs can almost certainly be excluded from solving any of the dark matter problems. In order to avoid being disrupted by collisions within the age of the Universe, they must have a mass of at least 1g (Hegyi & Olive 1983). On the other hand, they are excluded by the upper limit on the frequency of encounters with interstellar meteors between 1g and 10^7g, by the number of impact craters on the Moon between 10^7g and 10^{16}g, and by the fact that no interstellar comet has crossed the Earth's orbit in the last 400 years between 10^{15}g to 10^{22}g (Hills 1986). These mass limits apply for disk objects; they are somewhat stronger for halo objects because of their larger velocities. De Rújula et al. (1992) have claimed an even stronger limit on the grounds that snowballs smaller than 10^{26}g would be evaporated within the age of the Universe by their own heat. We have seen that EROS microlensing limits also exclude halo objects in the range $10^{26} - 10^{30}$g, which leaves no mass range at all.

BROWN DWARFS. These are objects between $0.001 M_\odot$ and $0.08 M_\odot$, which are never hot enough to ignite hydrogen. Such objects are hard to find but it would be surprising if the stellar mass function happened to cut off just above $0.08 M_\odot$ and there is now incontrovertible evidence for at least one brown dwarf in the form of Gliese 229B (Nakajima et al. 1995). In determining the contribution of brown dwarfs to the dark matter density, the best strategy is to study the mass function of stars just above the hydrogen-burning limit and infer whether its extrapolation would permit a lot of lower mass objects. Studies of the mass function of Population I stars (i.e. stars in the Galactic disc) by Kroupa et al. (1993) and Chabrier et al. (1995) suggest that brown dwarfs may dominate the *number* density - indeed Hawkins &

Jones (1996) may have already found such a population - but they can only contain about 1% of the mass. The situation is less clear when one considers Population II stars (i.e. stars in globular clusters and the Galactic Spheroid). Richer et al. (1991) and Richer & Falman (1992) claim these may have a steeper mass function but Mera et al. (1996) disagree. In any case, there may be no connection between the mass function of halo stars and Population II stars since they probably form at a different time and place. As discussed in Section 4, the most important signature of brown dwarfs would be intensity fluctuations in background sources due to their microlensing effects, an effect which may have already been observed in our own or other galactic halos.

M-DWARFS. These are stars below $0.1M_\odot$, which burn hydrogen but are very dim. Discrete source count constraints imply that such stars can comprise no more than 6% of the halo dark mass and 15% of the disk dark mass (Bahcall et al. 1995b, Flynn et al. 1996). They would also seem to be excluded by infrared measurements of other galaxy halos. The K-band mass-to-light ratio exceeds 100 for M87 (Boughn & Saulson 1983) and 140 for NGC 100 (Casali & James 1995). Since the mass-to-light ratio is less than 60 for stars bigger than $0.08M_\odot$, the lower limit for hydrogen-burning (D'Antona & Mazzitelli 1985), this suggests that any hydrogen-burning stars are excluded. Although Sackett et al. (1994) and James & Casali (1996) have claimed to detect a faint red halo around NGC 5907, suggesting that there are some M-dwarfs, there are not enough to provide all the halo mass.

WHITE DWARFS. These would be the natural end-state of stars with initial mass in the range $0.8 - 8M_\odot$. They would be the most conservative candidate since white dwarfs certainly form prolifically today (Silk 1993). This scenario would have many interesting observational consequences, such as an abundance of cool white dwarfs today (Tamanaha et al. 1990). However, one needs a very contrived mass spectrum if white dwarfs make up galactic halos: the IMF must be restricted to between 2 and 8 $M_\odot$ to avoid producing too much light or too many metals (Ryu et al. 1990) and even then one must worry about excessive helium production. There are other problems with this scenario: the fraction of white dwarfs in binaries might produce too many type 1a supernovae (Smecker & Wyse 1991) and deep galaxy surveys may already exclude the bright early evolutionary phase which would be expected if WDs provided even 10% of the halo mass (Charlot & Silk 1995). WDs could still provide the dark matter in the Galactic disk (if real) but the observed mass function does not indicate this.

NEUTRON STARS. Although these would be the natural end-state of stars in some mass range above 8 $M_\odot$, the fact that the poorest Population I stars have metallicity of order 10^{-3} places an upper limit on the fraction of the Universe's mass which can have been processed through the stellar precursors and this probably precludes their explaining any of the dark matter problems (Carr et al. 1994). We have also seen that the lack of microlensing effects on the line-continuum ratio for distant quasars implies that compact objects in the mass range $0.01 - 20M_\odot$ (i.e. both white dwarfs and neutron stars) must have $\Omega_C < 0.1$.

STELLAR BLACK HOLES. Stars larger than 20 - 50 $M_\odot$ may leave black hole rather than neutron star remnants, with some of their nucleosynthetic products being swallowed. However, they will still return a lot of heavy elements through winds prior to collapsing

(Maeder 1992), so normal stellar black holes are probably excluded from explaining any of the dark matter problems. On the other hand, "Very Massive Objects" larger than 200 $M_\odot$ undergo complete collapse and so may be better dark matter candidates (Bond et al. 1984). However, since the precursors of VMOs would be highly luminous, an important constraint on the number of VMO black holes comes from background light limits. If the radiation from VMOs is affected only by cosmological redshift, then it would presently be in the near-infrared (Bond et al. 1986). In this case, the COBE constraints imply that the VMOs could only have the density required to explain galactic halos if they burn sufficiently early ($z > 200$) for their light to be redshifted beyond 10μ (where it would be hidden by interplanetary dust emission). However, in many circumstances, one would expect the VMO light to be reprocessed into the submillimetre band by pregalactic dust (Bond et al. 1991), in which case the strong constraints on the spectral distortion of the microwave background imposed by COBE exclude almost all scenarios (Wright et al. 1994). We have seen that lensing effects on the line-to-continuum ratio of quasars may also preclude black holes having a critical density below 300 $M_\odot$.

SUPERMASSIVE BLACK HOLES. Objects larger than $10^5 M_\odot$ would collapse directly to black holes without any nuclear burning due to relativistic instabilities, so they would not be excluded by either nucleosynthetic or background light constraints. However, we have seen that halo black holes would heat up the disk stars more than is observed unless they were smaller than about $10^6 M_\odot$, so they would have to lie in the narrow mass range $10^5 - 10^6 M_\odot$ and the disruption of globular clusters and dynamical friction effects may exclude even this range. SMO black holes could still reside outside halos but dynamical and lensing effects imply that any cluster holes must be smaller than $10^9 M_\odot$ and any intergalactic black holes must have a density parameter less than 0.4 between 10^7 and $10^9 M_\odot$.

7 Conclusions

There is good evidence that a large fraction of the Universe is dark and - unless one believes the high deuterium measurements in quasar absorption systems - many of the baryons must also be dark. Therefore at least some of the dark matter problems could have baryonic solutions. If the *local dark matter* is real, it is almost certainly baryonic and probably in the form of brown dwarfs or white dwarfs. However, observations of the Population I mass function give no reason for expecting this and, anyway, this would only explain a small fraction of the missing baryons. If the rest of the dark baryons are not contained in dwarf galaxies or an intergalactic medium, they are probably in the remnants of a first generation of pregalactic or protogalactic stars. The *halo dark matter* could consist at least partly of such remnants and brown dwarfs are currently the favoured candidate. Although observations of the Population II mass function may not support this suggestion, there may be no connection between halo stars and Population II stars anyway. Microlensing searches probably provide the best test of this scenario, although - rather perplexingly - the data currently indicate a lens mass in the white dwarf rather than brown dwarf range.

There has not been space here to discuss non-baryonic candidates but it should be stressed that the *background dark matter* must be mainly non-baryonic if one believes that inflation requires a critical density. The most natural candidate would seem to be a WIMP (i.e. some "cold" elementary particle). Although one cannot exclude primordial black holes - indeed it has been claimed that microlensing already provides evidence for this - such a proposal may require implausibly fine tuning. Unless one invokes an unconventional cosmological nucleosynthesis scenario, the *cluster dark matter* must also be mainly non-baryonic, although there would need to be some baryonic fraction if halos are themselves baryonic. Finally, it should be stressed that, if the non-baryonic dark matter consists of WIMPs, it would naturally collect into galactic halos. One would therefore expect halos to comprise a mixture of MACHOs and WIMPS, so WIMP searchers should not be discouraged by the microlensing results.

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