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Autor: Mukhanov, V.

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INFLATION AND COSMOLOGICAL PERTURBATIONS

By V. Mukhanov

Institute for Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich

Abstract. The inflationary paradigm is discussed. The definiteness of the most important prediction of inflation, namely, the prediction of initial inhomogeneities is analised critically. Some open problems of inflationary cosmology are analised.

I. Introduction

At the end of the seventies most of the cosmologists being confronted with some fundamental questions on the origin of the universe would rather just refer them to the subject of metaphysics. In particular it was unclear whether we even may ask *why* the universe at the beginning was prepared in extremely homogeneous, isotropic state in causally disconnected regions. Furthermore, *why* initial state was so special that the universe survived for 10 billion years and does not become empty and *what* was the reason for Big Bang. One more important problem concerns the origin of small initial inhomogeneities, without which the observed structure of the universe in the form of galaxies and their clusters would never be formed.

Of course one could always say that taking the appropriate initial conditions we will be able to fit the observations. Therefore one of the tasks of "old" cosmology was to find out which initial state permits us to describe in the best manner the available observational data. However, the required initial conditions were so untypical that it was hard to believe that God had no special reason to realize them. Anthropic principle which states that in a completely different universe nobody would raise these questions, could not give satisfactory answer. Actually one can easily imagine the universe which, for instance, is not so isotropic as ours, but where the life nevertheless would be possible.

The favorable way to avoid all these questions was to refer them to the yet unknown quantum theory of gravity.

The situation became more dramatic when the particle physicists applied their new theories to describe the early stages of the universe. For instance, according to most of these theories the phase transitions with formation of the topological defects (monopoles etc.) were inevitable in early times. However the predicted abundances of these defects were in conflict with observations. It could mean one of two possibilities: either these theories are wrong or we do not know any significant features of the evolution of early universe.

In 1981, A. Guth realized and explicitly stated that the problems with homogeneity and flatness of the universe can be solved if we assume that the universe went through an inflationary stage of expansion. During this stage all "old values" were wiped out and the "new ones" were created. Since, as we believe, inflation took place at the energy scale below the Planckian, one can study it basing on the well established physical ideas without referring to unknown quantum gravity.

About 15 years passed since inflation was invented. During this period a lot of work has been done and the subject was well reflected in many review papers and monographs.

Therefore, the purpose of this paper is not to write one more review on inflation, but it is rather an attempt to analyze which facts and ideas of inflationary cosmology have chance to survive in future and to discuss some unsolved problems directly related to the inflationary scenarios. Since the presentation will unavoidably reflect the author's (subjective) point of view, we apologize in advance to those who feel differently with regard to the topics under consideration. The paper covers only a very limited number of topics directly related to the inflationary paradigm. Needless to say, not all of the important contributions to the field will be mentioned here.

II. Homogeneity and flatness from inflation

Let us consider isotropic universe with the line element

$$ds^2 = dt^2 - a^2(t)\,\gamma_{ik} dx^k\, dx^k, \tag{1}$$

where γ_{ik} is the metric of 3D space the constant curvature $\mathcal{K} = 0, \pm 1$. The scale factor $a(t)$ obeys the Einstein equations

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{8\pi}{3}\,\varepsilon, \tag{2}$$

$$\ddot{a} = -\frac{4\pi}{3}\,(\varepsilon + 3p)a, \tag{3}$$

where a dot denotes the time derivative, $H = \dot{a}/a$ is the Hubble constant and ε and $p(\varepsilon)$ are, respectively, the effective energy density and pressure of the matter. We will use everywhere the natural units in which $c = \hbar = G = 1$.

It is easy to check that the conservation law for the matter

$$d\varepsilon = -3(\varepsilon + p)d\ln a \tag{4}$$

is the consequence of the equations (2), (3).

The size of causally connected region in the universe at the moment t is defined by particle horizon:

$$l_p(t) = a(t)\int^t dt'/a(t') \tag{5}$$

Thus, the size of the at present ($t = t_0$) observed universe is about $l_o(t_0) \sim l_p(t_0) \sim (1/H_0)$,where H_0 is the current value of Hubble constant. Recalculated to some very early initial moment of time t_i it was $l_o(t_i) \sim a_i/H_0\, a_0$, where a_i and a_0 are the values of scale factor at the moments t_i and t_0, respectively. In all "old" cosmological models the scale of causally connected region at the moment t_i can be estimated from (5) as $l_c(t_i) \sim l_p(t_i) \sim 1/H_i$. Thus one gets

$$\left(\frac{l_o}{l_c}\right)_i \sim \frac{H_i a_i}{H_0 a_0} = \frac{\dot{a}_i}{\dot{a}_0}\,. \tag{6}$$

Since at present the observed universe is extremely homogeneous in big scales, it should be also the case in the scales $\sim l_o(t_i)$ at t_i. The usual gravity is an attractive force. Therefore, the "velocity" of expansion $\dot{a}(t)$ can only monotonically decrease with the expansion, that is $\dot{a}_i > \dot{a}_0$. Then, as it follows from (6), the size of the homogeneous isotropic region from which the universe originated at $t = t_i$ was bigger than the size of the causally connected region. In "old" models the typical value for the ratio (6) is bigger than 10^{28} if we take t_i close to the Planckian time $t_{Pl} \sim 10^{-43} sec.$. The natural question is: who cared to prepare the universe at the initial moment in exactly the same state in a tremendously big number ($> (10^{28})^3$) of causally disconnected regions? Physical processes which are causal ones cannot be responsible for that and the probability that such a state arose spontaneously is vanishingly small. This is the so called *horizon* problem.

To formulate the *flatness* problem let us write the equation

$$\Omega_i - 1 \equiv \frac{\frac{8\pi}{3}\varepsilon_i - H_i^2}{H_i^2} = (\Omega_0 - 1)\frac{(a_0 H_0)^2}{(a_i H_i)^2} \sim \left(\frac{\dot{a}_0}{\dot{a}_i}\right)^2 \tag{7}$$

which immediately follows from (2). Since the ratio $\dot{a}_0/\dot{a}_i \lesssim 10^{-28}$ and $\Omega_0 = O(1)$ in the "old" cosmological models, we see that the initial values of H_i^2 and ε_i should be extremely fine tuned to an accuracy better than of one part in 10^{56}, while the natural expected value for $|\Omega_i - 1|$ would be of order one.

Only in such a case the universe survives for 10 billion years and does not become empty. One more question which can be addressed here concerns the origin of the extremely big initial velocities $\propto \dot{a}_i$ (Big Bang), which then decrease till the current moderate values.

All these questions would disappear only if the initial "velocity" $\dot{a}_i$ would be of the order of $\dot{a}_0$ or less. This occurs in the inflationary models of the evolution of universe. The most important feature of them is that according to these models during some stage (inflation) the gravity acts effectively as repulsive (antigravitational) force. As it is obvious from equation (3) this can only happen if the energy dominance condition is somewhere violated, i.e., $(\varepsilon + 3p) < 0$. Then the acceleration $\ddot{a}$ becomes positive and the "velocity" $\dot{a}$ increases with time. One can start from the very small initial "velocities" $\dot{a}_i \lesssim \dot{a}_0$, and then the universe acquires big"velocities" via the stage of accelerated expansion.

The important property of the expanding universe with the violated energy-dominance condition is the existence of event horizon. Actually, the size of the event horizon at $t = t_i$ is given by the formula

$$r_e(t_i) = a(t_i) \int_{t_i}^{\infty} \frac{dt}{a(t)} = a(t_i) \int_{a_i}^{\infty} \frac{da}{\dot{a}a} \tag{8}$$

and the integral obviously converges at $a \to \infty$ since $\dot{a}$ increases with a. In such a universe the observer will never know what happened at the distances $r > r_e(t_i)$ from him at the moment t_i. Consequently, if the universe was homogeneous and isotropic in some domain with the scales $r > r_e(t_i)$, then near the center of this domain the universe can expand as a homogeneous, isotropic one forever, even if it was very inhomogeneous outside. Moreover, if during inflationary stage $\varepsilon + p \ll \varepsilon$ then, as it follows from (4), the energy density ε changes insignificantly compared to the relative change of scale factor, i.e. $\varepsilon_i/\varepsilon(t) \ll a(t)/a_i$. Regorously speaking this consideration refers only the the eternal inflationary stage. If inflation finally ends then the above arguments should be a little modified. However the main idea remains basically the same also for inflation with finite duration.

Thus we see an obvious indication that homogeneity and horizon problems can be solved in inflationary model . Actually one can start with a small causally connected homogeneous region in an inhomogeneous universe and increase tremendously its size during the inflation without significant change of the energy density in it. The particle horizon will also increase tremendously. It is important that if the size of this region is originally bigger than the size of the event horizon then the inhomogeneities which present outside this region will not disturb the evolution near the center of it. If the matter finally converts, e.g., into a radiation or dust as a result of the transition *synchronized by the cosmic time* t, one gets a big piece of an isotropic Friedmann universe.

If inflation occurred very early ($t_i \sim 10^{-35} \div 10^{-43} sec$) and after that the universe evolved according to the "standard" scenario, the velocity at the end of the inflation $\dot{a}_f$ should be $\sim 10^{28}\dot{a}_0$. To avoid the problem with causality and have no time tuning for $(\Omega_i - 1)$ one should have $\dot{a}_i \lesssim \dot{a}_0$ at the beginning of inflation (see eqs (6),(7)).

So only if $\dot{a}_f/\dot{a}_i$ is not less than 10^{28}, inflation solves the horizon and flatness problems. When the "velocity" $\dot{a}$ increases significantly more than in 10^{28} time (no fine tuning for the duration of inflation) then, $\dot{a}_i \ll \dot{a}_0$. Inspecting the formula (6) we see that in this case the size of a region where we could expect homogeneity at $t = t_0$, is much bigger than the size of the observed universe. Thus one can say that inflation predicts that the universe is isotropic in the scales much bigger than the present horizon. (Unfortunately this can only be checked in many billion years.) The other (verifiable) prediction directly follows from eq. (7). If $|\Omega_i - 1| \sim O(1)$ and $\dot{a}_i \ll \dot{a}_0$, then $|\Omega_0 - 1| \ll 1$, i.e. the energy density at present should be very close to the critical density.

The problem with an overproduction of monopoles and other topological defects is also easily solved by inflation. Actually if monopoles were produced in a big amount before the inflation, their density drops to a negligible value after inflation.

III. Inflationary scenarios

In the previous consideration two questions were left completely open. One of them is how to realize the equation of state for which the energy-dominance condition is violated. The other one concerns the transition from inflation to the Friedmann stage. To answer these questions we should consider the concrete inflationary scenarios. At present there exist probably too many different scenarios, so it is hard to believe that we know the particular one which was actually realized in the very early universe. We just quote some of them: new, chaotic, extended, hybrid, natural, etc.. However, in spite of the fact that, at a first glance, they look different, all these scenarios have the same main physical features. Hence one can hope that we got, at least, the right underlying physical idea of what kind of processes were happening in the very early universe.

The simplest way to break down the energy dominance condition is to consider the matter with the effective equation of state $p \approx -\varepsilon$, corresponding to the cosmological term in the Einstein equations. Of course, p and ε in such a case, can still be slowly varying functions of time. There are two natural ways to get such an equation of state: either using classical scalar field or considering vacuum polarization of quantum fields in the external gravitational field. Vacuum polarization effects induce the corrections in Einstein equations. Such a theory is conformally equivalent to Einstein theory with the extra scalar fields (B. Whitt, 1984). Therefore in both cases the inflationary models look very similar.

From the very beginning it was clear that the concrete model suggested by A. Guth (1981) ("old" inflation) does not work, since in this model the transition from inflation to the Friedmann universe is not smooth enough to avoid big inhomogeneities. However, in those days, there already existed a working model based on vacuum polarization effects. This model was proposed by A. Starobinsky (1980), who hoped to solve the problem of singularity in this way. The present author and G. Chibisov (1981) investigated the vacuum metric fluctuations in the Starobinsky model. They found that the phase transition is due to the longwave vacuum fluctuations and can lead to a smooth Friedmann universe. A somewhat similar model relying on the scalar field with Coleman-Weinberg type potential ("new" inflation) was invented by A. Linde (1982) and A. Albrecht and P. Steinhardt (1982). In both of these models the initial conditions necessary for inflation were of very delicate nature. In particular, to start inflation it was necessary to put the initial field in a particular state (maximum of the potential). In addition, the models required very unnatural values for parameters of the potential.

The situation changed when A. Linde (1983) realized that the inflationary expansion is not special, but rather a general property of models with scalar fields and proposed chaotic inflationary scenario. This kind of scenarios can also be easily realized in higher derivative gravity without scalar fields. They are different from the previous ones mainly in two aspects. First, they do not require the very special shape of the potential and the initial values for scalar field. Second, the transition from inflation to the Friedmann era occurs as a result of the classical evolution of the field.

After this realization, the building of various new models for inflation became to some extent a purely technical matter. It became clear that the inflationary stage arises under rather natural initial conditions.

The main features which distinguish the existing inflationary scenarios are the following. First of all, it is the mechanism responsible for the emergence of the state in which the energy-dominance condition is violated. Such a state can be realized either via the classical scalar fields (or scalar condensates), or due to the vacuum polarization effects. The existence of the fundamental scalar fields is *not* a necessary condition for inflation. Second, the particular models are characterized by the type of transitions from inflationary to Friedmann epoch. The transition can be very violent accompanied by the formation of bubbles or very smooth. The reason for transition could be the following: underbarrier tunneling, instability due to the quantum fluctuations or just usual classical evolution of the scalar field. According to the most successful scenarios the transition is very smooth. There exist more complicated models which combine several features. Among them it is worth mentioning the extended inflation (D. La and P. Steinhardt, 1989).

The answers to the important questions about the energy scale of inflation and reheating after the inflation directly depend on the concrete model for inflation.

Among all scenarios the chaotic inflation is the simplest and most representative one. It has features that have a lot in common with many inflationary models. Therefore, concluding this section, we would like briefly remind this scenario. The purpose of this consideration is just to give an idea about dynamical evolution of scale factor and scalar field during inflation. Therefore for simplicity we consider only the case of flat expanding universe filled by homogeneous scalar field $\varphi(t)$ with the potential $V(\varphi)$. The evolution of the scale factor $a(t)$ and the field $\varphi(t)$ can be described by the 0-0 Einstein equation and the equation for scalar field

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) \tag{9}$$

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + V_{,\varphi} = 0 \tag{10}$$

where $V_{,\varphi} = \partial V/\partial\varphi$.

The equation for the scalar field looks similar to the equation for the particle moving in the medium with the viscosity $3H$. Therefore one can suspect that if the viscosity is large then there exists the solution for which $\varphi(t)$ changes not very fast, that is $\dot{\varphi}^2 \ll V$, $|\ddot{\varphi}| \ll H\dot{\varphi}$. Actually skipping the terms $\dot{\varphi}^2$ in the eq. (9) and $\ddot{\varphi}$ in the eq. (10) one easily gets the solution of the simplified system of equations. For instance, for a polynomial potential

$V(\varphi) = \frac{\lambda}{n}\varphi^n$, it can be written down as

$$a(\varphi) = a_i \exp\left[\frac{4\pi}{n}(\varphi_i^2 - \varphi^2(t))\right] = a_f \exp[-\frac{4\pi}{n}\varphi^2(t)] \; . \tag{11}$$

The scalar field $\varphi(t)$ slowly decreases with time. In particular, in the case of a massive scalar field ($V = \frac{1}{2}m^2\varphi^2$):

$$\varphi(t) = \varphi_i - \frac{m}{(12\pi)^{1/2}}\, t \; . \tag{12}$$

It is easy to check that $\dot{\varphi}^2 \ll V$, $|\ddot{\varphi}| \ll 3H\dot{\varphi}$ for the solution (11) only when $\varphi \gg 1$. Thus the stage described by (11) will be over when the field φ drops to the Planckian value $\varphi_f \sim 1$. During this stage $p+\varepsilon \propto \dot{\varphi}^2$ is much smaller than $\varepsilon \propto V(\varphi)$ and we have accelerated expansion-inflation. If the initial value of the scalar field was $\varphi_i \gg 1$, then the scale factor and velocities increase tremendously during the inflation $\dot{a}_f/\dot{a}_i \sim a_f/a_i \sim \exp(\varphi_i^2)$. If we want to stay outside the quantum gravity domain, then the initial value of the potential should not exceed the Planckian scale, that is, $V_i < 1$. Therefore, for the massive scalar field, the scale factor on inflation changes in $(a_f/a_i)_{max} \sim \exp(1/m^2)$ times, while the energy changes only in $1/m^2$ times. If $m \sim 10^{-6}\ m_{Pl} \sim 10^{-13}$GeV, then $(a_f/a_i) \sim \exp(10^{12}) >> 10^{28}$ and the inflation ends at the energy scale $H \sim m \sim 10^{13}$GeV.

IV. Quantum fluctuations

The valuable extra bonus from inflation is the explanation of the origin of small initial inhomogeneities responsible for the observable structure in the universe. It is supposed that they emerged from the initial vacuum fluctuations, which are unavoidable in any quantum theory. The spectrum of primordial inhomogeneities was first calculated in the inflationary model based on the vacuum polarization by the author of this paper and G. Chibisov (1981). Later several groups of authors analyzed the fluctuations in the "new" inflationary model based on the Coleman-Weinberg type potential for scalar field (A. Guth and S.-Y. Pi, 1982; S. Hawking, 1982; A. Starobinsky, 1982; J. Bardeen et al., 1983). In both cases it was found that the amplitude of metric fluctuations generated on inflation depends logarithmically on the scale in a huge scale range. Thus the fluctuations near the Planckian length and in much bigger scales corresponding to galaxies and their clusters have comparable amplitudes after inflation. It opens the possibility to give quantum mechanical explanation to the origin of the galaxies and their clusters. The universality of the scale invariant spectrum of fluctuations was further confirmed in the other inflationary scenarios. The only things which depend on the particular model are the power of the logarithmic dependence and the amplitude of the spectrum. In some models (with exponential potential, or in extended inflation) it is not logarithm but some small power. Hence, the scale invariant spectrum can be taken very seriously as the universal and important prediction of inflation. Only in more complicated (usually fine tuned) inflationary scenarios the spectrum can have a more complicated shape. For further references and review on the theory of cosmological perturbations see, for instance V. Mukhanov et al., (1992). To give an underlying physical idea of the origin of the fluctuations we consider here the simplified derivation of the spectrum of fluctuations in the chaotic inflationary models. To simplify the formulae we will ignore the numerical coefficients $\sim O(1)$.

Let us consider small inhomogeneities in the distribution of a scalar field in a flat isotropic universe, that is $\varphi = \varphi_0(t) + \delta\varphi(x,t)$, where $|\delta\varphi| \ll \varphi_0$. They induce the disturbances in the metric (1), which in a linear approximation can be written down as

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Psi)\delta_{ik}dx^i dx^k \; . \tag{13}$$

Here we consider only scalar type metric perturbations and fix the gauge to be a "longitudinal" one. It follows from the $i \neq j$ Einstein equations that the functions Φ and Ψ should be equal in the case of scalar field matter, that is $\Phi = \Psi$. To study the behavior of Φ and $\delta\varphi$ let us write down the linearized 0-0 Einstein equation and the equation for scalar field

$$\frac{1}{a^2}\Delta\Phi - 3H\dot{\Phi} - 3H^2\Phi = 4\pi[-\dot{\varphi}_0^2\Phi + \dot{\varphi}_0\delta\dot{\varphi} + V_{,\varphi}\,\delta\varphi], \tag{14}$$

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} - \frac{1}{a^2}\Delta\delta\varphi + V_{,\varphi\varphi}\,\delta\varphi - 4\dot{\varphi}_0\dot{\Phi} + 2V_{,\varphi}\,\Phi = 0 \; . \tag{15}$$

Taking the plane wave perturbation: $\Phi, \delta\varphi \propto e^{i\bar{k}\bar{x}}$ we will study its behavior when it is an inside as well as an outside *event* horizon on the inflationary stage. The horizon scale during inflation is about H^{-1}.

Inside horizon. In this case $\lambda_{ph} \sim a/k \ll H^{-1}$ (or $k \gg Ha$), and we can neglect the last three terms in eq. (15). Actually the gravitational potential is proportional to $(Ha/k)^2$ and $V_{,\varphi\varphi} \ll H^2$ during the inflation. Then the equation for the scalar field fluctuations similar to the equation for the massless scalar field in an expanding quasi de Sitter universe and $\delta\varphi$ can be quantized in a usual manner. The main result of the quantization is that the vacuum scalar field fluctuations cross the horizon with the typical amplitudes of the order of H, that is (for details see, e.g., Mukhanov et al., (1992))

$$\delta^{\varphi}_{k\sim Ha} \;=\; \sqrt{k^3\delta\varphi_k^2} \sim H_{k\sim Ha} \; . \tag{16}$$

The index $k \sim Ha$ means here that the appropriate quantity should be estimated at the moment of horizon crossing ($k \sim Ha$). We will use this result as the initial condition for the perturbations which cross the horizon.

Outside horizon. Since the scale factor is increasing with time, while H does not change too much, the perturbations with the appropriate comoving wave number k finally will cross the horizon. Therefore we now consider their evolution after horizon crossing ($k \ll Ha$). The system of eqs. (14-15) has two independent solutions. One of them decays very fast. To find the leading nondecaying solution we neglect in the equations (14-15) subdominant terms. After the solution will be found one can easily check that the animated terms are actually negligible. This is the justification for the whole procedure. If we are interested only in the behavior of a dominant mode on inflation, the equations (14-15) can be simplified as

$$\Phi \;\simeq\; -\frac{1}{2}\frac{V_{,\varphi}}{V}\,\delta\varphi \tag{17}$$

$$3H\delta\dot{\varphi} + V_{,\varphi\varphi}\,\delta\varphi + 2V_{,\varphi}\,\Phi \simeq 0\,. \tag{18}$$

Substituting Φ from (17) in (18) and replacing the time derivative by the derivative with respect to $\varphi_0(t)$ with the help of the equations for the background one gets

$$\frac{d\,\delta\varphi}{d\,\varphi_0} = \left(\frac{V_{,\varphi\varphi}}{V_{,\varphi}} - \frac{V_{,\varphi}}{V}\right)\delta\varphi \tag{19}$$

which can be easily integrated in

$$\delta\varphi(\varphi_0) \simeq C\,\frac{V_{,\varphi}}{V}\,. \tag{20}$$

For this solution the skipped earlier terms are proportional to the small parameters k/Ha, $\dot{H}/H^2 \ll 1$. To find the constant of integration in (20) we claim that the amplitude of perturbations at the moment of the horizon crossing was $\sim H_{k\sim Ha}$ (see eq. (16)). Thus finally for the perturbations of scalar field with $k \ll Ha$ one gets

$$\delta_k^{\varphi} \simeq \left(H\,\frac{V}{V_{,\varphi}}\right)_{k\sim Ha}\frac{V_{,\varphi}}{V} \sim \left(\frac{V^{3/2}}{V_{,\varphi}}\right)_{k\sim Ha}\frac{V_{,\varphi}}{V} \tag{21}$$

and for the metric perturbations, correspondingly,

$$\delta_k^{\Phi} \sim \left(\frac{V^{3/2}}{V_{,\varphi}}\right)_{k\sim Ha}\left(\frac{V_{,\varphi}}{V}\right)^2. \tag{22}$$

The above formulae are valid practically till the end of the inflation, when $\varphi_f \sim 1$. Expressing $(V^{3/2}/V_{,\varphi})_{k\sim Ha}$ in terms of k with the help of formula (11) in the case of massive scalar field, one obtains the following spectrum of the metric perturbations at the end of the inflation ($\varphi_f \sim 1$):

$$\delta_k^{\Phi} \sim m\ln\left(\frac{a_f H}{k}\right) \sim m\ln(\lambda_{ph}H)\,. \tag{23}$$

This formula is valid in the scales

$$Ha_f \gg k \gg Ha_i \tag{24}$$

where $a_f = a_i\,\exp(\varphi_i^2)$. This is the desired scale invariant spectrum of scalar metric perturbation. The potential Φ stays practically constant during the subsequent Friedmann evolution in the scales exceeding the "Friedmann particles horizon". After recombination it induces the temperature fluctuations in the angular scales $\theta > 1^o$ according to the formula $\delta T/T = \frac{1}{3}\,\Phi$. Such kind of fluctuations were recently observed by COBE at the level $10^{-5} \div 10^{-6}$. To fit the COBE observations one should take the mass of scalar field to be $m \sim 10^{-6} m_{pl} \sim 10^{13} GeV$.

Gravitational waves. The gravitational waves, which satisfy the equation

$$\ddot{h}_{ik} + 3\,\frac{\dot{a}}{a}\,\dot{h}_{ik} - \frac{1}{a^2}\,\Delta h_{ik} = 0 \tag{25}$$

will also be generated on inflation. Since eq. (25) is similar to equation (15), for $\delta\varphi$, one can expect that their spectrum will also be scale-invariant. Eq. (25) is different from (15) by a few extra terms which are responsible for the subsequent slow growing of the amplitude of scalar field after the perturbation crosses the horizon. Therefore as one can see directly from (25) after skipping the last term in it, the leading mode stays constant for the longwave ($k \ll Ha$) gravitational waves.

Taking the vacuum quantum fluctuations as initial conditions, for the longwave gravitational waves one obtains

$$\delta_k^h \sim H_{k\sim Ha} \, . \tag{26}$$

This result was first derived by A. Starobinsky (1979) and then it was reanalyzed in application to various inflationary scenarios in many papers (see Mukhanov et al., (1992) for refs.). Comparing (26) to (22), we see that the amplitude of the gravitational waves at the end of inflation ($\varphi_f \sim 1$) is $(V/V_{,\varphi})_{k\sim Ha}$ times smaller than the amplitude of scalar metric perturbations in an appropriate scale. For instance, for the massive scalar field this factor is $\varphi_{k\sim Ha} \sim \ln^{1/2}(a_f H/k)$. Hence the gravitational waves contribute less than the scalar metric perturbations to the anisotropy of microwave background. However, these longwave gravitational waves, in principle, can be indirectly detected in a not very far distant future.

V. Backreaction problem and self-reproduction of the universe

The amplitude of the fluctuations δ_k^φ takes the maximum value in the scales $k \sim Ha_i$ which left the horizon at the very beginning of inflation, $a(t_i) = a_i$. In the case of a massive scalar field this amplitude at the end of the inflation ($\varphi_f \sim 1$) is (see (23)),

$$\delta_{max}^\varphi \sim m \ln\left(\frac{a_f}{a_i}\right) \sim m\,\varphi_i^2 \, . \tag{27}$$

If the initial value φ_i is bigger than $m^{-1/2}$, then the inhomogeneities in the scales $k \sim Ha_i$ become too big ($\delta\varphi \approx \varphi_0$) before the end of inflation to consider them perturbatively . Therefore, even if the universe was completely homogeneous at the beginning of inflation, it will become absolutely inhomogeneous at the end of inflation in the scales bigger than $\lambda_{ph}(t_f) \sim H^{-1}\exp(\frac{1}{m})$.

However, for a small mass m the scale of quasihomogeneous pieces in this very inhomogeneous universe is still very big to explain in the observed isotropic "piece" of the universe. In every quasihomogeneous piece there will be small fluctuations with the spectrum (23).

Moreover, if inflation starts at $\varphi_i > m^{-1/2}$ then it will never end (see A. Linde et al., (1994) and refs. there). Actually, at $\varphi > m^{-1/2}$ "quantum diffusion" of a scalar field due to quantum fluctuations dominates over the classical "rolling down". This is the reason why scalar fields instead of going down to the minimum of the potential start to increase in some of the regions, which always exist if $\varphi_i > m^{-1/2}$. Thus the universe always reproduces itself. This rather general picture is valid practically in all realistic inflationary scenarios. To study the behavior of scalar field in the regions with scales $1/H$ by taking into account the backreaction of the quantum fluctuations, the diffusion equation approach was intensively applied. Many new interesting results concerning the global structure of the universe were obtained (A. Linde et al., 1994). However much more work has to be done here to clarify completely the global properties of the inflationary universe.

Now we can just say that it is quite hard to avoid the global inhomogeneity and self-reproduction of the universe in the inflationary models.

As we have seen the backreaction of quantum fluctuations on the behavior of the background model is absolutely important if inflation starts at $\varphi_i > m^{-1/2}$. Could one conclude from here that it can be ignored if $\varphi_i < m^{-1/2}$? The answer to this question is *no*. Actually, the produced fluctuations are relatively unimportant for the dynamics of background only if their energy-momentum tensor (EMT) stays always smaller than EMT of the classical field which drives the inflation.

Let us estimate the contribution to the energy of the induced longwave fluctuations ε_{ind}, coming from the "potential" term

$$\varepsilon_{ind} \sim m^2 \langle \delta\varphi^2 \rangle \sim m^2 \int \delta\varphi_k^2 \, k^3 \, \frac{dk}{k} \, . \tag{28}$$

At the end of the inflation the spectrum of perturbation $(\delta_k^{\varphi})^2 = \delta\varphi_k^2 k^3$ is given by (23). Hence one gets

$$\varepsilon_{ind}(t_f) \sim m^4 \int_{Ha_i}^{Ha_f} \ln^2\left(\frac{Ha_f}{k}\right) d\left(\ln \frac{Ha_f}{k}\right) \sim m^4 \varphi_i^6 \, . \tag{29}$$

The energy of the classical scalar field at t_f (when $\varphi_f \sim 1$) is about $\varepsilon_{cl}(t_f) \sim m^2\varphi_f^2 \sim m^2$. Thus the backreaction of produced fluctuations becomes important before the end of inflation $(\varepsilon_{ind}(t_f) > \varepsilon_{cl}(t_f))$ if

$$\varphi_i > m^{-1/3} \, . \tag{30}$$

This scale is lower than the scale of self-reproduction $\sim m^{-1/2}$. It seems that if $m^{-1/2} > \varphi_i > m^{-1/3}$ then inflation finally ends everywhere. However the fluctuations can significantly disturb the homogeneity of the universe because of the cumulative effect from many perturbations with different scales. This problem needs further investigation. Now we can only conclude that the universe is certainly quasihomogeneous in the scales $\sim H^{-1}\exp(m^{-2/3})$.

For $m \sim 10^{-6}$ this scale $\sim \exp(10^4)$ is much smaller than the scale associated with self-reproduction $\exp(m^{-1}) \sim \exp(10^6)$. Nevertheless it is still big enough to have no problem with an isotropy of the observed universe.

Concluding this Section, we would like to stress once again that the problem of backreaction of quantum fluctuations is a very crucial one in the inflationary scenarios even at the energy scales much below the Planckian ones.

VI. Open problems

In spite of the great success of the inflationary model, many questions have to be answered before we complete the picture of the evolution of the early universe. Actually at present the status of inflation is to some extent paradoxical. There is a nice idea on which the model is based, however there is no concrete scenario of which we could more or less be certain that it was indeed realized in nature. There exist too many competitive models. Therefore we would like to discuss briefly certain open problems which are worth being tackled in the future.

The scenario

At present it is not clear what was the particular mechanism responsible for inflation. For instance, up to now we haven't proceeded too much selecting one concrete model which would obviously be preferable over the others. One just could say that the most attractive scenarios are based on the idea of chaotic inflation. However, even if we accept this point, much freedom is left to choose a concrete scalar field potential in particular elementary particle theory. In addition it cannot be excluded that the chaotic inflation was actually due to the vacuum polarization.

Questions which could not be answered before we solve the above mentioned problem are the following. How long was inflation and at which energy scales did it happen? There is a big variety of quite different possibilities. The scales can vary from Planckian to electroweak scales. An other related question concerns the transition from inflationary stage to Friedmann era and the problems of reheating after inflation. Although the reheating was intensively investigated recently (see, e.g., L. Kofman et al., 1994; R. Brandenberger et al., 1994 and refs. there) and it was shown that there is no principle problem of recreating the radiation after inflation, it is not yet clear how this actually happened in the early universe.

One more problem refers to the explanation of a small parameter in the theory (sometimes this is called fine tuning problem) which is necessary to build a relatively small number 10^{-4} which characterizes the required dimensionless amplitude of the metric fluctuations. For instance, in the case of a massive scalar field this parameter is the mass which should be $\sim 10^{-6} m_{pl}$ to avoid contradiction with observations, while in the case of $\lambda\varphi^4$-potential, the coupling constant λ should be extremely small $\sim 10^{-12} \div 10^{-14}$. We believe that a significant progress here can only be achieved in parallel to the further progress in elementary particle physics. On the other hand it seems obvious that cosmology can significantly help particle theory in selecting a realistic model of fundamental interactions at high energies.

How certain are the predictions?

Since the definite model of elementary interactions at high energies is not available at present, one cannot answer the question how many inflationary stages took place in the early universe. Most of the popular theories involve many scalar fields. Therefore the concrete scenario for the evolution of the early universe can be more complicated than just a simple inflationary model. This ambiguity related to the number of scalar fields "participating in the game" leads to the variations in predictions of simple inflation.

Fortunately if the last inflationary stage lasted long enough everything about the previous evolution of the universe is wiped out in the observable scales. Therefore, the explanation of isotropy of the observed universe, furthermore the predictions of $\Omega_0 = 1$, and the scale invariance of perturbation spectrum are not much influenced by uncertainties in inflationary scenarios. However even in the case of a long final inflationary stage there is still uncertainty about the nature of the produced perturbations. They should be adiabatic if the inflaton field is responsible for them. In such a case there exist strong limits on the energy scales of final inflationary stage. However, if besides of the inflaton field there exist some other relatively stable scalar fields, one can easily obtain either isocurvature fluctuations which lately were converted into adiabatic ones (see, e.g., L. Kofman and A. Linde, 1987) or even entropy perturbations (V. Mukhanov and C. Schmid, in preparation). For entropy perturbations COBE normalization for the gravitational potential differs from the case of adiabatic perturbations. In the latter case the restrictions on the energy scales of the final inflation are not very restrictive.

Moreover, if the final stage of inflation takes place only in some definite short time interval (fine tuning for duration of inflation), then the spectrum of perturbations in a double inflationary model, for instance, can differ from the scale invariant one in an interesting galactic scales (V. Mukhanov and M. Zel'nikov, 1991; D. Polarski and A. Starobinsky, 1992).

Now about prediction of $\Omega_0 = 1$. For a long time it was believed that this prediction of inflation is unavoidable. However, it was found recently that it can be avoided and one can also get in an inflationary model $\Omega_0 < 1$ if we assume that the final inflation lasted for less than seventy Hubble times and at the end of the previous long inflation (which we need to solve the isotropy problem), the subbarrier transition took place. It can either be done in the model with a complicated potential for one scalar field (M. Bucher et al., 1994) or in relatively simple models with few scalar fields (A. Linde, 1995). The pessimism which could arise as a consequence of the above mentioned ambiguities can be easily dispersed if we take into account that all possibilities discussed can only be realized in fine tuned inflationary models. A simple inflation does not lead to these ambiguities. However, we should keep open all options and therefore further investigations on this track should be useful.

Transition from quantum to classical?

When we look at the sky we see the galaxies in certain positions. If these galaxies were formed from original quantum fluctuations, a natural question arises: how a galaxy, e.g., the Magellanic cloud, occurred at a *certain* place. Actually, the initial vacuum state was translationally invariant, hence there were no preferable positions in the space. The subsequent unitary evolution of the initial state does not destroy this invariance.

The answer to the above question is directly related to the problem of transition of quantum fluctuations to classical inhomogeneities. The necessary condition for this is the emergency of a decoherence for amplified cosmological perturbations. The appearance of this decoherence can easily be understood either in terms of interaction with "environment" or just as a result of "course graining". A related issue concerns the entropy of cosmological perturbations (R. Brandenberger et al., 1992; M. Gasperini and M. Giovanini, 1993).

However, the decoherence is a necessary but *not sufficient* condition to explain the breaking of translational invariance of the initial quantum state. Actually, as a result of the unitary evolution of initial state, one gets the state which can be represented as a superposition of many states, each corresponding to a particular macroscopical realization of galaxy distribution. Such a state is a close analogy of the "Schrödinger cat". Therefore, to pick up from this superposition the particularly observed macroscopic state we should appeal either to the Bohr's reduction of the state vector or to the many-world interpretations of quantum mechanics (H. Everett, 1957).

Global structure of the universe and singularity problem.

As we have seen earlier inflation leads to a nontrivial global structure of the universe. It predicts that the universe in the scales much bigger than the present horizon should be quite inhomogeneous. Of course, one could raise the question why we should care at all about so big scales which are not observable. There is a tendency to refer this question to metaphysics.

We don't think this is a fruitful attitude. Actually the problem of initial conditions cannot be considered as completely solved by inflation. The inflation only increases tremendously the "measure" of initial conditions which could lead to our type universe. However, the notion of the "measure" is not well defined there and it is not obvious from which pre-initial state the inflation could start off. On one hand this question should probably be referred to the subject of quantum cosmology. On the other hand it seems that we could move far enough even if we stay outside of the quantum gravity domain. Significant progress in the description of the global properties of the Universe was recently achieved (see A. Linde et al., 1994 and refs. there). However, many questions still remain to be answered. One of the important problems for instance, is how to define the measure for initial conditions in the context of stochastic approach to inflation. It cannot be excluded that further progress in this field can also lead to definite observational predictions (A. Linde et al., 1995; A. Vilenkin, 1995).

To emphasize the need for further investigations of the global structure of the universe, we feel obliged to mention the problem of singularity. Without solving this problem one cannot consider the problem of initial conditions to be completed. It seems that stochastic approach to inflation opens new perspectives to attack the problem of singularity which J. Wheeler characterized as "the crisis of the modern physics". It was also found that with the aid of inflation the singularity in isotropic homogeneous universe might be avoided (V. Mukhanov and R. Brandenberger, 1992). At present the number of open problems in this field greatly exceeds the number of solved problems.

Alternatives

Cosmic strings: Cosmic strings and textures provide us with the mechanisms for the origin of initial inhomogeneities which compete with inflation. However, they cannot help us to solve the problem of isotropy and flatness of the universe. Therefore these theories can seriously be considered only in combination with inflation which should take place before the formations of topological defects. We feel that this unjustifiably complicates the inflationary model, which itself can explain the origin of inhomogeneities. However, on the basis of the present observations one cannot yet rule out this possibility. Therefore the further investigations of topological defects and future observations are needed to decide which scenario was actually realized in nature.

Fundamental strings: An interesting model for the early universe was proposed recently in the context of fundamental string theory by G. Veneziano et al.(for review see G. Veneziano, 1994). According to their approach the universe went through the stage of superinflationary expansion. This model is quite different from the usual inflationary models, since the universe at the superinflationary stage expands (in Brans-Dicke frame) much faster than according to standard inflationary scenarios. It needs further examination before it can be seriously compared with inflationary models. In particular, one of the hardest problems to be solved there is the problem of graceful exit - transition from superexpansion to Friedman stage (R. Brustein and G. Veneziano, 1994).

Summarizing the previous considerations one can say that inflation solves in a nice way several mysterious and at the first glance unrelated to each other puzzles of the "old" cosmology on the basis of a simple assumption about breaking down of the energy-dominance condition in the very early universe. Namely, it *explains* in a natural way the origin of the big initial velocity, the homogeneity-isotropy and flatness of the observed universe, the emergency of small initial inhomogeneities responsible for galaxy formation. Inflationary cosmology *predicts* the nearly scale-invariant spectra for the energy density fluctuations and gravitational waves, the present value of the cosmological parameter Ω_0, which should be very close to one. All these predictions are highly independent of the particular inflationary scenarios.

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