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FRW model with vector fields in N=1 supergravity

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Abstract. A FRW model obtained from N=1 supergravity with supermatter is analysed in this paper. The matter content is restricted to a vector supermultiplet. The Lorentz and supersymmetry constraints are derived. Non-trivial solutions (no-boundary and wormhole states) are then found.

1 Introduction

Research in supersymmetric quantum cosmology using canonical methods started about 10 years or so [1]. A review on quantum N=1,2 supergravity is found in ref. [2].

Finding and identifying physical states in minisuperspaces obtained from N=1 supergravity with supermatter constitutes an important assignment. A FRW model in the presence of a scalar supermultiplet (constituted by complex scalar fields, ϕ , $\bar{\phi}$ and their spin $-\frac{1}{2}$ partners, χ_A , $\bar{\chi}_{A'}$) and a vector supermultiplet (formed by a gauge vector field $A_{\mu}^{(a)}$ and its supersymmetric partner) was analysed in ref. [3]. However, the results found there were disapointing: the only allowed physical state was $\Psi = 0$.

The main purpose of this paper and ref. [4] is to initiate a discussion on the parodoxical situation found in ref. [3]. We will study a FRW model where supermatter is restricted to a vector supermultiplet. In section 2 we will address the ansätze for the field variables employed in ref. [3]. In section 3 we derive the quantum constraints. In contrast with ref. [3], non-trivial solutions are obtained. We identify a component of the Hartle-Hawking (no-boundary) solution [8]. These results support our approach. Finally, our discussions and conclusions close this paper in section 4.

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2 Ansätze for the field variables

The action for our model is obtained from the more general theory of N=1 supergravity with gauged supermatter [6]. We put all scalar fields and corresponding supersymmetric partners equal to zero. Our field variables will be the tetrad, $e_{\mu}^{AA'}$, the gravitino fields, $\psi_{\mu}^{A}, \bar{\psi}_{\nu}^{A'}$, a gauge spin-1 field, $A_{\mu}^{(a)}$, ((a) is a gauge group index) and the spin- $\frac{1}{2}$ partners, $\lambda_{A}^{(a)}, \bar{\lambda}_{A'}^{(a)}$. The restriction to a closed FRW model requires specific ansätze for these fields.

We choose the geometry to be that of a k=+1 Friedmann model with S^3 spatial sections. The ansatz for the tetrad can then be written as

$$e_{a\mu} = \operatorname{diag}(N(\tau), a(\tau)) , \qquad (2.1)$$

where \hat{a} and i run from 1 to 3. $E_{\hat{a}i}$ is a basis of left-invariant 1-forms on the unit S^3 with volume $\sigma^2 = 2\pi^2$. The Lagrange multipliers ψ^A_0 and $\bar{\psi}^{A'}_0$ are taken to be functions of time only. The ansatz for the gravitino field further includes

$$\psi^{A}{}_{i} = e^{AA'}{}_{i}\bar{\psi}_{A'} , \ \bar{\psi}^{A'}{}_{i} = e^{AA'}{}_{i}\psi_{A} , \tag{2.2}$$

where we introduce the new spinors ψ_A and $\bar{\psi}_{A'}$ which are functions of time only.

In the case of pure N=1 supergravity, ansätze (2.1), (2.2) are preserved by a combination of local coordinate, Lorentz and supersymmetry transformations. This holds provided that the generators of Lorentz, coordinate and supersymmetry transformations satisfy specific conditions. The Lorentz constraint $J_{AB} == 0$ also has to be imposed.

Let us then consider the case of a gauge group $\hat{G} = SU(2)$.

The simplest choice would be to take $A_{\mu}^{(a)}$, $\lambda_{A}^{(a)}$, $\bar{\lambda}_{A'}^{(a)}$ as time-dependent only. However, this is not sufficient in ordinary quantum cosmology with Yang-Mills fields. Special ansätze are required for $A_{\mu}^{(a)}$ [8, 5]. The ansatz described in ref. [8, 5] for $A_{\mu}^{(a)}$ (and also employed in [3]) is the simplest one that allows vector fields to be present in a FRW geometry. The spin-1 field is taken to be

$$\mathbf{A}_{\mu}(t) \ \omega^{\mu} = \left(\frac{f(t)}{4} \varepsilon_{(a)i(b)} \mathcal{T}^{(a)(b)}\right) \omega^{i} \ . \tag{2.3}$$

Here $\{\omega^{\mu}\}=\{dt,\omega^i\}$, $\omega^i=\hat{E}^i{}_{\hat{c}}dx^{\hat{c}}$ $(i,\hat{c}=1,2,3)$ and $\mathcal{T}_{(a)(b)}$ are the generators of the SU(2) gauge group. We will use the more general choice for the fermionic partner of $A^{(a)}_{\mu}$ as $\lambda^{(a)}_A=\lambda^{(a)}_A(t)$. Is then possible to see that these choice of field configurations are invariant for a specific and rather restrictive combination of Lorentz, gauge, supersymmetry and local coordinate transformations (see ref. [4] for more details.

3 Quantum constraints and solutions

We chose $(\bar{\lambda}_{A'}^{(a)}, \psi_A, a, f)$ to be the coordinates of the configuration space and $(\lambda_A^{(a)}, \bar{\psi}_A, \pi_a, \pi_f)$ to be the momentum operators in this representation. Hence $\lambda^a_A \to -\frac{\partial}{\partial \bar{\lambda}^{(a)}A}$, $\bar{\psi}_A \to \frac{\partial}{\partial \psi^A}$, $\pi_a \to \frac{\partial}{\partial a}$, $\pi_f \to -i\frac{\partial}{\partial f}$.

The supersymmetry constraints for our FRW model have the differential operator form

$$S_{A} = -\frac{1}{2\sqrt{6}}a\psi_{A}\frac{\partial}{\partial a} - \sqrt{\frac{3}{2}}\sigma^{2}a^{2}\psi_{A} - \frac{1}{8\sqrt{6}}\psi_{B}\psi^{B}\frac{\partial}{\partial\psi^{A}}$$

$$- \frac{1}{4\sqrt{6}}\psi^{C}\bar{\lambda}^{(a)}_{C}\frac{\partial}{\partial\bar{\lambda}^{(a)A}} + \frac{1}{3\sqrt{6}}\sigma^{a}_{AB'}\sigma^{bCC'}n_{D}^{B'}n_{C'}^{B'}\bar{\lambda}^{(a)D}\psi_{C}\frac{\partial}{\partial\bar{\lambda}^{(b)B}}$$

$$+ \frac{1}{6\sqrt{6}}\sigma^{a}_{AB'}\sigma^{bBA'}n_{D}^{B'}n_{A'}^{E}\bar{\lambda}^{(a)D}\bar{\lambda}^{(b)}_{B}\frac{\partial}{\partial\psi^{E}} - \frac{1}{2\sqrt{6}}\psi_{A}\bar{\lambda}^{(a)C}\frac{\partial}{\bar{\lambda}^{(a)C}}$$

$$+ \sigma^{a}_{AA'}n^{BA'}\bar{\lambda}^{(a)}_{B}\left(-\frac{\sqrt{2}}{3}\frac{\partial}{\partial f} + \frac{1}{8\sqrt{2}}(1 - (f - 1)^{2})\sigma^{2}\right) + \frac{3}{8\sqrt{6}}\bar{\lambda}^{a}_{A}\lambda^{(a)C}\frac{\partial}{\partial\psi^{C}} \quad (3.1)$$

and Hermitian conjugate.

When matter fields are taken into account we have $J_{AB} = \psi_{(A} \bar{\psi}^{B'} n_{B)B'} - \lambda_{(A}^{(a)} \bar{\lambda}^{(a)B'} n_{B)B'} = 0$. The Lorentz constraint J_{AB} implies that a physical wave function should be a Lorentz scalar:

$$\Psi = A + B\psi^{C}\psi_{C} + d_{a}\lambda^{(a)C}\psi_{C} + c_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C} + e_{ab}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\psi^{D}\psi_{D}
+ c_{abc}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\psi_{D} + c_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D} + d_{abcd}\bar{\lambda}^{(a)C}\bar{\lambda}^{(b)}_{C}\bar{\lambda}^{(c)D}\bar{\lambda}^{(d)}_{D}\psi^{E}\psi_{E}
+ \mu_{1}\bar{\lambda}^{(2)C}\bar{\lambda}^{(2)}_{C}\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_{D}\bar{\lambda}^{(1)E}\psi_{E}
+ \mu_{2}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(3)D}\bar{\lambda}^{(3)}_{D}\bar{\lambda}^{(2)E}\psi_{E} + \mu_{3}\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)D}\bar{\lambda}^{(3)E}\psi_{E}
+ F\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)}_{D}\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)E}_{E} + G\bar{\lambda}^{(1)C}\bar{\lambda}^{(1)}_{C}\bar{\lambda}^{(2)D}\bar{\lambda}^{(2)D}\bar{\lambda}^{(3)E}\bar{\lambda}^{(3)E}\psi_{F}$$
(3.2)

where A, B,...,G are functions of a, f only.

From $S_A \Psi = 0$, $\bar{S}_A \Psi = 0$ we obtain

$$A = e^{-3\sigma^2 a^2} e^{\frac{3}{16}\sigma^2 \left(-\frac{f^3}{3} + f^2\right)}, \ G = e^{3\sigma^2 a^2} e^{\frac{3}{16}\sigma^2 \left(\frac{f^3}{3} - f^2\right)}. \tag{3.3}$$

The last solution in (3.3) is present in the Hartle-Hawking (no-boundary) solution of ref. [8]. However, the wave function (3.3) represents only one of the components of the wave function in ref. [8]. The first solution in (3.3) has wormhole features. The Dirac bracket of the supersymmetry constraints induces an expression whose bosonic sector corresponds to the (decoupled) gravitational and vector field components of the Hamiltonian constraint in ref. [8].

4 Discussions and Conclusions

Summarizing our work, we considered the canonical formulation of the more general theory of N=1 supergravity with supermatter [6] subject to a k=+1 FRW geometry. Ansätze for the the gravitational and gravitino fields, the gauge vector field A^a_{μ} and fermionic partners were introduced. The scalar fields and their partners were set equal to zero.

Concerning the ansätze employed here (and also in ref. [3]), the form of the tetrad and gravitinos were consistent with the FRW geometry. Supersymmetry invariance was achieved for $A^{(a)}_{\mu}$ and $\lambda^{(a)}_{A}$ if further conditions were imposed.

Interesting physical features were derived in secction 3. After a dimensional reduction, we obtained the supersymmetric constraints. We found a non-trivial solution that can be interpreted as a (Hartle-Hawking) no-boundary solution. This result were quite supportive. Namely, the Hartle-Hawking solution found here corresponded to a component of a solution found from a Wheeler-DeWitt equation in ordinary quantum cosmology [8].

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