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Large Black holes have no Hair

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Abstract. We show that, under very general conditions, the region with non-trivial structure of the non-linear matter fields in hairy black holes, must extend beyond 3/2 the horizon radius, independently of all other parameters present in the theory. We argue that this is a universal lower bound that applies in every theory where hair is present. This *no short hair conjecture* is then put forward as a more modest alternative to the original *no hair conjecture*, the validity of which now seems doubtful. Moreover, we argue that, for theories with massive fields, this can be interpreted instead as *large black holes have no hair*, a statement that is very suggestive in the context of attempts to identify black holes with some sort of elementary particles

The belief that stationary black holes were completely specified by the conserved charges that can be measured at asymptotic infinity, the *"no hair conjecture"* (NHC) [1], was based on both the famous rigorous results known as black hole uniqueness theorems in Einstein Maxwell Theory, as well as some other "no hair theorems" for other specific theories, and on the physical argument that suggested that all matter fields present in a black hole space time would be, eventually, either radiated to infinity, or "sucked" into the black hole, except when those fields were associated with conserved charges defined at asymptotic infinity.

The discovery of the "color black holes" (*i. e.*, static black hole solutions in Einstein-Yang–Mills (EYM)[2] theory that require for their complete specification, not only the value of the mass, but also an additional integer, and the subsequent discovery of similar solutions in Einstein-Skyrme (ES) [3], Einstein–Yang–Mills–Higgs (EYMH) [4], Einstein–non Abelian–Procca (ENAP) [4], and others, make clear the lack of the validity of the original form of the NHC [6] (other versions invoking stability also seem to be invalid).

The lack of validity of the NHC naturally gives raise to the question: What happened to the physical arguments put forward to support it? It seems clear now that the non-linear character of the matter content of the examples discussed plays an essential role: The interaction between the part of the field that would be radiated away and that which would be sucked in is responsible for the failure of the argument and, thus, for the existence of black hole hair. This suggests, on the other hand, that the non-linear behavior of the matter fields must be present both, in a region very close to the horizon (a region from which presumably the fields would tend to be sucked in), and in a region relatively distant from the horizon (a region from which presumably the fields would tend to be radiated away), with the self interaction being responsible for binding together the fields in these two regions.

In this paper, we show a result (originally reported in [7])that gives support to the heuristic argument mentioned above, by establishing the existence of a lower bound for the size of the "Hairosphere" (the region with nontrivial structure, to be defined below). We do this by proving a theorem that applies to all theories in which black hole hair has been found, and which states that this lower bound is parameter and theory independent, and has the universal value given by 3/2 the horizon radius. Furthermore, the result also applies to black holes in theories whose matter content is any combination of the matter fields corresponding to those theories.

We will focus on asymptotically flat static spherically symmetric black hole space–times and matter fields, and write the line element as

$$ds^2 = -e^{-2\delta}\mu\, dt^2 + \mu^{-1}\, dr^2 + r^2\, d\Omega^2 \ , \tag{1}$$

where δ and $\mu = 1 - 2m(r)/r$ are functions of r only, and we assume that there is a regular event horizon at r_H, so $m(r_H) = r_H/2$, and $\delta(r_H)$ is finite. Asymptotic flatness requires, in particular, that $\mu \to 1$ and $\delta \to 0$, at infinity.

We want to give a unified treatment of the hairy black holes, so we will consider only the energy momentum tensor associated with the fields, and not the fields themselves. So, instead of the field equations, we will deal only with the conservation equation $T^{\mu}{}_{\nu;\mu} = 0$.

We say that in a given theory there is black hole hair when the space time metric and the configuration of the other fields of a stationary black hole solution are not completely specified by the conserved charges defined at asymptotic infinity. Since the charges defined at asymptotic infinity are associated with the r^{-2} behavior of the fields, and, in general, the energy momentum tensor is at least quadratic in these fields, we will impose that $r^4\, T^r{}_r$ goes to zero at infinity, as a way to enforce the requirement that there is an additional conserved charge associated with the fields.

We find convenient to introduce the term "Hairosphere" to refer to the loosely defined region where the non–linear behavior of the fields is present, in contrast to the asymptotic region where the behavior of the fields is dominated by linear terms in their respective equations of motion. On the grounds of the discussion above, it appears, that we could take as a suitable definition of the "Hairosphere" the complement of the region where $r^4\, T^r{}_r$ monotonically approaches zero.

The conservation equation for the configurations of interest has only one non–trivial component: $T^{\mu}{}_{r;\mu} = 0$.

Einstein's equations give

$$\mu' = 8\,\pi\,r\,T^t{}_t + \frac{1-\mu}{r}, \qquad \delta' = \frac{8\,\pi\,r}{2\,\mu}\,(T^t{}_t - T^r{}_r), \tag{2}$$

where prime stands for differentiation with respect to r.

Using the Einstein's equations (2) in $T^{\mu}{}_{r;\mu} = 0$, it is straight forward to obtain:

$$e^{\delta}\,(e^{-\delta}\,r^4\,T^r{}_r)' = \frac{r^3}{2\,\mu}\left[(3\,\mu - 1)\,(T^r{}_r - T^t{}_t) + 2\,\mu\,T\right], \tag{3}$$

where T stands for the trace of the stress energy tensor.

We will be making the assumption that the matter fields satisfy the weak energy condition (WEC), that in our context means that the energy density, $\rho \equiv -T^t{}_t$, is positive semi–definite, and that it bounds the pressures, in particular, $|T^r{}_r| \leq -T^t{}_t$. We will also assume that $T < 0$. We note that in all theories where hair has been found, (EYM, ES, EYMH, ENAP, and Einstein–Yang–Mills–Dilaton, with or without and additional potential term [8]), the above conditions hold, so our results apply to all these theories. Furthermore, the additivity of the energy momentum tensor ensures that, in a theory involving a collection of any number of these fields (with the same type of anzats), these conditions will continue to hold.

Now we are in a position to obtain the following:

Theorem: Let equation (1) represent the line element of an asymptotically flat static spherically symmetric black hole space time, satisfying Einstein's equations, with matter fields satisfying the WEC, and such that the trace of the energy momentum tensor is nonpositive, and such that the energy density, $T^t{}_t$ goes to zero faster than r^4. Then the function $\mathcal{E} = e^{-\delta}\,r^4\,T^r{}_r$ is negative semi-definite at the horizon and is decreasing between r_H and r_0 where $r_0 > \frac{3}{2}\,r_H$, and from some $r > r_0$, the function $\mathcal{E}$ begins to increase towards its asymptotic value, namely 0.

Proof: First we note that the proper radial distance is given by $dx = \mu^{-\frac{1}{2}}\,dr$, so equation (3) can be written as

$$\frac{d}{dx}(e^{-\delta}\,r^4\,T^r{}_r) = \frac{e^{-\delta}\,r^3}{2}\left[\mu^{\frac{1}{2}}\,(3\,(T^r{}_r - T^t{}_t) + 2\,T) - \mu^{-\frac{1}{2}}\,(T^r{}_r - T^t{}_t)\right]. \tag{4}$$

Since the components $T^r{}_r, T^t{}_t, T^{\theta}{}_{\theta}$ must be regular at the horizon (*i. e.* the scalar $T_{\mu\nu}\,T^{\mu\nu} = (T^r{}_r)^2 + (T^t{}_t)^2 + 2\,(T^{\theta}{}_{\theta})^2$ is regular at the horizon), and, since x is a good coordinate at the horizon, the left hand side of equation (4) must be finite in the limit $r \longrightarrow r_H$, and since $\mu(r_H) = 0$, we find: $T^r{}_r(r_H) = T^t{}_t(r_H) = -\rho(r_H)$, so $\mathcal{E}(r_H) \leq 0$. Next, inspecting equation (3), we note that the left hand side is negative definite, unless $(3\,\mu - 1) > 0$. This follows from the WEC, which requires $(T^r{}_r - T^t{}_t) > 0$, and the assumption that $T < 0$. Thus $\mathcal{E}$ is a decreasing function, at least up to the point where $3\,\mu - 1$ becomes positive, and this occurs at $r_1 = 3\,m(r_1)$, therefore $r_0 > r_1$. Since $m(r)$ is an increasing function (as follows from the WEC and the fact that eq. (2) can be written as $m' = 4\,\pi\,r^2\,\rho > 0$), we then have: $r_0 > 3\,m(r_1) > 3\,m(r_H) = \frac{3}{2} r_H$. Q. E. D.

We see that under the conditions of the theorem, the asymptotic behavior of the fields can not start before r is sufficiently large, since this behavior is characterized by the fact that $T^r{}_r$ approaches zero, at least as r^{-4}. And, in particular, in the asymptotic regime $\mathcal{E}$ is not simultaneously negative and decreasing.

We interpret the theorem as stating that, under the conditions of its hypothesis, the matter fields start their asymptotic behavior at some $r > \frac{3}{2}\,r_H$, and thus that the "Hairosphere" must extend beyond this point.

One might "recognize" the value $r = 3/2 r_H$ as the radius of the circular orbits of photons, however this result holds only in the Schwarzchild spacetimes, and not in general in the hairy black hole configurations for which the condition of circular orbits for photons is $3\mu - 1 = 8\pi r^2 T^r_r$, which, in general, differs from the former.

In view of the evidence shown here, and based on the physical arguments described at the beginning, we are lead to conjecture that for all stationary black holes in theories in which the matter content is described via a Lagrangian in terms of self interacting fields, the "Hairosphere", if it exists, must extend beyond the above mentioned distance. In short: *If a black hole has hair, then it can not be shorter that* 3/2 *the horizon radius.* We believe, these set of results should generalize to the stationary black hole cases where we expect that the "Hairosphere" should also be characterized by the length $r_{Hair} = 3/2\sqrt{\frac{A}{4\pi}}$, where A is the horizon area.

It is interesting to note that if we take the natural expectation that in theories with massive fields (as in EYMH, ES, ENAP and Einstein–Yang–Mills–Dilaton (EYMD) with an additional potential term [8]), stationary configurations will correspond to fields that decrease rapidly within a Compton length of the horizon, *i. e.*, that the "Hairosphere" lies within $r_{hair} < \frac{1}{mass}$, and combine it with the result presented above, we obtain an upper bound for the size of hairy black holes, namely $r_H < \frac{2}{3}\,r_{hair} < \frac{2}{3}\,\frac{1}{mass}$.

Thus, **big black holes will have no hair**. Actually, the numerical investigations [5] of all massive theories known to present hair show evidence for such an upper bound on the size of hairy black holes.

Moreover, viewed in this light, the result is very suggestive in the context of attempts to identify black holes with some sort of elementary particles, as it illustrates the possibility that small black holes (of subnuclear size) might be richer in variety and features than large astrophysical black holes. In this respect, we should also note that for these hairy (or "dirty") black holes the Hawking temperature will be substantially reduced, as has been pointed out in [9].

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