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The Quantum Theory of General Relativity at Low Energies

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Abstract. In quantum field theory there is now a well developed technique, effective field theory, which allows one to obtain low energy quantum predictions in "non-renormalizable" theories, using only the degrees of freedom and interactions appropriate for those energies. Whether or not general relativity is truly fundamental, at low energies it is automatically described as a quantum effective field theory and this allows a consistent framework for quantum gravity at ordinary energies. I briefly describe the nature and limits of the technique.

Effective field theory is a calculational technique in quantum field theory which has become fully developed within the past decade[1]. It is now in everyday use in a variety of contexts. Anyone who cares about quantum field theory should be familiar with the methods and insights of effective field theory.

The goal of this talk is to convince you that a consistent quantum theory of general relativity exists at energies well below the Planck mass, and that it is necessarily of the form that we call effective field theory. Given all the work that has gone into quantum gravity, I feel that this is a significant result. Indeed, the gravitational effective field theory[2,3] is likely the full quantum content of pure general relativity.

The gravitational effective field theory is a completion of the program started by Feynman and DeWitt[4], 'tHooft and Veltman[5] and many others. The previous work focused on the divergence structure and the problems at high energy. What the effective field theory techniques do is to shift the focus to low energy, which is the reliable part of the theory.

The low energy particles and interactions lead to quantum effects which are distinct from whatever physics is going on at very high energies. For example, the long range quantum correction to the gravitational potential is determined by the low energy interactions of the massless particles in the theory (gravitons, photons, and neutrinos) and is reliably calculable. For a particular definition of the potential, it has the form[3,6]

$$V_{1pr}(r) = -\frac{Gm_1m_2}{r}\left[1 - \frac{135+2N_\nu}{30\pi^2}\frac{G\hbar}{r^2c^3} + \ldots\right] \tag{1}$$

where N_ν is the number of massless neutrinos. I will here focus on the general nature and limits of the gravitational effective field theory.

First let's describe effective field theory in general. Once you understand the basic ideas it is easy to see how it applies to gravity. The phrase "effective" carries the connotation of a low energy approximation of a more complete high energy theory. However, the techniques to be described don't rely at all on the high energy theory. (Moreover, even if you believe that general relativity is exact and fundamental at all scales, these techniques are still appropriate at low energy.) It is perhaps better to focus on a second meaning of "effective", "effective" $\sim$ "useful", which implies that it is the most effective thing to do. This is because the particles and interactions of the effective theory are the useful ones at that energy. An "effective Lagrangian" is a local Lagrangian which describes the low energy interactions. There is an old fallacy that effective Lagrangians can be used only at tree level. This sometimes still surfaces despite general knowledge to the contrary. "Effective field theory" is more than just the use of effective Lagangians. It implies a specific full field-theoretic treatment, with loops, renormalization etc. The goal is to extract the full quantum effects of the particles and interactions at low energies.

The key to the separation of high energy from low is the uncertainty principle. When one is working with external particles at low energy, the effects of virtual heavy particles or high energy intermediate states involve short distances, and hence can be represented by a series of local Lagrangians. This is true even for the high energy portion of loop diagrams. In contrast, effects that are non-local, where the particles propagate long distances, can only come from the low energy part of the theory. From this distinction, we know that we can represent the effects of the high energy theory by the most general local effective Lagrangian. The second key is the energy expansion, which orders the infinite number of terms within this most general Lagrangian in powers of the low energy scale divided by the high energy scale. To any given order in this small parameter, one needs to deal with only a finite number of terms (with coefficients which in general need to be determined from experiment). The lowest order Lagrangian can be used to determine the propagators and low energy vertices, and the rest can be treated as perturbations. When this theory is quantized and used to calculate loops, the usual ultraviolet divergences will share the form of the most general Lagrangian (since they are high energy and hence local) and can be absorbed into the definition of renormalized couplings. There are however leftover effects in the amplitudes from long distance propagation which are distinct from the local Lagrangian and which are the quantum predictions of the low energy theory.

This technique can be used in both renormalizable and non-renormalizable theories, as there is no need to restrict the dimensionality of terms in the Lagrangian. (Note that the terminology is bad: we *are* able to renormalize non-renormalizable theories!) One example, which is amusing to describe to die-hard loyalists who insist on the renormalizable paradigm, is Heavy Quark Effective Theory[7]. Here we have a perfectly good renormalizable field theory (QCD), yet we choose to turn it into a non-renormalizable effective field theory by a field redefinition which isolates the most important heavy quark degree of freedom. This is the effective thing to do because the properties of heavy quarks become readily apparent and the difficult parts of QCD are contained in a few universal parameters or functions. The effective field theory which is most similar to general relativity is chiral perturbation theory[8], which describes the theory of pions and photons which is the low energy limit of QCD. The theory is highly nonlinear, with a lowest order Lagrangian which can be written with the exponential of the pion fields

$$\begin{aligned} \mathcal{L} &= \frac{F_\pi^2}{4} Tr\left(\nabla_\mu U \nabla^\mu U^\dagger\right) + \frac{F_\pi^2 m_\pi^2}{4} Tr\left(U + U^\dagger\right) \\ U &= exp\left(i\frac{\tau^i \pi^i(x)}{F_\pi}\right) \quad , \end{aligned} \tag{2}$$

with τ^i being the SU(2) Pauli matrices and $F_\pi = 92.3 MeV$ being a dimensionful coupling constant. This theory has been extensively studied theoretically, to one and two loops, and experimentally. There are processes which clearly reveal the presence of loop diagrams. In a way, chiral perturbation theory is the model for a complete non-renormalizable effective field theory in the same way that QED serves as a model for renormalizable field theories.

At low energies, general relativity automatically behaves in the way that we treat effective field theories. This is not a philosophical statement implying that there must be a deeper high energy theory of which general relativity is the low energy approximation (however, more on this later). Rather, it is a practical statement. Whether or not general relativity is truly fundamental, the low energy quantum interactions must behave in a particular way because of the nature of the gravitational couplings, and this way is that of effective field theory.

The Einstein action, the scalar curvature, involves two derivatives on the metric field. Higher powers of the curvature, allowed by general covariance, involve more derivatives and hence the energy expansion has the form of a derivative expansion. (The renormalized cosmological constant is small on ordinary scales and so I neglect it, although it could possibly be treated as a perturbation itself, as the pion mass is treated in chiral perturbation theory.) The higher powers of the curvature in the most general Lagrangian do not cause problems when treated as low energy perturbations.[9] The Einstein action is in fact readily quantized, using gauge-fixing and ghost fields ala Feynman, DeWitt, Faddeev, Popov[4]. The background field method used by 'tHooft and Veltman[5] is most beautiful in this context because it allows one to retain the symmetries of general relativity in the background field, while still gauge-fixing the quantum fluctuations. The dimensionful nature of the gravitational coupling implies that loop diagrams (both the finite and infinite parts) will generate effects at higher orders in the energy expansion[10]. The one and two loop counterterms for graviton loops are known[5,11] and go into the renormalization of the coefficients in the

Lagrangian. However, these are not really predictions of the effective theory. The real action comes at low energy.

How in practice does one separate high energy from low? Fortunately, the calculation takes care of this automatically, although it is important to know what is happening. Again, the main point is that the high energy effects share the structure of the local Lagrangian, while low energy effects are different. When one completes a calculation, high energy effects will appear in the answer in the same way that the coefficients from the local Lagrangian will. One cannot distinguish these effects from the unknown coefficients. However, low energy effects are anything that has a different structure. Most often the distinction is that of analytic versus non-analytic in momentum space. Analytic expressions can be Taylor expanded in the momentum and therefore have the behavior of an energy expansion, much like the effects of a local Lagrangian ordered in a derivative expansion. However, non-analytic terms can never be confused with the local Lagrangian, and are intrinsically non-local. Typical non-analytic forms are $\sqrt{-q^2}$ and $\ln(-q^2)$. These are always consequences of low energy propagation.

A conceptually simple (although calculationally difficult) example is graviton-graviton scattering. This has recently been calculated to one-loop in an impressive paper by Dunbar and Norridge[12] using string based methods. Because the reaction involves only the pure gravity sector, and $R_{\mu\nu} = 0$ is the lowest order equation of motion, the result is independent of any of the four-derivative terms that can occur in the Lagrangian (R^2 or $R_{\mu\nu}R^{\mu\nu}$)[5]. Thus the result is independent of any unknown coefficient to one loop order. Their result for the scattering of positive helicity gravitons is

$$\begin{aligned}
\mathcal{A}(++\to++) &= 8\pi G\frac{s^4}{stu}\{\,1 \\
&+ \frac{G}{\pi}[\,\left(t\ln(\frac{-u}{\delta})\ln(\frac{-s}{\delta})+u\ln(\frac{-t}{\delta})\ln(\frac{-s}{\delta})+s\ln(\frac{-t}{\delta})\ln(\frac{-u}{\delta})\right) \\
&+ \ln(\frac{t}{u})\frac{tu(t-u)}{60s^6}\left(341(t^4+u^4)+1609(t^3u+u^3t)+2566t^2u^2\right) \\
&+ \left(\ln^2(\frac{t}{u})+\pi^2\right)\frac{tu(t+2u)(u+2t)}{2s^7}\left(2t^4+2u^4+2t^3u+2u^3t-t^2u^2\right) \\
&+ \frac{tu}{360s^5}\left(1922(t^4+u^4)+9143(t^3u+u^3t)+14622t^2u^2\right)\,]\,\}
\end{aligned} \tag{3}$$

where $s = (p_1+p_2)^2$, $t = (p_1-p_3)^2$, $u = (p_1-p_4)^2$, $(s+t+u=0)$ and where I have used δ as an infrared cutoff. One sees the non-analytic terms in the logarithms. Also one sees the nature of the energy expansion in the graviton sector - it is an expansion in GE^2 where E is a typical energy in the problem. I consider this result to be very beautiful. It is a low energy theorem of quantum gravity. The graviton scattering amplitude *must* behave in this specific fashion no matter what the ultimate high energy theory is and no matter what the massive particles of the theory are. This is a rigorous prediction of quantum gravity.

The other complete example of this style of calculation is the long distance quantum correction to the gravitational interaction of two masses. Again the result is independent of any unknown coefficients in the general matter and gravity Lagrangian, because the effect of such analytic terms is to lead to a short range delta-function interaction[3,13]. Only the propagation of massless fields can generate the nonanalytic behavior that yields power-law corrections in coordinate space[2,3]. Since the low energy couplings of massless particles are determined by Einstein's theory, these effects are also rigorously calculable. Besides classical corrections[14], one obtains the true quantum correction as quoted in the introduction above. Note that this calculation is the first to provide a quantitative answer to the question as to whether the effective gravitational coupling increases or decreases at short distance due to quantum effects. While there is some arbitrariness in what one defines to be G_{eff}, it must be a universal property (this eliminates from consideration the Post-Newtonian classical correction which depends on the external masses) and must represent a general property of the theory. The diagrams involved in the above potential are the same ones that go into the definition of the running coupling in QED and QCD and the quantum corrections are independent of the external masses. If one uses this gravitational interaction to define a running coupling one finds

$$G_{eff}(r) = G\left[1 - \frac{135 + 2N_\nu}{30\pi^2}\frac{G\hbar}{r^2c^3}\right] \tag{4}$$

The quantum corrections *decrease* the strength of gravity at short distance, in agreement with handwaving expectations. (In pure gravity without photons or massless neutrinos, the factor $135 + 2N_\nu$ is replaced by 127.) An alternate definition including the diagrams calculated in [6] has a slightly different number, but the same qualitative conclusion. The power-law running, instead of the usual logarithm, is a consequence of the dimensionful gravitational coupling.

These two results do not exhaust the predictions of the effective field theory of gravity. In principle, any low energy gravitational process can be calculated[15]. The two examples above have been particularly nice in that they did not depend on any unknown coefficients from the general Lagrangian. However it is not a failure of the approach if one of these coefficients appears in a particular set of amplitudes. One simply treats it as a coupling constant, measuring it in one process (in principle) and using the result in the remaining amplitudes. The leftover structures aside from this coefficient are the low energy quantum predictions.

The effective field theory techniques can be applied at low energies and small curvatures. The techniques fail when the energy/curvature reaches the Planck scale. There is no known method to extend such a theory to higher energies. Indeed, even if such a technique were found, the result would likely be wrong. In all known effective theories, new degrees of freedom and new interactions come into play at high energies, and to simply try to extend the low energy theory to all scales is the wrong thing to do. One needs a new enlarged theory at high energy. However, many attempts to quantize general relativity ignore this distinction and appear misguided from our experience with other effective field theories. While admittedly we cannot be completely sure of the high energy fate of gravity, the structure of the theory itself hints very strongly that new interactions are needed for a healthy high energy theory. It is likely that, if one is concerned with only pure general relativity, the effective field theory is the full quantum content of the theory.

It is common to hear that gravity is different from all our other theories because gravity and quantum mechanics do not go together, that there is no quantum theory of gravity. This is not really the case, as there is no conflict between gravity and quantum mechanics at low energy. We also expect that all of our other theories, despite being renormalizable, are modified by new interactions at high energy. Nevertheless, we are content to make predictions with them in the region where they are valid. While gravity at low energies has a somewhat different structure than other theories, it is not that a quantum theory does not exist. Rather the more accurate statement is that the quantum theory of gravity reveals itself as an effective field theory at low energies and signals that we need a more elaborate theory at high energies.

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