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Pair Creation and Evolution of Black Holes in Inflation

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Abstract. We summarise recent work on the quantum production of black holes in the inflationary era. We describe, in simple terms, the Euclidean approach used, and the results obtained both for the pair creation rate and for the evolution of the black holes.

Introduction One usually thinks of black holes forming through gravitational collapse, and so it seems that inflation is not a good place to look for black holes, since matter is hurled apart by the rapid cosmological expansion. We will show, however, that it is possible to get black holes in inflation through the quantum process of pair creation [1, 2]. There are two physical motivations that might lead us to expect this: First of all, quantum fluctuations can be very large during inflation, which leads to large density perturbations. Secondly, in order to pair create any objects, whether particles or black holes, one needs a force to pull them apart. Think of electron-positron pair creation: unless there is a force pulling them apart, the virtual particles will just fall back and annihilate. But if they are in an external electric field, the field pulls them apart and provides them with the energy to become real particles. Similarly, whenever one pair creates black holes, one needs to do it on a background that will pull them apart. This could be, for example, Melvin's magnetic universe, where oppositely charged black holes are separated by the background magnetic field, or a cosmic string, which can snap with black holes sitting on the bare terminals, pulled apart by the string tension. For the black holes we shall consider, the necessary force will be provided by the rapid expansion of space during inflation. So this expansion, which we naively thought would prevent black holes from forming, actually enables pair creation.

Inflation In quantum cosmology, one expects the universe to begin in a phase called chaotic inflation. In this era the evolution of the universe is dominated by the vacuum energy $V(\phi)$ of some inflaton field ϕ. V starts out at about the Planck value, and then decreases slowly while the field rolls down to the minimum of the potential. During this time the universe

behaves like de Sitter space with an effective cosmological constant $\Lambda_{\text{eff}} \approx V$. Like the scalar field, Λ_{eff} decreases only very slowly in time, and for the purposes of calculating the pair creation rate, we can take Λ to be fixed [1].

Instanton method An instanton is a Euclidean solution of the Einstein equations, i.e., a solution with signature (++++). Instantons can be used for the description of non-perturbative gravitational effects, such as spontaneous black hole formation. What follows is a kind of kitchen recipe for this type of application. We must consider two different spacetimes: de Sitter space without black holes (i.e., the inflationary background), and de Sitter space containing a pair of black holes. For each of these two types of universes, we must find an instanton which can be analytically continued to become this particular Lorentzian universe. The next step is to calculate the Euclidean action I of each instanton. According to the Hartle-Hawking no boundary proposal [3], the value of a wave function Ψ is assigned to each universe. In the semi-classical approximation $\Psi = e^{-I}$, neglecting a prefactor. $P = |\Psi|^2 = e^{-2I^{\text{Re}}}$ is then interpreted as a probability measure for the creation of each particular universe. (Note that P depends only on the real part of the Euclidean action.) The pair creation rate of black holes on the background of de Sitter space is finally obtained by taking the ratio $\Gamma = P_{\text{BH}}/P_{\text{no BH}}$ of the two probability measures. One can also think of Γ as the ratio of the number of inflationary Hubble volumes containing black holes to the number of empty Hubble volumes.

de Sitter We begin with the simpler of the two spacetimes, an inflationary universe without black holes. In this case the spacelike sections are round three-spheres. In the Euclidean de Sitter solution, the three-spheres begin at zero radius, expand and then contract in Euclidean time. Thus they form a four-sphere of radius $\sqrt{3/\Lambda}$. The analytic continuation can be visualised as cutting the four-sphere in half, and then joining to it half the Lorentzian de Sitter hyperboloid, where the three-spheres expand exponentially in Lorentzian time. The real part of the Euclidean action for this geometry comes from the Euclidean half-four-sphere only: $I^{\text{Re}}_{\text{no BH}} = -3\pi/2\Lambda$. Correspondingly, the probability measure for de Sitter space is

$$P_{\text{no BH}} = \exp\left(\frac{3\pi}{\Lambda}\right). \tag{1}$$

Schwarzschild-de Sitter Now we need to go through the same procedure with the Schwarzschild-de Sitter solution, which corresponds to a pair of black holes immersed in de Sitter space. The spacelike sections in this case have the topology $S^1 \times S^2$. This can be seen by the following analogy: Empty Minkowski space has spacelike sections of topology $\mathbf{R}^3$. Inserting a black hole changes the topology to $S^2 \times \mathbf{R}$. Similarly, if we start with de Sitter space (topology S^3), inserting a black hole is like punching a hole through the three-sphere, thus changing the topology to $S^1 \times S^2$. In general, the radius of the S^2 varies along the S^1. The maximum two-sphere corresponds to the cosmological horizon, the minimum to the black hole horizon. Clearly, the black hole may not be larger than the cosmological horizon.

What we need is a Euclidean solution that can be analytically continued to contain this kind of spacelike slice. It turns out that such a smooth instanton does not exist in general for the Lorentzian Schwarzschild-de Sitter spacetimes. The only exception is the degenerate

case, where the black hole has the maximum possible size, and the radius of the two-spheres is constant along the S^1. The corresponding Euclidean solution is just the topological product of two round two-spheres, both of radius $1/\sqrt{\Lambda}$ [4]. It can be analytically continued to the Lorentzian Schwarzschild-de Sitter solution by cutting one of the two-spheres in half, and joining to it the 2-dimensional hyperboloid of $1+1$ dimensional Lorentzian de Sitter space. In the Lorentzian regime the S^1 expands exponentially, while the two-sphere just retains its constant radius. Thus the Euclidean approach predicts the size with which the black holes will be nucleated:

$$r_{\rm BH} = \sqrt{\frac{1}{\Lambda}}. \tag{2}$$

The real part of the Euclidean action for this instanton is given by $I^{\rm Re}_{\rm BH} = -\pi/\Lambda$, and the corresponding probability measure is

$$P_{\rm BH} = \exp\left(\frac{2\pi}{\Lambda}\right). \tag{3}$$

Pair creation rate Now we can take the ratio of the two probability measures, and obtain the pair creation rate:

$$\Gamma = \exp\left(-\frac{\pi}{\Lambda}\right). \tag{4}$$

Let us interpret this result. The cosmological constant is positive and no larger than order unity in Planck units. This means that black hole pair creation is suppressed. When $\Lambda \approx 1$ (early in inflation), the suppression is week and one can get a large number of black holes. However, by Eq. (2), they will be very small (Planck size). For smaller values of Λ (which are attained later in inflation), the black holes would be larger, but their creation becomes exponentially suppressed. This result, which was obtained from the no boundary proposal, is physically very sensible.

Tunnelling proposal According to Vilenkin's tunnelling proposal [5], the wave function is given by e^{+I}, rather than e^{-I}. If we tried to apply this prescription to our problem, the signs would get reversed in all the exponents, and we would get the inverse result for Γ. Thus black hole creation would be enhanced, rather than suppressed. Even worse, the bigger the black holes were, the more likely they would be to nucleate. As a consequence, de Sitter space would be catastrophically unstable. This prediction is obviously absurd. Thus, the consideration of cosmological black hole pair creation provides strong evidence in favour of the no boundary proposal.

Classical evolution What happens to black holes that have been pair created during inflation? In the above instanton solution they would just retain their constant size $r_{\rm BH} = 1/\sqrt{\Lambda}$. But in this case it is important to take into account that during inflation, the effective cosmological constant isn't fixed, but decreases slowly. With this correction, the black hole radius during inflation is given by $r_{\rm BH} = 1/\sqrt{\Lambda_{\rm eff}}$. As the inflaton field rolls down, $\Lambda_{\rm eff}$ decreases, and the black hole grows slowly, becoming quite large by the end of inflation. This growth can be explained by the First Law of black hole mechanics, which states that the increase in a black hole's horizon area, multiplied by its temperature, is equal to four

times the increase in its mass. The mass increase comes from the flux of energy-momentum of the inflaton field across the black hole horizon, as the field rolls down.

Quantum evolution There are some quantum effects on the evolution which we have not yet taken into account. It is well known that both the black hole and the cosmological horizon emit radiation. The temperature of each horizon is approximately proportional to its inverse radius. In the instanton solution the radii of the two horizons will be equal, and, therefore, also their radiation rates. The black hole loses as much mass due to Hawking radiation as it gains from the incoming cosmological radiation, and it would seem to be stable. Because of quantum fluctuations, however, the radius of the two-spheres will vary slightly along the one-sphere. Then the black hole will be smaller and hotter than the cosmological horizon. It starts to lose mass and evaporates. Only if it was created very late in inflation would it be massive and cold enough to grow classically and survive into the radiation era. But such black holes are highly suppressed. The tiny, hot black holes created early in inflation will all evaporate immediately. Therefore there will be no significant number of neutral black holes after inflation ends.

Magnetically charged black holes There also are instantons that correspond to the creation of magnetically charged black holes. Such black holes cannot evaporate altogether, because there are no magnetically charged particles they could radiate. Therefore they are still around today. A detailed calculation shows, however, that they are so suppressed, and so strongly diluted by the inflationary expansion, that there won't even be a single charged primordial black hole in the observable universe. (This is a sensible prediction, since we don't observe any.) In dilatonic theories of inflation, however, their number could be significantly larger; this is currently being investigated.

Summary Semi-classical calculations indicate that tiny black holes are plentifully produced at the Planck era. The creation of larger black holes is exponentially suppressed. During inflation, the black holes can grow classically, but will mostly evaporate due to quantum effects. Magnetically charged black holes cannot evaporate, but their number today is exponentially small. Generally, in the context of cosmological pair creation of black holes, the no boundary proposal gives physically sensible results, while the tunnelling proposal does not seem to be applicable.

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