Zeitschrift: Helvetica Physica Acta
Band: 69 (1996)
Heft: 3

Artikel: Numerical investigation of black hole interiors

Autor: Droz, Serge

DOI: https://doi.org/10.5169/seals-116933

Helv Phys Acta
Vol. 69 (1996)

0018-0238/96/030257-04$1.50+0.20/0

Numerical investigation of black hole interiors

By Serge Droz

Department of Physics
University of Alberta
Edmonton, Alberta
Canada T6G 2J1

Abstract. Any realistic black hole possesses an unstable inner horizon: Initially regular perturbations cause the formation of a lightlike scalar curvature singularity. In a generic setting the form of these perturbations and hence the structure of the singularity is determined by a pair of coupled nonlinear partial differential equations. Solving these equations numerically shows that the singularity is characterized by an divergence of the hole's (quasi-local) mass function, similar to the spherical case.

It has long been known that a realistic black hole possesses an unstable inner horizon [1]. Small perturbations, present in every collapse to a black hole (described in more detail in Figure 1a) experience an infinite blueshift. This blueshift is expected to lead to the formation of a scalar curvature singularity. However the type of singularity formed depends on the evolution of the initially regular perturbations as they fall deeper into the hole. In the spherical case it has been shown [2] that the resulting singularity is null and fully characterized by an exponential divergence of the hole's mass function. The analysis of the inner horizon in the nonspherical case is much more difficult. To investigate the effects of these perturbations in detail we have constructed a model that catches the essential physics and can easily be solved numerically.

There are two important points to note when attempting to describe the inner horizon. First, "Decent into a black hole is fundamentally progress in time" (remember that the radial coordinate r becomes timelike behind the horizon). Thus, we don't need to know the (possibly quantum) structure of the deeper layers of the hole. Second the inner horizon has a *finite* radius. Mathematically this allows us replace the spherical line element

$ds^2 = e^\lambda dudv + r^2 d\Omega^2$ by the "flattened out" metric

$$ds^2 = e^\lambda dudv + r^2 \left(dx^2 + dy^2\right), \tag{1}$$

where u and v are a pair of null coordinates.

The curvatures of these two metrics differ only by terms proportional to $1/r^2$ so this approximation should be good as long as curvatures are big (like near a singularity) and $r \gg 0$. However the metric (1) is still too simple to model gravitational perturbations. To accommodate these we have to introduce shear. The metric (1) then becomes

$$ds^2 = e^\lambda dudv + r^2 \left(e^{2\beta}\cosh\gamma\, dx^2 + e^{-2\beta}\cosh\gamma\, dy^2 + 2\sinh\gamma\, dxdy\right), \tag{2}$$

where for simplicity we assume the functions λ, r^2, β and γ to depend only on u and v. The metric (2) describes a plane wave spacetime. The shear, which is proportional to $\beta_{,A}$ and $\gamma_{,A}$ $(A = u, v)$, then describes the gravitational perturbations.

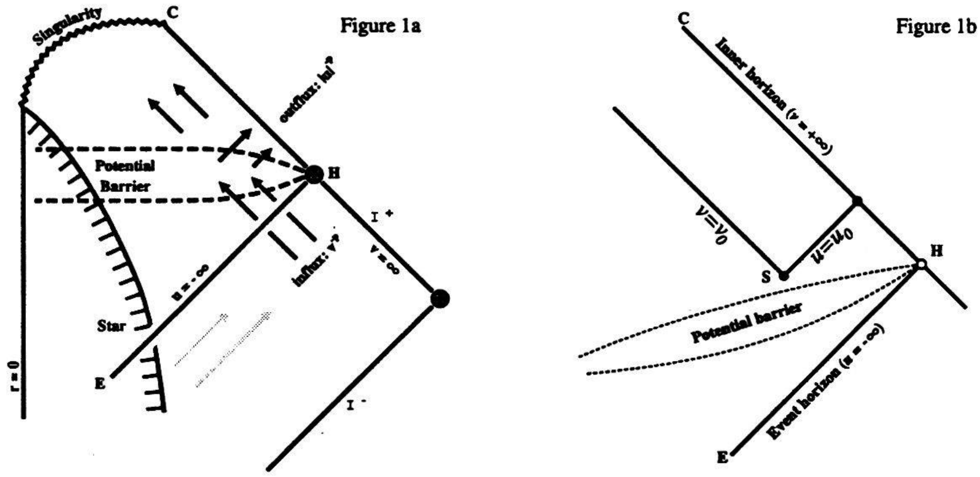

Figure 1. a) As a black hole forms it gets rid of its hair (gray arrows) by emitting decaying tails of gravitational radiation [3]. Part of this radiation gets scattered at the exterior gravitational potential (not shown) of the hole and falls into the black hole. Part of this scattered tail will experience another scattering, this time at the inner potential barrier. As the influx falls towards the inner horizon it gets infinitely blueshifted leading to a singularity along the inner horizon (CH). This singularity slowly contracts to zero radius where it presumably meets the much stronger central space like singularity at the core of the black hole.
b) Characteristic initial data for equations (5-6), mimicking a cross flow of radiation, is placed on the characteristics $v = v_0$ and $u = u_0$ *behind* the potential barrier.

The metric (2) is in fact an excellent approximation to a generic inner horizon. One can show [4] that it describes the leading contribution in an inverse curvature expansion of the spacetime near the inner horizon.

Solving Einstein's equations, is now equivalent to solving a characteristic initial value problem for the metric functions in (2). The vacuum field equations are: The two constraints

$$(\lambda + \log r)_{,A} = \left((r^2)_{,AA} + 2r^2(\beta_{,A})^2\cosh^2\gamma + r^2(\gamma_{,A})^2/2\right)/(r^2)_{,A} \quad A = u, v \tag{3}$$

and the wave equations

$$(r^2)_{,uv} = 0 \tag{4}$$

$$2r^2\beta_{,uv} + (r^2)_{,u}\beta_{,v} + (r^2)_{,v}\beta_{,u} = -2\,r^2\tanh\gamma\;(\gamma_{,u}\beta_{,v} + \gamma_{,v}\beta_{,u}) \tag{5}$$

$$2r^2\gamma_{,uv} + (r^2)_{,u}\gamma_{,v} + (r^2)_{,v}\gamma_{,u} = 4\,r^2\sinh(2\gamma)\beta_{,u}\beta_{,v} \tag{6}$$

plus a trivial wave equation for λ which we don't need.

Before discussing these equations in detail we have to specify the initial data and the gauge we want to use. If we choose u and v to be the standard retarded and advanced time coordinates, Price's analysis [3] implies that the infalling perturbations decay like inverse powers of v, and similarly for the backscattered radiation.

Equation (4) obviously has the solution $r^2 = r_0^2 + F(v) + G(v)$, where the two nonzero functions F and G are pure gauge. If we choose u and v to be the standard retarded and advanced time coordinates and compare to the spherical case [2, 5] we find $F \sim v^{-q+1}$ and $G \sim |u|^{-p+1}$ where $q = p + 2 = 4l + 6$ corresponds to the multipole moment l of the infalling perturbation.

The free gravitational data is encoded in the two functions β and γ. Price's initial data then translates to $\beta_{,v} \sim \gamma_{,v} \sim v^{-q/2}$ on the initial characteristic $u = u_0$ and $\beta_{,u} \sim \gamma_{,u} \sim |u|^{-p/2}$ on the initial characteristic $v = v_0$. We can now use the constraints (3) to calculate initial data for λ. We can then, in principle, solve the wave equations for λ, β and γ.

Let us calculate λ on the characteristic $u = u_0$. Solving the constraint (3) with $A = v$ for a asymptotic series we find $\lambda = -\kappa^2 v - q\log v + O(1/v)$ as $v \longrightarrow \infty$ where κ is a constant. This indeed corresponds to the announced curvature singularity. Inspecting for example the Kretschmann invariant shows $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \sim \exp(-2\lambda) \longrightarrow \infty$ in the above limit. Thus *regular* initial data leads to the formation of a curvature singularity at least on the initial characteristic $u = u_0$.

The difficult question now is: Do β and γ maintain their inverse powerlaw behaviour away from the initial surfaces? If these functions start to diverge a very different type of singularity might start to develop [6].

While it is believed that no closed form solution of the system (5,6) exists it is rather straightforward to solve these equations numerically.

This was done using a second order characteristic algorithm. In a first series of run we placed Price power law initial data of the type mentioned above on the initial surfaces $u = u_0$ and $v = v_0$ for the functions β and γ. In a first step we then calculate λ from its initial value, given on the two-sphere S (see Figure 1b), using the constraint equations (3). We then integrate the wave equations for β and γ using a second order method. The function λ is calculated by solving the $R_{vv} = 0$ constraint (Eq. (3) with $A = 0$), which is equivalent to solving the wave equation for λ. The remaining $R_{uu} = 0$ constraint is used to monitor the numerical error which is smaller than a few percent. The result for β and $\sigma = \lambda + \log r$ of such a run is shown in Figure 2. Note that the quasi-local mass functions behaves like $m \sim exp(-2\lambda)(r^2)_{,u}(r^2)_{,v}$.

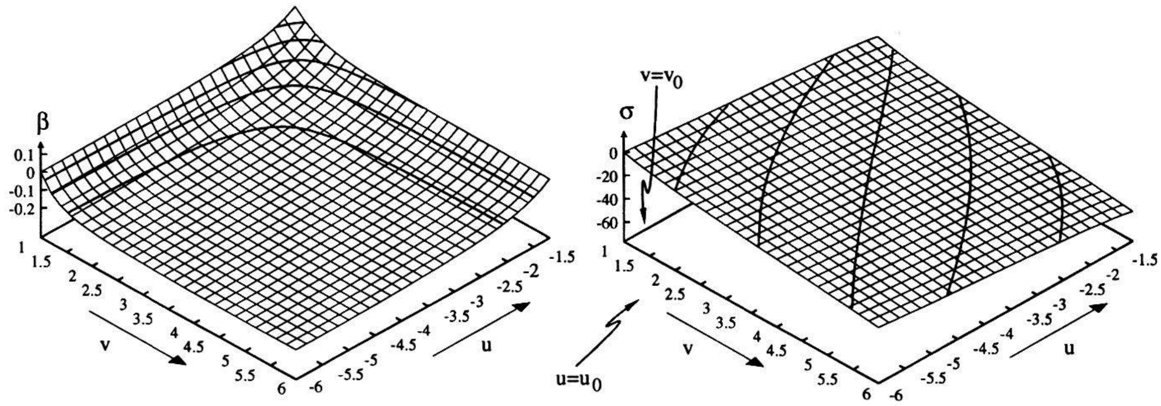

Figure 2. This figure shows the results of a numerical solution of equations (3-6). Displayed are the functions β and $\sigma = \lambda + \log r$. The initial data was of the Price tail form. Clearly σ and thus λ approaches large negative values while the function β exhibits the expected decay.

To test the stability of the mass inflation scenario we have, in a second series of runs, replaced the power law initial data for γ by a Gaussian pulse. While the quantitative behaviour of the solution changed it kept its qualitative features. In particular there were no divergences in either β or γ and the function σ kept decreasing towards the inner horizon.

In conclusion we can say that, at least in this simple model, mass inflation is generic. Regular initial data will generally lead to a mass inflation type singularity as predicted by the spherical models.

References

[1] Penrose, R. (1968), in *Battel Rencontres*, C. de Witt and J. Wheeler, eds., p. 222. W.A. Benjamin (1968).

[2] Poisson, E. and Israel, W. (1989), Phys. Rev. Lett. **63**, 1663.

[3] Price, R. H. (1972), PR **D5**, 2419.

[4] Brady, P. R., Droz, S., Israel, W. and Morsink, S. M. (1996), preprint **to appear**.

[5] Bonanno, A., Droz, S., Israel, W. and Morsink, S. (1994), PR **D50**, 7372.

[6] Yurtsever, U. (1993), CQG **10**, L17.