**Zeitschrift:** Helvetica Physica Acta
**Band:** 69 (1996)
**Heft:** 3

**Artikel:** A decrumpling model of the universe
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**DOI:** https://doi.org/10.5169/seals-116928

Helv Phys Acta
Vol. 69 ( 1996)

0018-0238/96/030237-04$1.50+0.20/0


# A Decrumpling Model of the Universe

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*Abstract.* Assuming a cellular structure for the space-time, we propose a model in which the expansion of the universe is understood as a decrumpling process, much like the one we know from polymeric surfaces. The dimension of space is then a real dynamical variable. The generalized Friedmann equation, derived from a Lagrangian, and the generalized equation of continuity for the matter content of the universe, give the dynamics of our model universe. This leads to an oscillatory non-singular model with two turning points for the dimension of space.

The picture we are proposing for the space-time is a generalization of polymeric or tethered surfaces, which are in turn simple generalizations of linear polymers to two-dimensional connected networks [1,2]. Visualizing the universe as a piece of paper, then the crumpled paper will stand for the state of the early universe [3]. As we are not going to develop yet

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a statistical mechanical model for our decrumpling universe, the dynamical model we are proposing in this paper could as well be interpreted as a generalization of fluid membranes [4]. In this case we can visualize the universe as a clay, which can be formed to a three dimensional ball, to a two dimensional disc, or even to a one dimensional string. In each case the effective dimension of the universe is a continuous number between the dimension of the embedding space and some $D_0$ which could be 3. To study the crumpling phenomenon in statistical physics one needs to define an embedding space, which does not exist in our case. Therefore, we assume an embedding space of arbitrary high dimension $\mathcal{D}$, which is allowed to be infinite. This is necessary, because the crumpling is highly dependent on the dimension of the embedding space.

Now, imagine the model universe to be the time evolution of a D-space embedded in a space with arbitrary large, maybe infinite, dimension $\mathcal{D}$. To model the crumpling, we assume the D-space to be consisted of cells with characteristic size of about the Planck length, denoted by $\delta$. The cells, playing the role of the monomers in polymerized surfaces, are allowed to have as many dimensions as the embedding space. Therefore, the cosmic space can have a dimension as large as the embedding space, like the polymers in crumpled phase. The radius of gyration of the crumpled cosmic space will play the role of the scale factor in a FRW cosmology in $D+1$ dimensional space-time, where $D$ is the fractal dimension of the crumpled space in the embedding space and could be as high as $\mathcal{D}$. The expansion of space is understood now as decrumpling of cosmic space. In the course of decrumpling the fractal dimension of space changes.

To formulate the problem we write down the Hilbert-Einstein action for a FRW metric in $D$ space dimension. This is along the same line as the formulation of homogenous cosmologies using the minisuperspace. Now in our toy model not only the scale factor $a$, but also the dimension $D$ of space, are dynamical variables. The above mentioned cell structure of the universe brings in the next simplification which is a relation between $a$ and $D$. It turns out that these generalized field equation admits the FRW model as a limit. For the sake of simplicity we confine ourselves to the flat, $k=0$, case.

Let us begin with a $D+1$ dimensional space-time $M \times R$, where $M$ is assumed to be homogeneous and isotropic. The space-time metric is written as

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \qquad i,j = 1, ..., D \tag{1}$$

The gravitational part of the Lagrangian, assuming $D$ to be a constant, becomes

$$L_G = -\frac{1}{2\kappa}D(D-1)\left(\frac{\dot{a}}{a}\right)^2 a^D, \tag{2}$$

where $a^D$ is the volume of $M$, and we have used the homogeneity of the metric to integrate the Lagrangian density.

To couple this Lagrangian to the source we use the well-known procedure in general relativity: write first the matter Lagrangian as

$$L_M = \frac{1}{2}\theta^{\mu\nu}g_{\mu\nu}, \tag{3}$$

where

$$\theta^{00} = \tilde{\rho} := \rho a^D, \tag{4}$$

and

$$\theta^{ij} = \tilde{p}\delta^{ij} := pa^{D-2}\delta^{ij}. \tag{5}$$

Now for the complete Lagrangian we obtain

$$L = -\frac{1}{2\kappa}D(D-1)\Big(\frac{\dot{a}}{a}\Big)^2 a^D + \Big(-\frac{\tilde{\rho}}{2} + \frac{\tilde{p}Da^2}{2}\Big). \tag{6}$$

We have to vary this Lagrangian with respect to $a$, considering $\tilde{\rho}$ and $\tilde{p}$ as constants. Therefore just one equation of motion is obtained. The continuity equation for the matter content of the cosmological model is the other one which we are going to use. It can be shown that these two equations give us the familiar 2 Friedmann equations.
To implement the idea of dimension as a dynamical variable, we assume a cellular structure for space: the universe consists of $N$ $D_0$-dimensional cells. To make them eligible to construct higher dimensional configurations, we assume our cells to have an arbitrary number of extra dimensions, each having a characteristic length scale $\delta$. Then the following relation holds between the $D$- and $D_0$-dimensional volume of the cells:

$$\mathrm{vol}_D(\mathrm{cell}) = \mathrm{vol}_{D_0}(\mathrm{cell})\ \delta^{D-D_0} \tag{7}$$

Now, taking $a$ as the radius of gyration of our decrumpling universe[5], we may write

$$a^D = N\ \mathrm{vol}_D(\mathrm{cell})\ \delta^{D-D_0} = N\ \mathrm{vol}_{D_0}(\mathrm{cell})\ \delta^{D-D_0} = a_0^{D_0}\delta^{D-D_0}, \tag{8}$$

or

$$\Big(\frac{a}{\delta}\Big)^D = e^C, \tag{9}$$

where $C$ is a constant. Here $D_0$ is a constant dimension which could be assumed to be 3, and $a_0$ is the corresponding length scale of the universe, i.e. the present radius of the universe. Now, we go on to start with the Lagrangian (6), letting the dimension $D$ to be any real number. It is then seen that the Lagrangian (6) suffers from the fact that its dimension is not a constant. To obtain a Lagrangian with a constant dimension we multiply (6) by $a_0^{D_0-D}$. Now, for our general case of variability of the space dimension, the constant part of this factor, $a_0^{D_0}$, can be omitted. Therefore, we finally arrive at the Lagrangian[6]

$$L = -\frac{D(D-1)}{2\kappa}\Big(\frac{\dot{a}}{a}\Big)^2\Big(\frac{a}{a_0}\Big)^D + \Big(-\frac{\hat{\rho}}{2} + \frac{\hat{p}Da^2}{2}\Big), \tag{10}$$

where

$$\hat{\rho} := \rho\Big(\frac{a}{a_0}\Big)^D \quad \text{, and} \quad \hat{p} := pa^{-2}\Big(\frac{a}{a_0}\Big)^D. \tag{11}$$

Variation of this Lagrangian with respect to $a$ and $D$ leads to a field equation for $a$ and one for $D$. But there is also the constraint equation (9). Taking this into account we arrive finally at the equations

$$(D-1)\left(\frac{\ddot{a}}{a} + \left[\frac{D^2}{2D_0} - 1 - \frac{D(2D-1)}{2C(D-1)}\right]\Big(\frac{\dot{a}}{a}\Big)^2\right) + \kappa p\Big(1 - \frac{D}{2C}\Big) = 0 \tag{12}$$

and

$$a\frac{dD}{da} = -\frac{D^2}{C}. \tag{13}$$

The field equation (12) is not sufficient to obtain $a$. A continuity equation, and an equation of state, are also needed. A dimensional reasoning leads to the following generalization of the continuity equation [6]:

$$\frac{d}{dt}(\rho a^D a_0^{D_0-D}) + p\frac{d}{dt}(a^D a_0^{D_0-D}) = 0, \tag{14}$$

or

$$\frac{d}{dt}\left[\rho\left(\frac{a}{a_0}\right)^D\right] + p\frac{d}{dt}\left(\frac{a}{a_0}\right)^D = 0. \tag{15}$$

The dynamics of our model universe is defined through (12), (13), (15), and an equation of state. This is a difficult system to be solved analytically. However, a first integral of motion can be derived which helps us to understand our model universe qualitatively. It leads to a potential which can be written for a radiation-like equation of state in the large $D$ limit in the form

$$U(D) \sim \frac{D_0}{2C}e^{-C}\left(\frac{D}{D_0}\right)^{\frac{C}{D_0}}. \tag{16}$$

Similarly, for $D$ near zero, assuming the pressure to remain finite (nonzero), it is obtained to be

$$U(D) \sim -C \ln D, \tag{17}$$

that is, $U$ grows unboundedly to infinity at $D \to \infty$, as well as $D \to 0$. It can be shown that the point $D = 2C$ is the point where the potential attains its minimum[6]. This means that there are two turning points, one above $D = 2C$, the other below it. The above discussion is valid, provided $T \geq 0$. However, the kinetic term (32) changes sign at $D = 1$. Therefore, to have two turning points, the constant $E := U + T$ must be sufficiently low to make the lower turning point greater than 1.

A study of the behavior of our model near the lower turning point shows that to have any effective change in the space dimension one has to go back in time as much as about some multiple of the currently assumed age of the universe, and that there is no sensible deviation from FRW models up to the Planck time.

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