Zeitschrift: Helvetica Physica Acta
Band: 69 (1996)
Heft: 3

Artikel: Causal dissipative cosmology

Autor: Beesham, Aroonkumar / Banerjee, Narayan

DOI: https://doi.org/10.5169/seals-116927

Helv Phys Acta
Vol. 69 (1996)

0018-0238/96/030233-04$1.50+0.20/0

Causal Dissipative Cosmology

By Aroonkumar Beesham[1]

Department of Applied Mathematics, University of Zululand,
Private Bag X1001, Kwa-Dlangezwa 3886, South Africa

and Narayan Banerjee

Relativity and Cosmology Research Centre, Department of Physics,
Jadavpur University, Calcutta 700 032, India

Abstract. The full version of the causal thermodynamics of non-equilibrium phenomena is discussed in the context of the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological models both in general relativity (GR) and in Brans-Dicke theory (BDT).

1 Introduction

In Einstein's field equations $R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}$, the left side has been well worked out, but there are many unanswered questions about the right side. Most studies in cosmology consider a perfect fluid with energy-momentum tensor

$$T_{ab} = (\mu + p)u_a u_b + pg_{ab} \tag{1.1}$$

The question that arises is whether the cosmological fluid is indeed perfect. There are several stages during the evolution of the universe when dissipative processes can be important: during GUT phase transitions, string creation, decoupling of neutrinos during the radiation era and the decoupling of matter and radiation during the recombination era. In addition,

[1]email: abeesham@pan.uzulu.ac.za

bulk viscosity can explain the entropy/baryon ratio, avoid the big-bang singularity, model quantum particle creation in a strong gravitational field and give rise to an inflationary scenario.

2 Eckart theory

The first candidate for a relativistic theory of thermodynamics was propounded by Eckart in 1940. It entails replacing the pressure p in equation (1.1) by $\bar{p} = p - 3\eta H$, where p is the perfect fluid contribution, $H = \dot{R}/R$ is the Hubble parameter and η is the coefficient of bulk viscosity. The problem with Eckart theory is that it is not relativistic as it is non-causal and equilibrium states are unstable (see [1] and references therein).

3 Truncated Israel-Stewart Theory (TIS)

This theory takes into account second order terms which are necessary to solve the problems in the first order Eckart theory. The pressure p in equation (1.1) is replaced by $\bar{p} = p + \sigma$ where the scalar viscous pressure σ satisfies the evolution equation

$$\sigma + \tau\dot{\sigma} = -3\eta H$$

where τ is the relaxation time for the irreversible process. The usual forms adopted for η and τ are

$$\eta = \eta_0 \mu^q \qquad \tau = \frac{\eta}{\mu} \tag{3.1}$$

where $\eta_0 \geq 0, q = const.$ The TIS solves the problems of Eckart theory as the second order terms transform the algebraic first order equations into differential evolution equations. The equilibrium and dissipative variables are placed on the same footing which is suitable for describing non-stationary processes.

A problem with the TIS is that it is possible to derive the following equation for the temperature: $T \sim (\tau/\eta)R^3$. Hence, in an expanding model, the temperature rises if (τ/η) does not decrease at least at the same rate as that with which the volume increases.

4 Full Israel-Stewart Theory (FIS)

In this case the pressure p in equation (1.1) has to be replaced by $\bar{p} = p + \Pi$ where Π is the bulk viscous stress which satisfies the evolution equation [1]

$$\Pi + \tau\dot{\Pi} = -3\eta H - \frac{\epsilon}{2}\tau\Pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\eta}}{\eta} - \frac{\dot{T}}{T}\right) \tag{4.1}$$

where T is the temperature. For $\tau = 0$, we recover Eckart theory, $\epsilon = 0$ yields the TIS, and $\epsilon = 1$ gives the FIS. Equation (4.1) is the simplest way which is linear in Π to satisfy the H-theorem (rate of generation of entropy production must be nonnegative).

For the $k = 0$ FLRW metric, Einstein's field equations yield

$$3H^2 = \mu \tag{4.2}$$

$$2\dot{H} + 3H^2 = -p - \Pi \tag{4.3}$$

Zakari and Jou [2] showed that these equations admit exponential expansion and Maartens [1] showed that it is possible to generate the right amount of entropy.

Do equations (4.2)-(4.3) admit power-law inflation? Assuming the relations (3.1) and the usual equation of state

$$p = (\gamma - 1)\mu \qquad 1 \leq \gamma \leq 2 \tag{4.4}$$

it is possible to find the following solution in the FIS ($\epsilon = 1$)

$$R = R_0 t^a \qquad T = T_0 t^{(2C-D)/C} exp(Et^{2q-1})$$

where $R_0, a = const$, $T_0 =$ constant of integration, $C = F/2$, $D = 2F - 9a^3 - (3FG/2)$, $E = G/[C(2q-1)]$, $F = a(2 - 3\gamma a)$ and $G = F\eta_0^{-1}(3a)^{1-q}$. The energy density and bulk viscous stress take the following forms

$$\mu = \frac{3a^2}{t^2} \qquad \Pi = \frac{a(2-3\gamma a)}{t^2}$$

The relevant constants have to be chosen so that the temperature T decreases with time. For further details, we refer to [3, 4, 5].

5 Brans-Dicke Theory

In BDT, the equations corresponding to (4.2) and (4.3) for the $k = 0$ FLRW metric are

$$3H^2 + 3H\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{\mu}{\phi} \tag{5.1}$$

$$2\dot{H} + 3H^2 + \frac{\ddot{\phi}}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2H\frac{\dot{\phi}}{\phi} = -\frac{p+\Pi}{\phi} \tag{5.2}$$

where w is the BD coupling parameter. In addition there is the following wave equation for the scalar field

$$\ddot{\phi} + 3H\dot{\phi} = \frac{1}{2\omega+3}(\mu - 3p - 3\Pi) \tag{5.3}$$

If we assume the usual equation of state (4.4), then equations (5.1)-(5.3) represent three equations in four unknowns. We assume that

$$\phi = AR^a \tag{5.4}$$

where $A, a = const$ and then obtain the following solution

$$R = R_0 t^{1/b} \qquad \mu = \mu_0 t^c \qquad \Pi = \Pi_0 t^c$$

where

$$\mu_0, R_0 = const \qquad b = \frac{\omega\eta_0^2 + 6\omega\eta_0 - 12}{2(\omega\eta_0 - 3)} \qquad c = \frac{\eta_0}{b} - 2$$

$$\Pi_0 = Aa^{\eta_0}\frac{2\eta_0 b + 4b - 6 - 4\eta_0 - (2+\omega)\eta_0^2}{2b^2} - (\gamma - 1)\mu_0$$

Adopting the usual relations (3.1), we find that equation (4.1) can now be integrated to yield

$$T = T_0 t^B exp\left(Dt^{c-cq+1}\right)$$

where

$$B = 3c + \frac{6\mu_0}{\pi_0 b} + \frac{3}{b} \qquad D = \frac{2\mu_0^{(1-q)}}{\eta_0(c - cq + 1)}$$

Our solution derived here does not have a corresponding analogue in GR. Indeed if we try to put $\phi = const$ to generate the GR solution, we find from equation (5.4) that $R = const$. For further details we refer to [6].

6 Conclusion

We have found some power law solutions for the scale factor in the $k = 0$ FLRW models with a causal viscous fluid in the FIS in both GR and BDT.

References

[1] R. Maartens, *Class. Quantum Grav.*, **12**, 1455 (1995)

[2] M. Zakari and D. Jou, *Phys. Rev. D*, **48**, 1597 (1993)

[3] N. Banerjee and Aroonkumar Beesham, *Pramana*, **46**, 213 (1996)

[4] R. Maartens and A. M. Kgathi, Unpublished (1994)

[5] A. A. Coley, R. J. van den Hoogen and R. Maartens, Qualitative Viscous Cosmology, preprint (1996)

[6] N. Banerjee and A. Beesham, *Aust. J. Phys.*, Brans-Dicke Cosmology with Causal Viscous Fluid, to appear (1996)