CMBR dipole from ultra large scale isocurvature perturbations

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Abstract. The observed CMBR dipole, generally interpreted as the consequence of the peculiar motion of the Sun with respect to the reference frame of the CMBR, can also be explained by the presence of ultra large scale (of the order of $100H_0^{-1}$) isocurvature perturbations. Moreover, the simplest model of inflation with several scalar fields, namely that of double inflation, with appropriate parameters, can produce such perturbations.

1 Introduction

The dipole moment of the Cosmic Microwave Background Radiation (CMBR) anisotropy is larger than the quadrupole, measured recently by the satellite COBE, by two orders of magnitude ([1]). In general, this dipole is interpreted as the peculiar motion of the Sun with respect to the CMBR "rest frame", whereas the other multipoles are seen as a consequence of primordial cosmological perturbations of a homogeneous and isotropic universe.

However, from a theoretical point of view, the dipole or part of it can also be of cosmological origin ([2], [3]). From an observational point of view, although there is a general trend to favour the Doppler effect, this question is not yet settled. This contribution summarizes the conclusions of two recent works ([3],[4]) on an alternative explanation for the dipole: superhorizon isocurvature perturbations.

One must be aware that the origin of the dipole affects the interpretation of the measured
quadrupole. Indeed, the Doppler effect, whose complete expression is
\[ T_{obs}(\vec{e}) = T(1 - v^2/c^2)^{1/2}(1 - \vec{e} \cdot \vec{u}/c)^{-1}, \]  
(\vec{e} gives the direction of observation on the celestial sphere) also induces a quadrupole, which must be subtracted from the observed dipole to obtain the “true cosmological quadrupole”. The latter would be modified if the dipole turns out not to be a Doppler dipole.

2 Ultra large scale isocurvature perturbations

The CMBR fluctuations at large angular scales can be written as the sum of two contributions: the intrinsic fluctuations on the last scattering surface; the Sachs-Wolfe fluctuations due to the presence of geometrical perturbations on the light trajectories. In a “scalarly” perturbed flat FLRW model, with the metric
\[ ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Phi)\delta_{ij}dx^i dx^j, \]
the Sachs-Wolfe contribution is simply
\[ \left( \frac{\Delta T}{T} \right)_{SW} (\vec{e}) = \frac{1}{3} [\Phi_{ls} - \Phi_0] - \vec{e} \cdot [\vec{v}_{ls} - \vec{v}_0], \]
where the subscript \( ls \) refers to the last scattering surface and the subscript \( 0 \) to the observer today. In a universe with radiative matter (with a density contrast \( \delta_r = \delta \rho_r/\rho_r \)) and pressureless matter (with a density contrast \( \delta_m = \delta \rho_m/\rho_m \)), matter perturbations can be decomposed into adiabatic perturbations, for which the intrinsic contribution is negligible, and isocurvature perturbations, for which
\[ \left( \frac{\Delta T}{T} \right)_{int} \approx -\frac{1}{3} S, \]
where \( S = \delta_m - (3/4)\delta_r \) is the entropy perturbation.

The observed CMBR temperature fluctuations are generally decomposed in spherical harmonics:
\[ \frac{\Delta T}{T}(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi). \]
Assuming that the cosmological perturbations can be described as a homogeneous random field, the power spectrum being defined by \( \langle \Phi_k \Phi_{k'} \rangle = 2\pi^2 k^{-3} P(\Phi)(k) \delta(k - k') \) (\( \Phi_k = (2\pi)^{-3/2} \int d^3 x \Phi(x) e^{-ik \cdot x} \)), the prediction for the harmonic components \( l \geq 2 \) on large angular scales, in the case of adiabatic perturbations, is
\[ \Sigma_l = \langle |a_{lm}|^2 \rangle = \frac{4\pi}{9} \int \frac{dk}{k} P(\Phi)(k) j_l^2(2k/a_0 H_0). \]
As shown in [3], if our peculiar velocity is ignored, the dipole from adiabatic perturbations can never be bigger than the quadrupole. This is due to a cancellation between the terms.
\( \frac{\Phi}{3} \) and \( \text{e.e.v} \) in the dipole. However, in the case of isocurvature perturbations, because of the large intrinsic contribution, the dipole can be made much larger than the quadrupole by considering perturbations larger than the present Hubble radius. The resulting “cosmological” dipole will then be \( [3] \)

\[
a_1^2 \equiv \frac{3}{4\pi} \Sigma_1^2 = \frac{1}{3} \int \frac{dk}{k} \mathcal{P}_s(k) \gamma_1^2 (2k/a_0 H_0).
\]  

(2.6)

The question now arises what could be the physical origin of such perturbations. The next section provides a possible answer, in the framework of double inflation.

3 The double inflation model

The simplest model of double inflation (see e.g. [5]), consisting of two minimally coupled massive scalar fields, is described by the Lagrangian

\[
\mathcal{L} = \frac{(4)^R}{16\pi G} - \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} m_l^2 \phi_1^2 - \frac{1}{2} \partial_\mu \phi_h \partial^\mu \phi_h - \frac{1}{2} m_h^2 \phi_h^2,
\]  

(3.1)

where \( \phi_1 \) and \( \phi_h \) are respectively the light and heavy scalar fields; \( ^R \) is the scalar spacetime curvature and \( G \) is Newton’s constant. In addition to \( m_l \) and \( m_h \), which from now on are assumed to satisfy \( m_h/m_l \gg 1 \), there is a third parameter \( s_0 \) corresponding to the initial conditions for the two scalar fields \( (s_0 = 4\pi G\phi_1^2 \) when \( \phi_h = \phi_1 \)). It is convenient to label any moment during inflation by the corresponding number of e-folds before the end of inflation, \( s \). There is a natural scale associated with it, which is the comoving Hubble radius at this moment.

The first phase of inflation, driven by the heavy scalar field, lasts until equality of the energies of the two scalar fields \( (m_h \phi_h = m_l \phi_1) \), which occurs for \( s_b \sim s_0 \). If \( s_0 > (m_h/m_l)^2 \), which will be assumed here, inflation goes on, now driven by the light scalar field. Meanwhile the heavy scalar field continues its slow-rolling until \( H \sim m_h \), corresponding to \( s = s_c \), after which it oscillates. Finally the Hubble parameter reaches the value \( m_l \) and inflation stops. In the “standard” model of double inflation, the scales \( \lambda_b \) and \( \lambda_c \) (one always has \( \lambda_b > \lambda_c \)) are within the Hubble radius today, denoted \( \lambda_H \), \( ([5]) \) whereas, here, the parameters of the model are chosen such that these two scales are outside the present Hubble radius \( \lambda_H \).

The spectrum of adiabatic perturbations today, which it is convenient to express in terms of \( \Phi \), is computed from the vacuum quantum fluctuations of the two scalar fields in the inflationary period. This spectrum has two plateaus, as explained in [5]. Since, here, the transition scale is larger than the Hubble radius today, only the lower plateau, corresponding to the second phase of inflation (driven by the light scalar field), and given by

\[
\mathcal{P}_s = \frac{12}{25\pi} s^2 \frac{m_l^2}{m_P^2},
\]  

(3.2)

will be accessible to observations \( (s(k) \approx \ln(k_e/k)) \). Assuming the quadrupole to be essentially due to adiabatic perturbations, the COBE measurement of \( Q_{rms} - P_s (n = 1) \approx 18 \mu K \) \( ([6]) \) implies \( m_l \approx 7.8 \times 10^{-5} m_P s_H \) (for \( s_H = 60 \), \( m_l \approx 1.3 \times 10^{-6} m_P \)).
In addition to adiabatic perturbations, this model can also produce isocurvature perturbations. This is the case for example if one assumes that the light scalar field, after inflation, will decay into ordinary matter, while the heavy scalar field remains decoupled from ordinary matter. The subsequent spectrum of isocurvature perturbations is given by ([4], [7])

$$\mathcal{P}_S = \frac{4G}{3\pi^2} m_i^2 \left( \frac{s_0}{s} \right)^{m_h^2/m_i^2}$$  \hspace{1cm} (3.3)

until \( s = s_c \), after which the production of isocurvature perturbations falls abruptly. Substituting this spectrum in (2.6) and introducing the variable \( X = \ln(k_c/k) \), the expected dipole reads

$$a_1^2 = \frac{16}{81\pi} \left( \frac{m_i}{m_P} \right)^2 e^{2(s_H-s_c)} \left( \frac{s_0}{s_c} \right)^{m_h^2/m_i^2} \int_0^\infty dX \frac{e^{-2X}}{(1 + X/s_c)^{m_h^2/m_i^2}},$$  \hspace{1cm} (3.4)

where has been used the fact that \( j_1(x) \sim x/3 \) for small \( x \). The integral in (3.4) is well approximated as \( (2 + (m_h/m_i)^2/s_c)^{-1} \). For consistency, one must also check that the quadrupole generated by isocurvature perturbations is smaller than the adiabatic quadrupole. This requires, as seen in [3], that

$$s_c > s_H + 4.6,$$  \hspace{1cm} (3.5)

i.e. that the cut-off scale \( \lambda_c \) is one hundred times larger than the Hubble radius today. The value \( s_H \) is of the order of 60, its exact value depending on the history of the universe. Because of the power law dependence in (3.4), the dipole can be much bigger than the quadrupole. Because of the condition (3.5), implying that \( m_h^2/m_i^2 \) must be large, the main constraint, surprisingly, comes from requiring that the dipole is not too high with respect to the quadrupole: this requires that \( s_0/s_c \) must be very close to 1. In this range of parameters, double inflation thus provides adequate spectra of perturbations, explaining the observed dipole and higher multipoles without a Doppler effect.

**References**


