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# Cosmology with bulk pressure

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*Abstract.* It is well known that cosmological particle production processes may phenomenologically be described in terms of effective viscous pressures. We investigate the consistency of this approach on the level of relativistic kinetic theory, using a Boltzmann equation with an additional source term that leads to a change in the number of gas particles. For a simple creation rate model we find the conditions for collisional equilibrium of a Maxwell-Boltzmann gas with increasing particle number and discuss their cosmological implications. We also comment on the possibility of a bulk pressure driven inflationary phase within causal, nonequilibrium thermodynamics.

## 1 Introduction

Entropy producing processes have played an important role in the evolution of the early universe. On the level of fluid cosmology, the simplest phenomenon connected with a nonvanishing entropy production is a bulk viscosity. A bulk viscous pressure is the only dissipative effect that is compatible with the symmetry requirements of the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker models. For the possible physical origin of bulk pressures in the expanding universe see [1] and references therein.

On the other hand, there exist attempts to use effective viscous pressures in order to model particle production proceses in the early universe (see references in [2]). For certain quantum processes in the early universe 'viscosity functions' have been calculated, opening the possibility of an effective imperfect fluid description of these phenomena.
Generally, any production process will give rise to source terms in the macroscopic fluid balances. The interpretation of these contributions as effective bulk pressures uses the fact that a source term in the energy balance of a fluid may formally be rewritten in terms of an effective bulk pressure. The advantage of a rewriting like this is obvious: An energy-momentum balance with a nonzero source term violates the integrability conditions of Einstein's equations, while the energy-momentum tensor of an imperfect fluid is a reasonable right-hand side of the field equations. Moreover, this approach allows us to calculate the backreaction of the corresponding process on the cosmological dynamics.
We shall investigate here whether the approach of regarding particle production processes as equivalent to viscous pressures is consistent with a kinetic description. In section 2 we sketch the general kinetic theory of a simple gas with nonconserved particle number and introduce an effective rate approximation for the production term. The conditions for collisional equilibrium are found in section 3, while the above mentioned effective viscous pressure approach is established in section 4. The backreaction of particle production processes on the cosmological dynamics under the conditions of collisional equilibrium is considered in section 5. In section 6 we drop the equilibrium conditions and comment on the possibility of a bulk pressure driven inflationary phase within causal, nonequilibrium thermodynamics.

# 2 Basic kinetic theory

The one-particle distribution function $f = f\,(x,p)$ of a relativistic gas with varying particle number is supposed to obey the equation [2]

$$L\,[f] \equiv p^i f_{,i} - \Gamma^i_{kl} p^k p^l \frac{\partial f}{\partial p^i} = C\,[f] + S\,(x,p) \ . \tag{2.1}$$

$L\,[f]$ is the Liouville operator and $C[f]$ is Boltzmann's collision term. The source term $S$ takes into account the fact that the distribution function $f$ may additionally vary due to creation or decay processes, supposedly of quantum origin. On the level of classical kinetic theory we regard this term as a given input quantity. Assuming a linear coupling of the source term $S$ to $f$ and requiring a macroscopic description of the production process in terms of the zeroth, first and second moments of $f$ only, provides us with the expression

$$S\,(x,p) = \left(-\frac{u_a p^a}{\tau\,(x)} + \nu\,(x)\right) f\,(x,p)\,, \tag{2.2}$$

for the source term in (2.1), where $u^a$ is the macroscopic 4-velocity. The creation process is characterized by the spacetime functions $\tau$ and $\nu$.

# 3 Collisional equilibrium

As in the case $S = 0$, the condition for collisional equilibrium implies the structure

$$f(x,p) = f^0(x,p) = \exp\left[\alpha + \beta_a p^a\right] , \tag{3.1}$$

of the distribution function. Using the latter expression together with (2.2) in (2.1) yields the equilibrium conditions

$$\dot{\alpha} = \frac{1}{\tau} , \quad h^a_b \alpha_{,a} = 0, \quad \beta_{(a;b)} = -\frac{\nu}{m^2} g_{ab} , \tag{3.2}$$

which are less restrictive than in the case of particle number conservation. Different from the case without particle production ($\tau^{-1} = \nu = 0$) we find that $\alpha$ may change along the fluid flow lines and that $\beta_a \equiv u_a/T$ obeys a conformal Killing equation for $m \neq 0$. The latter property implies the temperature behaviour $\dot{T}/T = -\Theta/3$ that coincides with the temperature law for massless particles (radiation) ($\Theta = u^a_{;a}$ is the fluid expansion). We conclude that *radiation and matter in the expanding Universe may be in equilibrium at the same temperature, provided the particle number of the matter component is allowed to change at a specific rate.* It is well known, that for conserved particle numbers an equilibrium between both components is impossible.
A further consequence of the equilibrium conditions (3.2) is a modification of Tolman's relation [2]. For nonvanishing particle production *the quantity* $T\sqrt{-g_{00}}$ *is allowed to be time dependent.* However, like $\alpha$ in (3.2), it has to be spatially constant.

# 4 The effective viscous pressure approach

Let, as usual, the energy-momentum tensor $T^{ik}$ be defined as the second moment of the distribution function (3.1). *The effective viscous pressure approach then amounts to the replacement of the nonconserved energy-momentum tensor* $T^{ik} = \rho u^i u^k + p h^{ik}$ *with* $\dot{\rho} + \Theta(\rho + p) = \rho/\tau + \nu n$ *by the conserved energy-momentum tensor* $\hat{T}^{ik} = \rho u^i u^k + (p + \pi) h^{ik}$ *with* $\dot{\rho} + \Theta(\rho + p + \pi) = 0$, *where* $\Theta\pi = -\rho/\tau - \nu n$. We find this approach to be consistent for homogeneous spacetimes but not necessarily for inhomogeneous ones [2].

# 5 Backreaction on the cosmological dynamics

A nonvanishing effective bulk pressure influences the entire cosmological dynamics. Restricting ourselves to the homogeneous, isotropic and spatially flat case, the relation $2\dot{H}/H = \dot{\rho}/\rho$ is valid, where $H$ is the Hubble parameter $H = \Theta/3$. The functions $\nu$ and $\tau$ are fixed by the conditions (3.2) and by the additional assumption of 'adiabatic' particle production [3]. With $H \equiv \dot{a}/a$, where $a$ is the scale factor of the Robertson-Walker metric, we find (see [3]) that for massive particles ($m \gg T$) the scale factor behaves like $a \propto t^{4/3}$ instead of the

familiar $a \propto t^{2/3}$ for $\tau^{-1} = \nu = 0$. *Massive particles with adiabatically changing particle number are allowed to be in collisional equilibrium only in a power-law inflationary universe.* Introducing a '$\gamma$-law' in the usual form $p = (\gamma - 1)\rho$, one obtains accelerated expansion, i.e., $\ddot{a}/a > 0$, in the interval $1 \leq \gamma \leq \gamma_{cr}$ with $\gamma_{cr} \approx 1.09$. In the opposite limit of massless particles the equilibrium conditions require $\tau^{-1} = \nu = 0$, i.e., there is no adiabatic production of relativistic particles.
We also investigated whether a phase of exponential inflation is consistent with the conditions for collisional equilibrium [3]. In our setting exponential inflation may be realized by 'nonadiabatic' particle production. It turns out that *the equilibrium conditions (3.2) are compatible with a de Sitter phase only for a time interval of the order of the Hubble time.* For larger times exponential inflation violates the second law of thermodynamics, i.e., the de Sitter phase becomes thermodynamically unstable.

# 6 Causal thermodynamics and inflation

Up to now our analysis was based on the equilibrium conditions (3.2). Now we drop these conditions and investigate the role of bulk pressures within causal, nonequilibrium thermodynamics (see [4, 5] and references therein). The viscous pressure becomes a dynamical degree of freedom on its own and the cosmological dynamics is determined by a second-order equation for the Hubble parameter $H$. A basic ingredient of the theory is a nonvanishing relaxation time. There exists a debate on whether deviations from thermodynamical equilibrium may drive a de Sitter phase. Assuming the applicability of the causal, nonequilibrium thermodynamics also far from equilibrium, it may be shown that *the causal evolution equation for $H$ admits stable, inflationary solutions with relaxation times of the order of the Hubble time, i.e., the nonequilibrium may be 'frozen in'.* Provided, the bulk pressure is again due to particle production processes, one finds that the temperature remains approximately constant during the de Sitter phase (no supercooling!). The latter feature is connected with an exponential increase of the comoving entropy. *While the standard scenario of an inflationary universe comprises an adiabatic de Sitter phase and a subsequent reheating period, in which all the entropy in the universe is produced, the entropy production takes place during the de Sitter stage itself in the present picture.*

# References

[1] W. Zimdahl, *Mon. Not. R. Astr. Soc.* **280**, 1239 (1996)

[2] J. Triginer, W. Zimdahl, D. Pavón, *Class. Quantum Grav.* **13**, 403 (1996)

[3] W. Zimdahl, J. Triginer, D. Pavón, submitted to *Phys. Rev. D*

[4] R. Maartens, *Class. Quantum Grav.* **12**, 1455 (1995)

[5] W. Zimdahl, *Phys. Rev.* **D 53**, 5483 (1996)