The kinetic stage of the Universe reheating

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Abstract.   Thermalization process during reheating of the Universe is investigated for the model of self-interacting inflaton field. Corresponding Boltzmann kinetic equation was solved numerically and the relaxation time is found.

The process of the Universe reheating is one of the most important issues in the inflationary cosmology. After the end of inflation the inflaton field begins to oscillate near the minimum of its potential. These oscillations lead to particle production, which usually was associated with friction-like terms in equations of motion [1].

However, it was realized recently, that parametric resonance could sufficiently change the reheating process [2, 3]. According to the new scenario of reheating there is stage of "preheating" during which occupation numbers of produced bosons grow exponentially due to a parametric resonance inside some resonance bands. Considerable part of inflaton energy goes to fluctuations during this stage. During the second stage of reheating, produced particles interact with each other and with classical inflaton field, approaching the state of thermal equilibrium eventually.

The semi-classical investigation of preheating stage was made in [4]. Contrary to previous approaches [2, 3], semiclassical description allows to include non-linear effects like rescattering of produced particles on zero momentum mode, as well as scattering back to zero momentum mode.

However, semiclassical approach fails to describe evolution of a system during late stages of thermalization. On the other hand, investigation of thermalization is important for estimates of the reheating temperature and possibility of out of equilibrium symmetry restoration

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in models with broken symmetry [5].

The thermalization stage can be studied in frameworks of kinetic equations. Some analytical results and estimates were obtained [6]. For more precise calculation it is necessary to solve complete kinetic equation. We report here preliminary results [7] of numerical integration of Boltzmann kinetic equations for the simplest model of self-interacting inflaton field. Let us consider inflaton field $\phi(x, t)$ with potential $V = \lambda \frac{\phi^4}{4}$. The corresponding equation of motion is

$$\ddot{\phi} - \Delta_x \phi + \lambda \phi^3 = 0. \tag{0.1}$$

The inflaton field $\phi(x, t)$ could be divided in the two parts:

$$\phi(x, t) = \varphi_0(t) + \varphi(x, t) , \tag{0.2}$$

where $\varphi_0(t)$ is the classical field of zero momentum mode, $\varphi(x, t)$ corresponds to fluctuations.

After the inflation field $\varphi_0(t)$ begins to oscillate, fluctuations start to grow exponentially in the first resonant band with momentum $p \sim \sqrt{\lambda}\varphi_0(0)$. By the time $t_0 = 200/(\sqrt{\lambda}\varphi_0(0))$ the regime of parametric resonance finishes [4]. After that the particle production rate becomes relatively slow and rescattering processes begin to dominate.

The corresponding stage of thermalization can be investigated in frameworks of Boltzmann kinetic equation which governs the evolution of one-particle distribution function $f(\vec{x}, \vec{p}, t)$. We can restrict ourselves to homogeneous system. Then the distribution function depends only on two variables: time t and energy ε, i.e. $\tilde{f} = \tilde{f}(\varepsilon, t)$. We can divide this function in two parts

$$\tilde{f}(\varepsilon, t) = f(\varepsilon, t) + (2\pi)^3 \delta^3(\vec{p}) n_c(t) . \tag{0.3}$$

The first term in Eq.(0.3) corresponds to fluctuations $\varphi(x, t)$ while the second one describes the condensate. The function $n_c(t)$ is the number density of particles in condensate and it is related to the amplitude of oscillations, $\varphi_0(t)$. We can define the effective mass of the field ϕ as

$$m^2 = 3\lambda < \varphi_0(t)^2 >_t + 3\lambda < \varphi(x, t)^2 >_x , \tag{0.4}$$

where $< ... >_x$ means space averaging and $< ... >_t$ is the average over period of oscillations of φ_0. If we neglect the slow change of m^2 during thermalization, we have $m^2 \approx 3/2\lambda\varphi_0^2(0)$.

Now we can write the system of kinetic equations for the distribution function (0.3) as (see also [8]):

$$\dot{n}_c(t) = -\frac{\lambda^2 n_c(t)}{64\pi^3 m} \int_m^\infty d\varepsilon_1' d\varepsilon_2' f(\varepsilon_1' + \varepsilon_2') , \tag{0.5}$$

$$\dot{f}(\varepsilon, t) = \frac{\lambda^2}{64\pi^3 \varepsilon} \int_\Omega \int d\varepsilon_1' d\varepsilon_2' \left([1 + f_1 + f_2]f_1' f_2' - [1 + f_1' + f_2']f_1 f_2\right) \frac{\min(p_i)}{p}$$

$$+ \frac{n_c(t)}{32\pi\varepsilon m p} \left(\int_m^\varepsilon [f_1' f_2' - f_1(1 + f_1' + f_2')]d\varepsilon_2' \right.$$

$$+ 2 \int_\varepsilon^\infty [f_2'(1 + f_1 + f_2) - f_1 f_2]d\varepsilon_2' \bigg) . \tag{0.6}$$

The system of equations Eqs.(0.5,0.6) has two integrals of motion – total number density of particles and total energy density[2]:

$$N_{\text{tot}} = n_c(t) + \int \frac{d^3\vec{p}}{(2\pi)^3} f(\varepsilon, t) , \qquad (0.7)$$

$$E_{\text{tot}} = mn_c(t) + \int \frac{d^3\vec{p}}{(2\pi)^3} \varepsilon f(\varepsilon, t) . \qquad (0.8)$$

Let us now define appropriate dimensionless variables. The overall amplitude of distribution function at resonance peak at time t_0 we define as f_0. Then distribution function of non-zero momenta particles will have a form $\overline{f}(\varepsilon, t) = f_0 f(\varepsilon, t)$. In the same way we can define dimensionless number of particles in condensate as $n_c(t) = n_0 m^3 n(t)$ where $m^3 n_0$ is condensate density at the moment t_0 and $n(t_0) = 1$. In addition we define

$$\tilde{p} = pm , \qquad \tilde{\varepsilon} = m\varepsilon = m\sqrt{1+p^2}$$

$$t = \tau \frac{64\pi^3}{m(f_0\lambda)(n_0\lambda)} \qquad (0.9)$$

We solved the set of Eqs.(0.5,0.6) in rescaled variables (0.9) on energy grid in region $1 < \varepsilon < 10$ with the following initial conditions: the distribution function strongly peaked around $\varepsilon = 2$, $f(\varepsilon, t_0) = \exp(-100|\varepsilon - 2|)$ with amplitude $f_0 = 10^3$. We require that 2/3 of total energy at time t_0 is in condensate. From Eq.(0.8) we have $n_0 = 15$.

The results of numerical integration are presented at Fig.1. On this figure the distribution function as a function of energy at different moments of time is shown. The upper curves correspond to latter moments of time. The lowest line corresponds to initial distribution function, which is peaked near $\varepsilon = 2$. Already at time $\tau = 10^{-3}$ we can see secondary peaks at energies $\varepsilon = N\varepsilon_0$, where N is integer number. This peaks appear due to rescattering of particles on condensate (for example, $\varepsilon_0 + \varepsilon_0 \to m + \varepsilon_1$). At times $\tau = 0.3, 2.3$ (the next two curves) they become larger. At latter times ($\tau > 2.5$), when $f(\varepsilon, t)$ grow and $n_c(t)$ decrease, the first term in Eq.(0.6) begin to dominate. Particles scatter on each other rather then on condensate and peaks disappear (the upper curve on Fig.1 at time $\tau = 3.3$). This is thermalization process. The time scale of further relaxation to equilibrium is much larger than $\tau = 1$. The picture obtained here is similar to what was observed in [4] as a direct solution of Eq. 0.1.

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[2]Let us note here that equation (0.7) is not valid in realistic model, where number of particles does not conserves due to processes like $4 \to 2$.
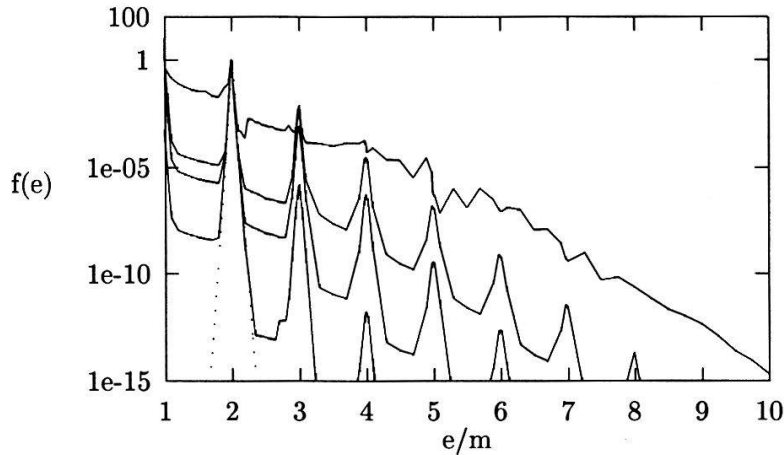
Fig.1. The distribution function is shown at different moments of time. The dotted line is the initial distribution.

In conclusion, let us note that for complete investigation of kinetic stage of reheating process one needs to include source term in kinetic equations to take into account particle creation during parametric resonance, as well as to include effects of expansion of the Universe. This work is in progress [7].

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