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Cosmological Perturbations of Ultrarelativistic Plasmas

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Abstract. Scalar cosmological perturbations of a weakly self-interacting plasma mixed with a perfect radiation fluid are investigated. Effects of this plasma are considered through order $\lambda^{3/2}$ of perturbative thermal-field-theory in the radiation dominated universe. The breakdown of thermal perturbation theory at vastly subhorizon scales is circumvented by a Padé approximant solution. Compared to collisionless plasmas the phase speed and subhorizon damping of the plasma density perturbations are changed. An example for a self-interacting thermal field is provided by the neutrinos with effective 4-fermion interactions.

The evolution of cosmological perturbations depends on the matter content of the universe. In the radiation dominated epoch *photons, electrons*, and *baryons* are coupled strongly, hence an effective description as a perfect radiation fluid is possible. The energy density of the perfect fluid is related to its pressure by the equation of state $\rho = 3p$. The cosmological perturbations [1] are determined by the linearized Einstein equations together with the perturbed equation of state $\delta p = c_s^2 \delta\rho$, where c_s is the sound speed of the perfect radiation fluid; the anisotropic pressure vanishes. *Neutrinos* interact weakly, therefore they decouple from the strongly interacting matter at a temperature ~ 1 MeV and propagate freely thereafter. This almost collisionless gas of neutrinos may be described by the linearized Einstein-Vlasov equations [2] or by a thermal-field-theoretic approach [3]. In the latter the perturbed energy-momentum tensor is calculated from the response of the ultrarelativistic plasma on a metric perturbation

$$\delta(\sqrt{-g}T^{\mu\nu})(x) = \int_y \frac{\delta^2\Gamma}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}(y)} \delta g_{\alpha\beta}(y) , \tag{1}$$

where Γ is the effective action of (thermal) matter. The Hubble rate $H \sim T^2/m_{\rm Planck}$ is much smaller than the temperature T, thus momenta $k^{\rm phys}$ of cosmological interest are much smaller than T as well. The second variation of the effective action, the gravitational

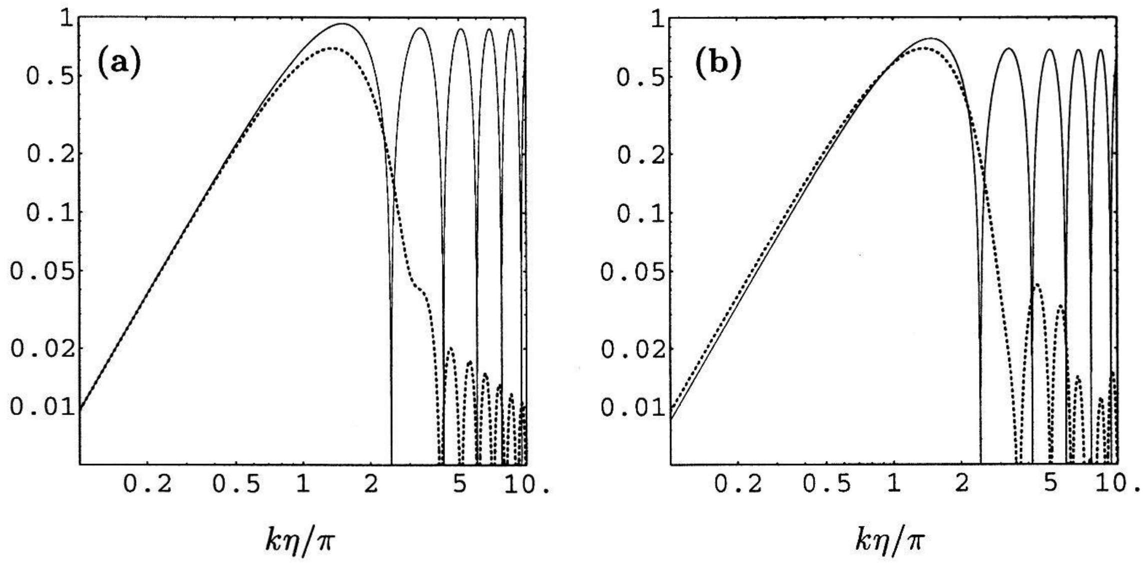

Figure 1: **(a)** The comoving density contrast grows as a function of the conformal time η on superhorizon scales ($k\eta \equiv k^{\rm phys}/H < \pi$) for the perfect fluid (full line) and the collisionless plasma (dotted line). On subhorizon scales ($k^{\rm phys}/H > \pi$) the high sound speed, $c_s = 1/\sqrt{3}$, of the perfect fluid gives rise to acoustic oscillations of constant amplitude. Collisionless matter perturbations are damped due to directional dispersion, the phase speed is one, and the oscillations are due to the gravitational interactions of the collisionless plasma with itself. **(b)** A mixture of 50% perfect radiation fluid (full line) and 50% collisionless plasma (dotted line) evolved together. On subhorizon scales the collisionless plasma feels the gravitational potential of the radiation fluid, its damping is modulated. The normalization is arbitrary.

polarization tensor, is evaluated in the high temperature limit, $T \gg k^{\rm phys}$. The thermal-field-theoretic formulation goes beyond classical kinetic theory. Eq. (1) and the linearized Einstein equations define a closed set of integro-differential equations for the metric perturbations, which may be solved by a power-series ansatz [3]. Fig. 1(a) shows the comoving density contrast δ_k as a function of conformal time η of a perfect radiation fluid and a collisionless plasma respectively. At temperatures below the muon threshold ($T < 35$ MeV) the ratio $\alpha \equiv \rho_\nu/\rho_{\rm tot} \approx 0.49$, whereas after e^+e^--annihilation at about 0.2 MeV $\alpha \approx 0.41$. Fig. 1(b) shows the density contrasts for $\alpha = 1/2$. Between temperatures of 1 MeV (decoupling of neutrinos) and 10 eV (the cold dark matter contributes only 1/10 of the total energy density) this is a good model for the dominant matter perturbations at that time. Neutrinos well above 1 MeV are included in the radiation fluid usually. How can we describe neutrinos near the decoupling temperature and what effects should we expect for cosmological perturbations?

In a recent work H. Nachbagauer, A. Rebhan, and myself [4] studied the behavior of cosmological perturbations for thermal $O(N)$-symmetric *scalar fields* with *weak self-interactions* $\lambda\phi^4$. We discovered an unexpected breakdown of thermal perturbation theory on vastly subhorizon scales ($k^{\rm phys} \gg H$). The full set of 2-loop diagrams, after resummation of thermal masses $m = \lambda^{1/2}T + \mathcal{O}(\lambda)$ and nonlocal gravitational vertices, has been included in the gravitational polarization tensor of Eq. (1). The resummation of an infinite subset of all

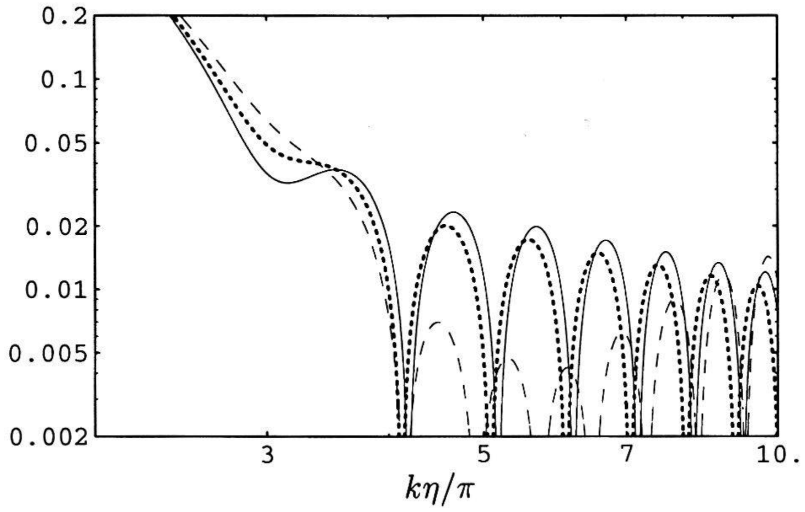

Figure 2: The subhorizon evolution of comoving density perturbations of a collisionless plasma (dotted line as in Fig. 1(a)) is compared with the density perturbations of a self-interacting scalar plasma with $\lambda = 1$ (dashed line). At $k\eta/\pi \approx 4$ the thermal perturbation theory, evaluated through order $\lambda^{3/2}$, starts to break down, which leads to unphysical growth of the perturbation asymptotically. The subhorizon behavior of the solutions has been improved by use of Padé approximants (full line). Vastly subhorizon the phase speed is smaller than one and the damping is weaker than $1/(k\eta)$.

diagrams is standard in hot QCD [5] and avoids infra-red divergences beyond the leading order λ. Due to the presence of a thermal mass the next-to-leading order is $\lambda^{3/2}$. On superhorizon scales the behavior of the growing mode is the same for any value of λ, but the decaying mode may show superhorizon oscillations depending on the values of α and λ [3, 4]. The subhorizon evolution of the density perturbation is shown in Fig. 2 for $\lambda = 1$. Thermal perturbation theory does not only break down for large values of the coupling λ, but also vastly subhorizon at perturbatively small values of λ. Increasing orders in $\lambda^{1/2}$ are increasingly infra-red singular in the high temperature limit as the external momentum k approaches the light-cone. For subhorizon scales $\eta \gg 1/k$ the contribution from λ^1 overtakes the contribution from λ^0, $\lambda^{3/2}$ overtakes λ^1, and so on. A way to improve the subhorizon behavior is to rewrite the perturbative series in $\lambda^{1/2}$ into a (2,1) Padé approximant

$$a + b\lambda + c\lambda^{3/2} + \ldots \simeq a + b\lambda \Big/ \Big[1 - (c/b)\lambda^{1/2}\Big] + \ldots \tag{2}$$

In the large N limit this gives an excellent approximation to the exact value of the thermal mass [6] even for large values of λ. A calculation of the most important 1-loop diagram of the polarization tensor to all orders in $\lambda^{1/2}$ has shown that the Padé approximant, in contrast to the perturbative result, describes the phases correct and provides a good approximation of the damping for large values of λ and η. The full 2-loop Padé improved solution for the density contrast with $\lambda = 1$ is shown in Fig. 2. The main conclusions for scalar perturbations (for the vector and tensor sector see [4]) are:

At superhorizon modes the anisotropic pressure decreases as the coupling λ is increased.

At subhorizon scales the phase speed of the damped oscillations is smaller than one, and the damping is weaker than $1/(k\eta)$.

For one scalar field we have at subhorizon scales the paradoxical situation that, although the interaction rate Γ may be much bigger than the Hubble rate, $\Gamma/H \sim \lambda^2 m_{\text{Planck}}/T$, when $T \ll m_{\text{Planck}}$ and λ is perturbatively small, the plasma perturbations behave almost collisionless. This happens because the class of diagrams dropping out in the large N limit (order $\lambda^2 \ln \lambda$ and higher) is not taken into account by the Padé approximant solution. In the large N limit, where the Padé approximant is close to the exact solution, the Hubble scale picks up a factor $\sqrt{N}$ from the relativistic degrees of freedom, thus the interactions are slower than the expansion. On superhorizon scales the interaction rate is irrelevant for the behaviour of the perturbations. The amount of superhorizon anisotropic pressure is determined by the initial conditions and gravity only.

What does this tell us about neutrinos? At low temperatures $T \ll 100$ GeV the weak interactions of neutrinos are well described by effective 4-fermion interactions. The topology of the corresponding loop-diagrams is the same as before. Although the details are quite different the qualitative features should be similar. Let me naively relate λ to $G_F T^2$. At neutrino decoupling this corresponds to $\lambda \approx 10^{-11}$, thus perturbation theory should work perfect. Our results for the scalar plasma suggest that neutrino perturbations behave almost collisionless even above decoupling! On subhorizon scales this is not reliable for two reasons: Again our result can only be trusted for a large number of neutrino flavors, and the neutrinos couple to matter which is strongly interacting and is described by a perfect fluid. Our results show that, depending on initial conditions, effects of anisotropic pressure on superhorizon scales can be important although the dominant plasma is collisional.

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