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The Ground-State Characteristics Of Deuteron Using Gaussian Potentials

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Abstract. The translation invariant shell model is applied to the deuteron nucleus with even number of quanta of excitations in the range $0 \leq N \leq 10$. The residual interparticle interactions for this nucleus consist of central, tensor, spin-orbit and quadratic spin-orbit forces with Gaussian radial dependences. The parameters of these interactions, are fitted to reproduce good agreements between the calculated and the experimental values of the deuteron binding energy, mean-square radius, D-state probability, magnetic dipole moment and electric quadrupole moment.

1. Introduction

In a nucleus, there is no external source to provide a force on the individual nucleon and, as a result, there is no fundamental one-body interaction. The only one-body operator in a nuclear Hamiltonian is the kinetic energy operator connected with the motion of each nucleon. The source of the effective one-body potential is, however, the two-body interaction between nucleons. On the other hand, it is not possible to rule out three-body and higher particle rank terms in the nuclear interaction. All the available evidence indicates that such forces, if present, must be very much weaker than two-body force.

A wide variety of nucleon-nucleon interactions has been used in the shell model calculations. Since the nucleon-nucleon interaction is not precisely known a large number of different interactions are available. In addition, the kind of interaction that can be used is restricted to some extent by the kind of model space used. Finally, unless the model space is complete, with a sufficiently large number of bases, the interaction is really only an effective interaction and cannot be uniquely defined. Moreover, unless one is very restrictive in

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defining the model space,it is necessary to diagonalize large matrices. There will be an inevitable conflict between the desire to keep a small model space and the hope to find an effective interaction with a wide range of applicability. In addition to these reasons,which can to a greater or lesser extent be justified by the physics of the problem,personal preference and ease of computation also play a role.

The structure of deuteron may well be harder to understand than many other nuclei because of its large radius and the fact that the tensor force is crucial,accounting for 70% of the binding interaction in spite of the small D-state admixture.

The nature of the deuteron quadrupole moment,Q_d and the ratio of the deuteron's D-wave to S-wave asymptotic normalization constants, $\eta(^2H)$,is well explicated by Ericson [9]. Based on the various local,energy-independent potential models of the two-nucleon interaction,consistent with the experimental effective range and the one-pion-exchange,it has been proposed that there is a linear relationship between Q_d and $\eta(^2H)$that can be tested with a higher precision determination of $\eta(^2H)$ [8,10].Based on the successful description of Q_d and $\eta(^2H)$,Ericson and Rosa-Clot suggest that the entire deuteron wave function be thought of as one-pion-exchange dominated,with the precise value of the deuteron binding energy set by the short-range part of the two-nucleon interaction.

Amado [1] emphasized that within a given model, the deuteron D-state probability P_D (^2H) has a fixed value, but a different dynamical model fitted to the same empirical data will generally yield a different value of P_D (^2H). Unitary transformations can be applied to wave functions and operators that leave observables unaltered but that change P_D (^2H). This approach was taken by Friar [11] to show that P_D (^2H) depends on the unitary transformation. Nevertheless, P_D (^2H) is a useful parameter that can be used to characterize a particular two-nucleon interaction.

Weller and Lehman [16] in their work on the manifestations of the D-state in light nuclei have pointed out that it seems clear that we are still at the threshold of understanding the importance of the tensor interaction in nuclear physics.

In an approach to tackle the deuteron properties the expansion of the square of the ratio of the deuteron radius to the triplet neutron-proton scattering length is generalized to include the D-state component of the deuteron wave function. This approach is recently applied by Dijk et.al.[3] to estimate the effect of the D-state on the deuteron radius.

The deuteron properties are also studied by using the one -pion exchange potential truncated at a radius R,with a constant interior potential. Sprung et. al. [14] discussed the relation of this model to more realistic models of the nucleon-nucleon interaction.

The aim of this work is to construct the ground-state wave function of deuteron and to evaluate a simple nucleon-nucleon potential that gives acceptable fit to the deuteron properties.

The methods of expanding the nuclear wave function in terms of a complete set of orthonormal functions, basis functions, have been used on a large scale. In principle, the predicted results for the nuclear characteristics should be independent of the particular bases chosen when the number of terms in the expansion is kept large enough.

In this paper we applied the translation invariant shell model [2,4,7,15]which has shown good results for the calculations of the ground-state characteristics and some of the excited-state characteristics of light nuclei [4-7]. Bases of this model with even number of quanta of excitations in the range $0 \leq N \leq 10$ are used to construct the ground-state wave function of deuteron. The residual interparticle interaction is assumed to have central, tensor, spin-orbit and quadratic spin-orbit interactions. The central part has symmetric and Serber forces. The radial dependences of these forces are taken as Gaussian forces which are suitable for our calculations of the matrix elements of the two-particle operators needed in our investigations as well as they lead to faster convergences of the series and integrals in our calculations. The Wigner, Majorana, Bartlett and Heisenberg constants of the central part of the interaction are taken in agreement with previous works [6,12,13].The central depth parameter V_C is allowed to vary in the range $25 \leq V_C \leq 55$ MeV with a step of one MeV and the corresponding range parameter r_C is allowed to vary in the range $1.5 \leq r_C \leq 3.0$ fm with a step of 10^{-3} fm. Each of the tensor, spin-orbit and quadratic spin-orbit parts of the interaction has one depth parameter, which is assumed to vary in the range from -50.0 to -5.0 MeV with a step of 0.01 MeV. The corresponding range parameter is assumed to vary in the range from 0.3 to 3.0 fm with a step of 10^{-3} fm. The oscillator parameter $\hbar\omega$ is allowed to vary in the range $10 \leq \hbar\omega \leq 25$ Mev in order to obtain the minimum value of the ground-state energy eigenvalue of the deuteron nucleus.

Accordingly, the Hamiltonian matrices for the ground state of the deuteron are diagonalized with respect to the oscillator parameter $\hbar\omega$, from which the binding energies and the nuclear wave functions are obtained for each set of values of the eight parameters of the potential, namely: V_C , r_C , V_T , r_T , V_S , r_S , V_L and r_L. The obtained nuclear wave functions are used to calculate the mean-square radius, the D-state probability, the magnetic dipole moment and the electric quadrupole moment of deuteron. The cases which gave results in good agreements with the corresponding experimental values are given and discussed.

2. The Hamiltonian Matrix And The Nuclear Wave function

The Hamiltonian $\mathcal{H}$ of the two-nucleon system can be written in the form

$$\mathcal{H} = \frac{1}{2m}\sum_{i=1}^{2} p_i^2 + V(|\vec{r}_1 - \vec{r}_2|) \tag{2.1}$$

By separating the center of mass kinetic energy, the Hamiltonian corresponding to the internal motion becomes

$$H = \mathcal{H} - \frac{1}{4m}\left(\sum_{i=1}^{2} \vec{p}_i\right)^2 \tag{2.2}$$

By adding and subtracting an oscillator potential referred to the center of mass, the internal Hamiltonian becomes

$$\begin{aligned} H = {} & \frac{1}{2m}\sum_{i=1}^{2} p_i^2 - \frac{1}{4m}\left(\sum_{i=1}^{2} \vec{p}_i\right)^2 + \frac{1}{2} m\omega^2 \sum_{i=1}^{2} (\vec{r}_i - \vec{R})^2 \\ & + V(|\vec{r}_1 - \vec{r}_2|) - \frac{1}{2} m\omega^2 \sum_{i=1}^{2} (\vec{r}_i - \vec{R})^2 \end{aligned} \tag{2.3}$$

This Hamiltonian can be recasted, in terms of the relative coordinates of the two nucleons, in the form

$$H = H_0 + V', \tag{2.4}$$

where

$$H_0 = \frac{1}{2}\left[\frac{(\vec{p}_1 - \vec{p}_2)^2}{2m} + \frac{1}{2} m\omega^2 (\vec{r}_1 - \vec{r}_2)^2\right], \tag{2.5}$$

is the translation-invariant shell model Hamiltonian of the two nucleon system and

$$\begin{aligned} V' &= V(|\vec{r}_1 - \vec{r}_2|) - \frac{m\omega^2}{4}(\vec{r}_1 - \vec{r}_2)^2 \\ &= V(r) - \frac{m\omega^2}{4} r^2, \end{aligned} \tag{2.6}$$

is the residual interaction

The energy eigenvalues and eigenfuctions of the Hamiltonian H_0 are given by

$$E_N^{(0)} = (N + \frac{3}{2})\hbar\omega, \tag{2.7}$$

$$|Nlm_\ell, sm_s tm_t> = R_{N\ell}(r) Y_{\ell m_\ell}(\theta, \phi) \chi_{sm_s} \tau_{tm_t}, \tag{2.8}$$

where $N=2n+\ell$, in which n is the radial quantum number of the inter-particle distance joining the two nucleons. The radial wave functions $R_{N\ell}(r)$ are given, with the usual notations, by

$$R_{N\ell}(r) = a_0^{-3/2} \sqrt{\frac{2\Gamma((N-\ell+2)/2)}{\Gamma((n+\ell+3)/2)}} \exp(-\rho^2/2)\, \rho^\ell L_{\frac{N-\ell}{2}}^{\ell+\frac{1}{2}}(\rho^2), \tag{2.9}$$

where $\rho = r/a_0$, $a_0 = \sqrt{\hbar/m\omega}$

By virtue of the basis functions (2.8) one constructs the ground-state wave function of the deuteron with $j=m_j=1$ and $t=m_t=0$, in the usual manner as follows

$$\begin{aligned} |j=m_j=1, t=m_t=0> &= \sum_{N\ell s} C_{N\ell s}^{1,0} \sum_{m_\ell + m_s = 1} (\ell m_\ell, sm_s|11) \\ &\quad x\,|N\ell m_\ell, sm_s 00>, \end{aligned} \tag{2.10}$$

where $C^{1,0}_{N\ell s}$ are the state-expansion coefficients and $(\ell m_\ell,\ sm_s|\,11)$ are Clebsch-Gordan coefficients of the rotational group R_3. In the summation (2.10) N assumes only even integers, since the ground state of the deuteron has positive parity, so that s=1 and ℓ =0,2. Four Clebsch-Gordan coefficients occur in equation (2.10), namely:

$$(00,11\,|\,11) = 1,\ (20,11\,|\,11) = \frac{1}{\sqrt{10}},\ (21,10\,|11) = -\sqrt{\frac{3}{10}},\ (22,1-1\,|11) = \sqrt{\frac{6}{10}}.$$

Hence, the ground-state wave function of the deuteron, in our model with N $\leq$ 10, takes the form

$$\begin{aligned}|j=m_j=1,t=m_t=0> &= C_{00}\,|000,1100> + C_{20}\,|200,1100> \\ &+ C_{22}\left[\frac{1}{\sqrt{10}}\,|220,1100> - \sqrt{\frac{3}{10}}\,|221,1000> + \sqrt{\frac{6}{10}}\,|222,1-100>\right] \\ &+ C_{40}\,|400,1100> + C_{42}\left[\frac{1}{\sqrt{10}}\,|420,1100> - \sqrt{\frac{3}{10}}\,|421,1000> + \sqrt{\frac{6}{10}}\,|422,1-100>\right] \\ &+ C_{60}\,|600,1100> + C_{62}\left[\frac{1}{\sqrt{10}}\,|620,1100> - \sqrt{\frac{3}{10}}\,|621,1000> + \sqrt{\frac{6}{10}}\,|622,1-100>\right] \\ &+ C_{80}\,|800,1100> + C_{82}\left[\frac{1}{\sqrt{10}}\,|820,1100> - \sqrt{\frac{3}{10}}\,|821,1000> + \sqrt{\frac{6}{10}}\,|822,1-100>\right] \\ &+ C_{10,0}\,|10,00,1100> + C_{10,2}\left[\frac{1}{\sqrt{10}}\,|10,20,1100> - \sqrt{\frac{3}{10}}\,|10,21,1000> + \sqrt{\frac{6}{10}}\,|10,22,1-100>\right].\end{aligned} \tag{2.11}$$

Here $C_{N\ell} = C_{N\ell 1}{}^{10}$, since s=1 for all bases.

The matrix elements of the operator $\frac{m\omega^2 r^2}{4}$, appearing in equation (2.6), taken over the basis functions $|N\ell m_\ell,\ 1m_s 00>$ are given by

$$\begin{aligned}<N\ell m_\ell, 1m_s 00\,|\frac{m\omega^2 r^2}{4}\,|N'\ell' m'_\ell, 1m'_s 00> &= \frac{\hbar\omega}{4}\Big[(2N+3)\delta_N^{N'} \\ &-\sqrt{N-\ell+2}\sqrt{N+\ell+3}\,\delta_{N+2}^{N'} - \sqrt{N-\ell}\sqrt{N+\ell+1}\,\delta_{N-2}^{N'}\Big]\delta_\ell^{\ell'}\delta_{m_\ell}^{m'_\ell}\,\delta_{m_s}^{m'_s}.\end{aligned} \tag{2.12}$$

The Hamiltonian matrix elements of the operator (2.4) calculated with respect to the basis functions $|N\ell m_\ell,\ 1m_s 00>$ are, finally, given by

$$<N\ell M_\ell, 1m_s 00\,|H|\,N'\ell' m_\ell', 1m_s' 00>$$

$$=\Big\{\Big[(2N+3)\,\delta_N^{N'}+\sqrt{N-\ell+2}\,\sqrt{N+\ell+3}\,\delta_{N+2}^{N'} \tag{2.13}$$

$$+\sqrt{N-\ell}\,\sqrt{N+\ell+1}\,\delta_{N-2}^{N'}\Big]\frac{\hbar\omega}{4}\,\delta_\ell^{\ell'}+I_{N\ell\,,\,N'\ell'}\Big\}\,\delta_{m_\ell}^{m_\ell'}\,\delta_{m_s}^{m_s'},$$

where $I_{N\ell\,,\,N'\ell'}$ are the radial integrals of the operator V(r). The Hamiltonian matrix elements of equation (2.13) are, obviously, independent of m_ℓ and m_s.

3. The Residual Interaction And The Radial Integrals

For each two-nucleon-state with orbital-angular momentum ℓ, spin momentum s and isotopic spin t, our potential assumes the form

$$V(r) = {}^{ts}X\, V_C \exp(-r^2/r_C^2) + V_T S_{12} \exp(-r^2/r_T^2)$$
$$+ V_S(\vec{\ell}\,.\,\vec{s}) \exp(-r^2/r_S^2) + V_L L_{12} \exp(-r^2/r_L^2), \tag{3.1}$$

where

$${}^{ts}X = C_W+(-1)^{s+t+1}C_M+(-1)^{s+1}C_B+(-1)^{t+1}C_H, \tag{3.2}$$

$$S_{12}=3(\vec{\sigma}_1\,.\,\vec{n})(\vec{\sigma}_2\,.\,\vec{n})-(\vec{\sigma}_1\,.\,\vec{\sigma}_2),\ \vec{n}=\vec{r}/r$$

$$L_{12}=(\vec{\sigma}_1\,.\,\vec{\sigma}_2)\,\ell^2-\frac{1}{2}\Big\{(\vec{\sigma}_1\,.\,\vec{\ell})(\vec{\sigma}_2\,.\,\vec{\ell})+(\vec{\sigma}_2\,.\,\vec{\ell})(\vec{\sigma}_1\,.\,\vec{\ell})\Big\}$$

In equ. (3.2) C_W, C_M, C_B and C_H are the Wigner, Majorana, Bartlett and Heisenberg constants, respectively. V_C, r_C; V_T, r_T; V_S, r_S and V_L, r_L are the depth and the range parameters for the central, tensor, spin-orbit and quadratic spin-orbit forces, respectively.

For the symmetric and the Serber forces, the exchange constants satisfy the well known normalization condition

$$C_W + C_M + C_B - C_H = -1.$$

For the symmetric forces they satisfy the additional conditions

$$C_M = 2C_B \text{ and } C_H = -2\,C_W,$$

while, they satisfy the conditions

$$C_M = C_W \text{ and } C_H = -C_B,$$

in the case of the Serber forces. Two sets of values which satisfy these conditions are

Case-1 (symmetric forces)

C_W = 0.1333, C_M = -0.9333, C_B= -0.4667, C_H = -0.2667,
which are known as the Rosenfeld constants [13], and

Case-2 (Serber forces)

C_W = -0.41, C_M= -0.41, C_B = -0.09, C_H= 0.09
which are taken in accordance with the Lederer potential [12].

Our potentials are, then, obtained by allowing the depth parameters V_C, V_T, V_S and V_L and the range parameters r_C, r_T, r_S and r_L to vary in the ranges discussed in section-1, with a total of 8.36071 x 10^{25} different potentials. The radial integrals $I_{N\ell\,,\,N'\ell'}$ appearing in equ. (2.13) are, then, evaluated by using the radial wave functions (2.9) and the potentials (3.1).

Although the ground-state of deuteron has s=1 and t=0, so that the central part of our potentials assume the form

$$V(r) = -V_C \exp(-r^2/r_C^2), \tag{3.3}$$

for both of the symmetric and the Serber forces, our potentials are written in such form (equ.(3.1)) so that they can be used in other calculations with light nuclei.

4. The Deuteron Characteristics

Diagonalizing the Hamiltonian matrix, with elements given by equ. (2.13), we get the deuteron ground-state energy eigenvalues and eigenfunctions for each set of values of the potential parameters. The resulting nuclear wave functions are used to calculate the different ground - state characteristics of the deuteron as follows:

(i) The D-state Probability

The D-state probability, P_D, is the sum of the squares of the expansion coefficients in the ground - state wave function of deuteron for which ℓ=2, so that in our calculations

$$P_D = C_{22}^2 + C_{42}^2 + C_{62}^2 + C_{82}^2 + C_{10,2}^2 \,. \tag{4.1}$$

(ii) The Mean - Square Radius

The mean - square radius, R, is defined as

$$R = \sqrt{r_p^2 + < R_{Nuc}^2 >}\,, \tag{4.2}$$

where r_p = 0.85 fm is the proton radius and the second term is the mean value of the operator

$$R_{Nuc}{}^2 = \frac{1}{A^2} \sum_{i<j=1}^{A} r_{ij}^2 = \frac{1}{4} r^2 \,. \tag{4.3}$$

The mean value of the operator R_{Nuc}^2 is then obtained by multiplying the matrix element of equ.

(2.12) by $\frac{1}{m\omega^2}$ from which the value of R is obtained.

(iii) The Magnetic Dipole Moment

The magnetic dipole moment, μ, is defined as the expectation value of the operator [15]

$$\hat{\mu} = \hat{\mu}_\sigma + \hat{\mu}_o , \tag{4.4}$$

calculated in a state with $m_j = j$. The spin - isospin dependence operator $\hat{\mu}_\sigma$ is defined by

$$\hat{\mu}_\sigma = \sum_{i=1}^{A} \left[(\mu_p + \mu_n) + 2(\mu_p - \mu_n)\, t_{oi} \right] s_{oi} , \tag{4.5}$$

where μ_p and μ_n are the proton and the neutron magnetic moments, respectively, and t_{oi} an s_{oi} are the z-components of the isospin and the spin momenta of the ith nucleon, respectively. Similarly the orbital-dependence operator $\hat{\mu}_o$ is defined by

$$\hat{\mu}_o = \frac{1}{2} \sum_{i=1}^{A} (1 - 2t_{oi})\, \ell_{oi} , \tag{4.6}$$

where ℓ_{oi} is the z-component of the orbital-angular momentum operator of the ith nucleon. In the translation invariant shell model one can prove that

$$\hat{\mu}_c = \mu_c^{[A]} + \mu_c^{[A-1,1]} \; ; \; c = \sigma , o . \tag{4.7}$$

For the two-nucleon system, the expectation values of the antisymmetric operators $\mu_\sigma^{[11]}$ and $\mu_0^{[11]}$ vanish and the expectation values of the symmetric operators $\mu_\sigma^{[2]}$ and $\mu_o^{[2]}$ become

$$\begin{aligned} &\left\langle N\ell m_\ell , 1\, m_s\, o\, o \left| \mu_\sigma^{[2]} \right| N'\, \ell'\, m'_\ell , 1\, m'_s\, o\, o \right\rangle \\ &= \left[1 + \sqrt{\ell(\ell+1)}\, g \right] (\mu_n + \mu_p)\, \delta_N^{N'}\, \delta_\ell^{\ell'}\, \delta_{m_\ell}^{m'_\ell}\, \delta_{m_s}^{m'_s} , \end{aligned} \tag{4.8}$$

and

$$\begin{aligned} &\left\langle N\ell m_\ell , 1\, m_s\, o\, o \left| \mu_o^{[2]} \right| N'\, \ell'\, m'_\ell , 1\, m'_s\, o\, o \right\rangle \\ &= -\frac{1}{2} \sqrt{\ell(\ell+1)}\, g\, \delta_N^{N'}\, \delta_\ell^{\ell'}\, \delta_{m_\ell}^{m'_\ell}\, \delta_{m_s}^{m'_s} , \end{aligned} \tag{4.9}$$

where

$$g = - \sqrt{3(2\ell+1)} \begin{Bmatrix} 1 & \ell & 1 \\ 1 & 1 & \ell \end{Bmatrix} (10,11 \,|\, 11) . \tag{4.10}$$

Substituting the values of the Clebsch - Gordan coefficient and the 6j - symbol, the g factors are then calculated for $\ell = 0$ *and 2 from which the magnetic dipole moment is easily obtained.*

(iv) The Electric Quadrupole Moment

The electric quadrupole moment, Q_A , of a nuclear state is defined as the expectation value of the operator [17]

$$Q_o = \sqrt{\frac{16\pi}{5}}\, e \sum_{i=1}^{Z} r_i^2\, Y_{20}\, (\theta_i,\phi_i)\ , \tag{4.11}$$

calculated in the substate of maximum m_j , so that the electric quadrupole moment of deuteron is given by

$$Q_d = \langle\, j=m_j=1,\ t=m_t=o\ |Q_o| j=m_j=1, t=m_t=o\ \rangle\ . \tag{4.12}$$

Substituting from equ. (2.11) into equ. (4.12), we get

$$\begin{aligned} Q_d = e\sqrt{\frac{\pi}{5}} \sum_{N',N} \Big\{ & C_{N'o}\, C_{No}\, \langle N'o\,|r^2|No\rangle\, \langle 00\,|Y_{20}\,|00\rangle \\ & + \frac{2}{\sqrt{10}}\, C_{N'o}\, C_{N2}\, \langle N'o\ |r^2\,|N2\rangle\, \langle 00\,|Y_{20}\,|20\rangle \\ & + C_{N'2}\, C_{N2}\, \langle N'2\ |r^2\,|N2\rangle \Big[\frac{6}{10}\langle 22\,|Y_{20}\,|22\rangle + \frac{3}{10}\langle 21\,|Y_{20}\,|21\rangle \\ & + \frac{1}{10}\langle\, 20\ |\ Y_{20}\ |\ 20\,\rangle \Big] \Big\} \end{aligned} \tag{4.13}$$

Here, we used the fact that $r_Z \equiv r_p = \frac{1}{2}\, r$. Substituting the values of the matrix elements of the spherical harmonics Y_{20} into equ. (4.13) we then get

$$\begin{aligned} Q_d = e \sum_{N',N} \Big[& \frac{\sqrt{2}}{10}\, C_{N'o}\, C_{N2}\, \langle\, N'o|r^2\,|N2\rangle \\ & - \frac{1}{20}\, C_{N'2}\, C_{N2}\, \langle N'2\,|r^2\,|N2\rangle \Big] \end{aligned} \tag{4.14}$$

Hence, the quadrupole moment of the deuteron nucleus is obtained by calculating the two radial integrals appearing in equ. (4.14).

5- Results And Conclusions

For each value of the depth parameter V_C there corresponds a set of values for the other seven parameters $r_C, V_T, r_T, V_S, r_S, V_L$ and r_L which reproduces good agreements between the calculated and the experimental value of the deuteron binding energy.

The analysis of the results where the minimum energy eigenvalues are in good agreements with the deuteron-binding energy show that for values of $V_C \leq 36$ MeV the values of the mean-square radius increase while the values of the magnetic dipole moment and electric quadrupole moment decrease. On the otherhand, for values of $V_C > 42$ MeV the values of the mean-square radius decrease with the increase in the values of the magnetic dipole moment and electric quadrupole moment.

Table-1 Parameters of The Potentials

Parameter / Potl.		I	II	III	IV	V	VI
V_C	(MeV)	37	38	39	40	41	42
r_C	(fm)	1.937	1.910	1.883	1.855	1.828	1.803
V_T	(Mev)	-11.46	-11.45	-11.45	-11.50	-11.80	-11.50
r_T	(fm)	2.912	2.912	2.912	2.920	2.854	2.900
V_S	(Mev)	-18.00	-16.00	-18.00	-15.00	-15.00	-20.60
r_S	(fm)	0.600	0.650	0.700	0.820	0.820	0.890
V_L	(Mev)	-11.00	-11.50	-12.00	-11.00	-9.00	-7.00
r_L	(fm)	1.100	1.200	1.240	1.400	1.200	1.000

In table-1 we present the parameters of the potentials for which the calculated values of the deuteron characteristics are in agreements with the corresponding experimental values.

In Fig.1 we present the variation of the range parameter r_C with the depth parameter V_C. The method of least-squares gives a straight-line relation between r_C and V_C of the form

$$r_C = -0.02697\, V_C + 2.93467 \tag{5.1}$$

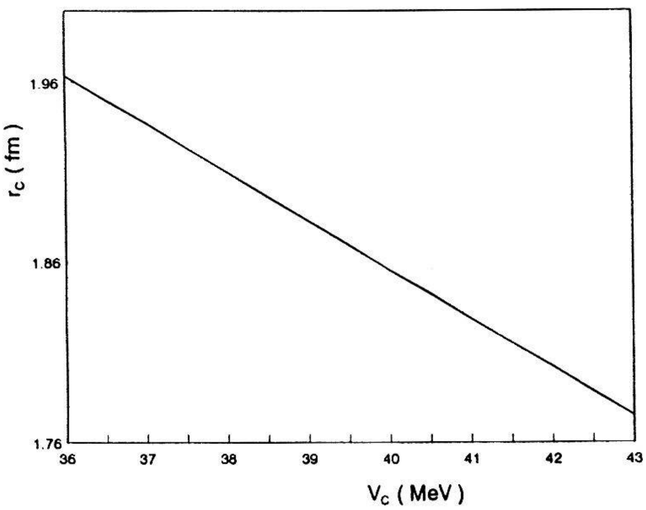

Fig.1 Variation of The Range Parameter r_C With The Depth Parameter V_C

Table-2 Deuteron Characteristics.

Case \ Charac.	B.E. (MeV)	R (fm)	P_D	μ_d (N.M.)	Q_d (e fm^2)	$\hbar\omega$ MeV	Refer.
exper.	2.22457	1.963	0.04-0.07	0.8574	0.2859	-	[17]
Potl-I	2.22450	2.0058	0.0434	0.8550	0.2868	19	
Potl-II	2.22447	2.0011	0.0428	0.8554	0.2910	19	
Potl-III	2.22421	1.9966	0.0425	0.8557	0.2953	19	
Potl-IV	2.22436	1.9932	0.0422	0.8557	0.2991	19	
Potl-V	2.22454	1.9802	0.0420	0.8559	0.3006	19	
Potl-VI	2.22456	1.9790	0.0420	0.8559	0.3050	19	
OPEP + Core							
V_0=0, R=0.0584	2.224575	1.9484	0.0750	-	0.28652	-	[14]
V_0=-10, R=0.8906	-	1.9366	0.0586	-	0.2751	-	[14]
V_0=-20, R=0.9096	-	1.9351	0.0572	-	0.2737	-	[14]
V_0=-50, R=1.0163	-	1.9275	0.0492	-	0.2650	-	[14]
V_0=-30, R=2.1894	-	2.0141	0.0627	-	0.1609	-	[14]
Yamaguchi (central)	-	1.934	-	-	-	-	[3]
Yamaguchi (with tensor)	-	1.943	-	-	-	-	[3]
OPEP (with tensor)	-	-	0.06±0.01	-	0.284	-	[10]

In Table-2 we present the calculated values of the deuteron binding energy (B.E.), mean-square radius (R), D-state probability (P_D), magnetic dipole moment (μ_d) and electric quadrupole moment (Q_d) in the cases of the six potentials together with the corresponding experimental values [17] and the values of the oscillator parameter $\hbar\omega$ which give the minimum energy eigenvalues. In Table-2 we give also previous results concerning some properties of deuteron by using other methods.

In figs. 2-6 we present the variations of the deuteron binding energy, mean-square radius, magnetic dipole moment, electric quadrupole moment, and D-state probability with respect to the oscillator parameter $\hbar\omega$ in the case of potential-II.

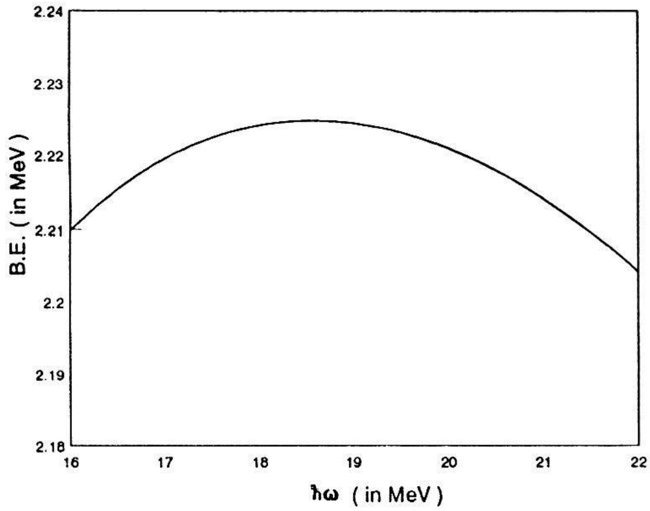

Fig.2 Variation of The Deuteron Binding Energy with $\hbar\omega$

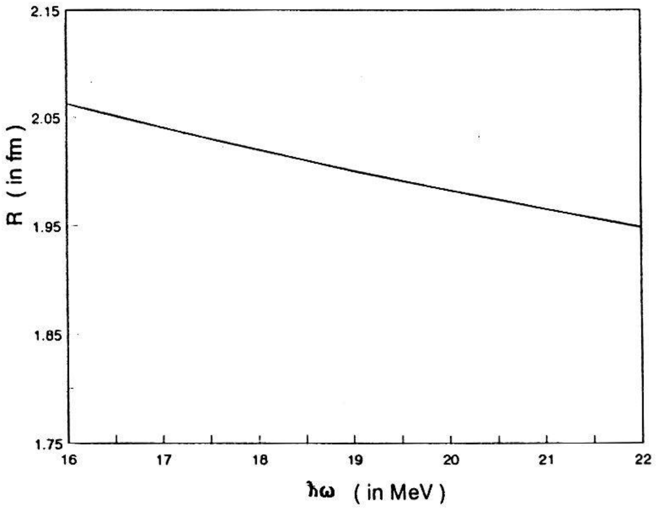

Fig.3 Variation of The Deuteron Mean-Square Radius with ħω

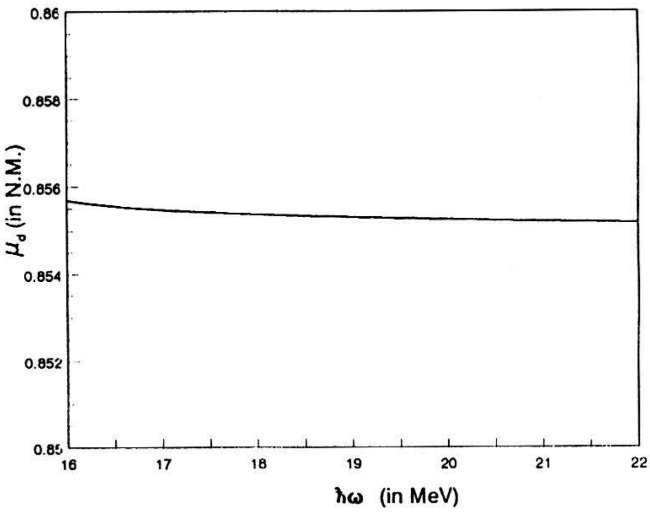

Fig.4 Variation of The Deuteron Dipole Moment with ħω

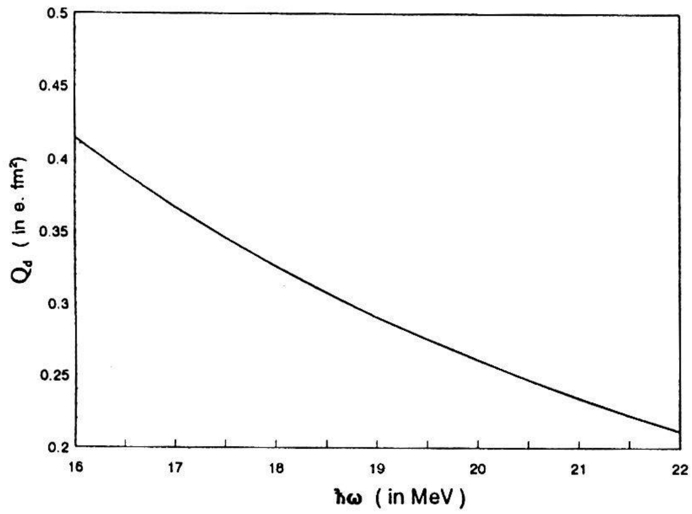

Fig.5 Variation of The Deuteron Quadrupole Moment with ħω

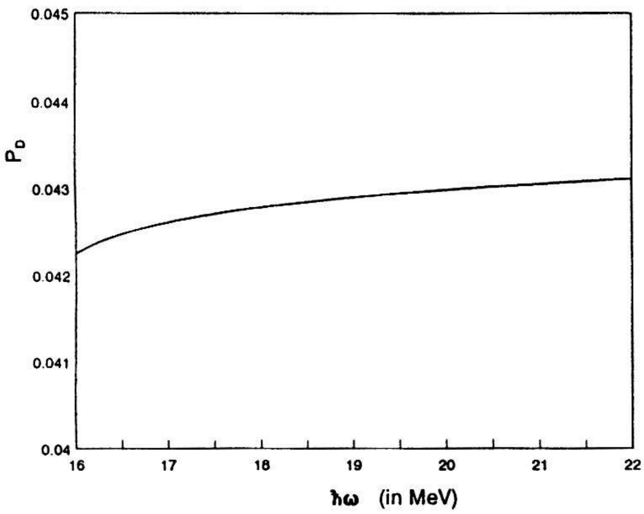

Fig.6 Variation of The Deuteron D-state probability with ħω

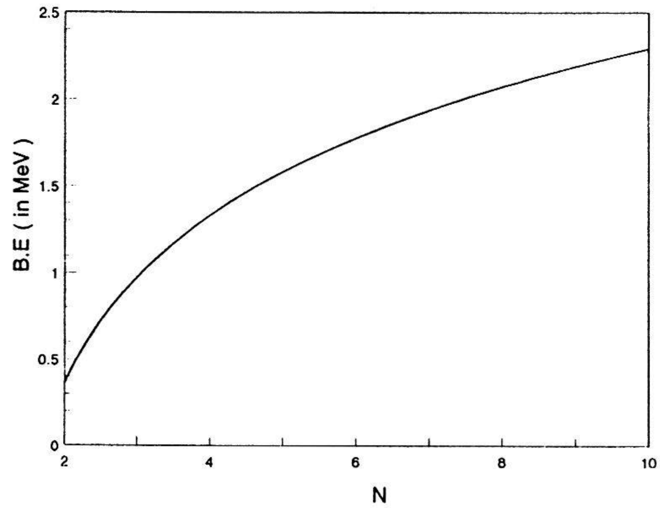

Fig.7 Variation of The Deuteron Binding Energy with N

In Figs. 7-11 we present the variations of the same characteristics with respect to the number of quanta of excitations N.

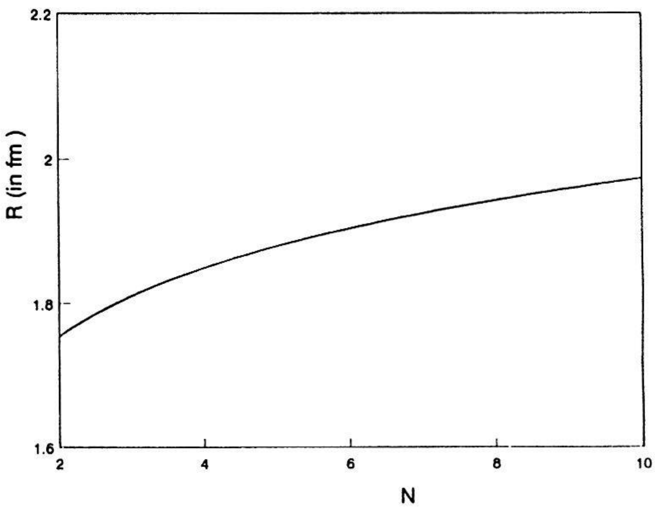

Fig.8 Variation of The Deuteron Mean-Square Radius with N

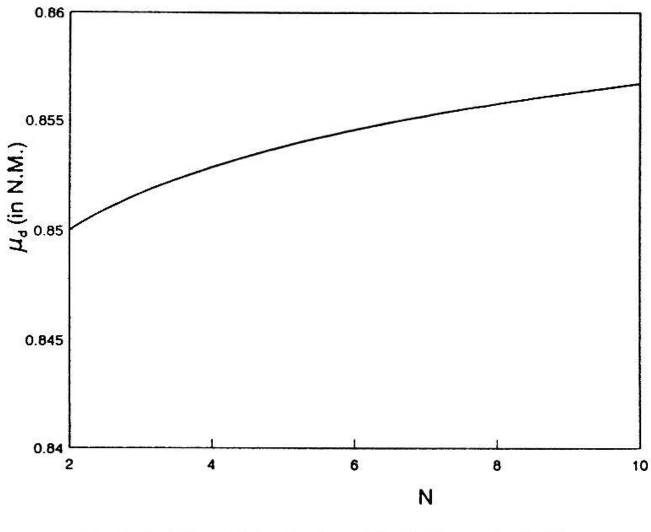

Fig.9 Variation of The Deuteron Dipole Moment with N

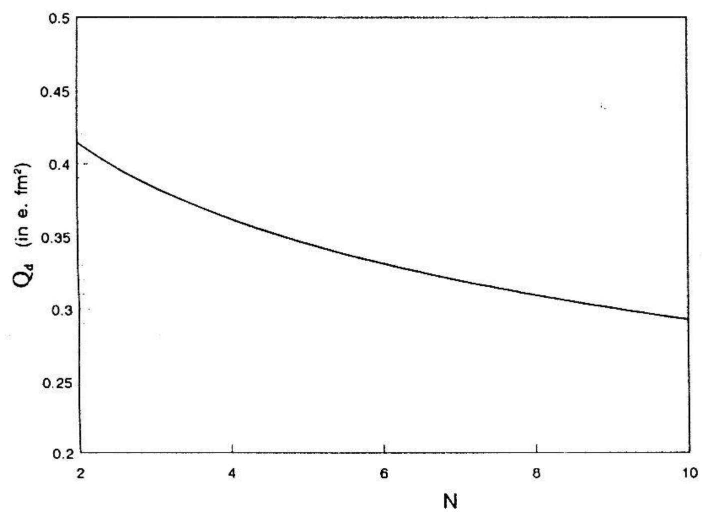

Fig.10 Variation of The Deuteron Quadrupole Moment with N

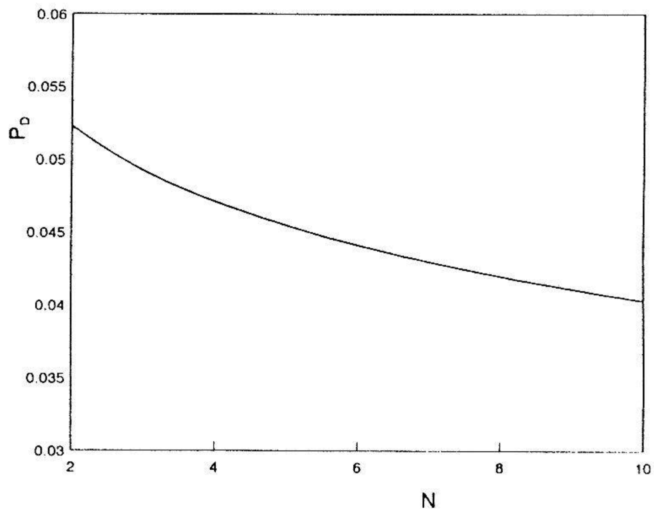

Fig.11 Variation of The Deuteron D-State Probability with N

In Table-3 we present the results of calculating the deuteron characteristics by using the possible superpositions of the different forces of potential-II. It is of interest to notice that any superposition of forces which does not include the tensor force does not add any thing to the results obtained by using the central force.

Table-3 Role of The Different Forces in Deuteron Characteristics.

Forces / **Character.**	V_C	V_C+ V_T	V_C + V_T+ V_S	V_C+V_T+ V_L	V_C+V_T+ V_S+V_L	Exper.
B.E. (MeV)	0.3064	2.2707	2.2679	2.2271	2.22447	2.22457
R (fm)	2.8403	1.9968	1.9971	2.0009	2.0011	1.963
P_D	0.0	0.0440	0.0439	0.0429	0.0428	0.04-0.07
μ_d (N.M.)	0.8798	0.8547	0.8547	0.8553	0.8554	0.8574
Q_d (e fm^2)	0.0	0.2954	0.2952	0.2912	0.2910	0.2859
$\hbar\omega$ (MeV)	8	19	19	19	19	--

It is seen from Table-2 that all the presented values of the binding energy and D-state probability of deuteron are in agreements with the corresponding experimental values. The calculated values of the magnetic dipole moment of deuteron are in agreements with the experimental value. Concerning the mean-square radius and the electric quadrupole moment their values depend mainly on the choice of the potential parameters as well as on the used nuclear wave function of deuteron and the improvement of one of them is at the expense of the other.

It is also of interest to notice from Figs. 3 and 5 that the calculated values of the deuteron mean-square radius and electric quadrupole moment can be improved by considering values of the oscillator parameter $\hbar\omega > 19$ MeV but this is rejected since we vary $\hbar\omega$ in order to obtain the best minimum energy eigenvalues in each case.

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