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Pulsed Laser Radiation — A Rigorous Model for the Collective Spontaneous Emission

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Abstract. A rigorous quantum mechanical model for the process of collective spontaneous emission (superradiance) is given. The model is based on a damped Dicke laser model for an arbitrary number of modes of the radiation field. A new result in the theory of one-parameter semigroups and their generators enables us, to obtain irreversible equations of motion for the macroscopic (i.e. classical) observables in the thermodynamic limit on the matter side, including damping reservoir terms. With help of these equations the influences of the reservoirs on the process are clarified. Equations for the emitted intensity pulses are derived and the radiation properties as well as the connections to the experiments and the literature are discussed.

1 Introduction

Since the first experimental discovery of collective spontaneous emission (CSE) or superradiance in quantum optics [1], some theoretical work has been done by means of various theoretical ansatzes (see [2] and [3] for comprehensive surveys). This goes from simple multi-spin treatment following the pioneering work of Dicke [4] up to complicated geometry-dependent calculations [5]. Some of these models bear strange features like pulse-like (i.e. irreversible) emission of intensity in a model with automorphic, i.e. reversible, time evolution (on the algebra of observables). This indicates that in such models some approximations, used to solve the problem, lead to qualitative changes in the physical consequences.

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In our approach we will follow different lines. We formulate the model in the thermodynamic limit in such a way that it can be solved without further assumptions with unforeseeable consequences. In this paper we give a treatment in the wellknown operator algebraic framework of quantum mean-field theory and quantum optics. The modified Dicke model is formulated in the Fock representation on the field side and as a combined microscopic *and* macroscopic irreversible mean-field quantum system on the matter side. The rigorous treatment results in the following advantages over the previous discussions.

First, since we are working on the basis of a mathematically rigorous treatment, we are able to obtain a new result in the theory of one-parameter semigroups, which is interesting for itself. Second, the only restriction for the (material) states of our model is, that they have to be permutation invariant with respect to the interchange of particles, i.e. we even can use mixed states. The permutation invariance seems to fit the situation in the experiment. Thirdly, by the thermodynamic limit we get rid of the number of interacting particles, which again is not a direct measureable quantity in a real experiment. This limit further leads to a reduction in the number of free parameters, that is to say to the *degree of excitation*, the *degree of cooperation* and a phase (not of interest in our case). These parameters are easy to control and have an obvious physical interpretation, which is given in the paper. Finally our results indicate clearly, that pulse-like emission can only be obtained via the influence of a reservoir (damping). This is in contrast to many models which were presented up to now.

The main results of our work can be summarised as follows: It is possible, in a rigorous way, to use a damped Dicke model for the description of the emission of coherent light pulses. With the new model it is further possible to discuss the essential mechanisms in detail, especially the action of various types of reservoirs, i.e. the damping forces.

Our work concerning the collective spontaneous emission is inspired by the investigations [6, 7] and [8, 9, 10], where problems and applications of the infinite Dicke model are discussed. However, by using only Hamiltonian dynamics for the matter resp. the field side, and the canonical Dicke interaction (compare eq. (16)) as the local (i.e. for a finite number of particles) interaction, they end with a description for cw-lasers, i.e. the continuous emission of coherent radiation. Although the number of atoms is going to infinity in thermodynamic limit, the interaction strength remains on the level of finitely many particles. Hence, in their model the coupling is too weak to allow a complete relaxation of all excited atoms into the ground state.

This paper is devoted to the *pulse-like* emission of radiation of the gas. Therefore we include the unavoidable losses in the gas into the model. This is done by using semigroup dynamics with Lindblad-like generators [11] on the matter side and hence irreversibility is obtained. Earlier works on dissipative versions of the Dicke model, [12] and most recently [13], lay their emphasis on the non-equilibrium phase transition in a damped and pumped system (i.e. a pumped cw-laser), thus they do not deal with pulsed radiation. The main differences between [12] [13] and our work are, first, we use a N^{-1} scaling for the interaction Hamiltonian whereas they scale with $N^{-1/2}$ (N number of particles in the system), and second, our radiation field consists of any (e.g. infinite, even continuous) number of modes, whereas theirs consists of a finite number of modes only.

Leaving the interaction and the free evolution of the field the same as in [6, 7] and [8, 9, 10], we show that the the inclusion of the above-mentioned damping terms is possible in a

rigorous way and is sufficient to obtain pulse-like emission. Since the detailed form of the generators is not fixed, we are able to discuss various choices for the reservoir terms, and thus get different influences on the collective behaviour of the emitting gas.

By defining the intensity via the time derivative of the (time-dependent) expectation value of the photon number-operator (in the Fock representation), i.e.

$$I(t) := \frac{d}{dt} \langle \mathcal{N} \rangle_t$$

we calculate the temporal development of the emitted radiation explicitly. Finally we prove that the emitted radiation is first order coherent in the sense of Glauber [14], thus being able to interfere constructively.

Starting with a compilation of the assumptions which lead to our model in Section 2, we state some established results of the theory of the Weyl algebra and mean-field quantum systems in Section 3. After this preparation we introduce our model and prove the main results in Section 4 and discuss and interpret the obtained results in the last Section.

2 Physical Assumptions for our Model

Before we state details of the mathematical description of our model, we want to give physical arguments for the assumptions which lead to our model. Therefore we start with an outline of the experimental setup (see [2] and [3] for details):

One has a macroscopic number (e.g. 10^{23}) of atoms or molecules confined in a (macroscopic) box. The level structure of the particles consists of two *working levels* which are separated from a ground level (or synonymous *state*) by an energy gap large enough to prevent thermal excitation from the ground state at room temperature.

With an incoherent intense pulse of light or a discharge a certain amount of electrons is carried from the ground state into the uppermost level (of the three which we are dealing with) leaving the intermediate level empty. If an immediate return into the ground state is forbidden (depending on the various ratios of the lifetimes) we have total population inversion with respect to the intermediate level. The process of excitation will be incorporated into the choice of the initial conditions, that is why it is sufficient to regard the particles as two-level systems.

If the conditions are right (which is by no means easy to prepare!), one can observe a pulse-like emission of coherent radiation (one obvious experimental condition is to make the transition to the intermediate level the most likely one).

Coherence in this situation is often understood as peak intensity proportional to the square of the number of excited particles. Obviously this is measured via the pressure dependence [15] (in the range of the ideal-gas-equation), the number of particles being inaccessible! This behaviour is then interpreted as constructive interference of *coherent* oscillators. We use the coherence concept of Glauber [14] to give this more heuristic argument a precise quantum-optical meaning.

The mirrors in ordinary lasers are necessary to increase stimulated emission. In CSE the time scale is too short to regard the phenomenon as a series of successive stimulated emissions

(i.e. one particle emitts spontaneously and then stimulates the others to emitt one after the other) (for details compare [2] and [3]). Therefore we do not have to include mirrors into our model.

Alltogether we want to give a model for the phenomenon of *pulse-like* and *coherent* emission of high intensity radiation from a *marcoscopic* system of two-level atoms or molecules by means of direct quantum correlations.

The main interest in this phenomenon lies in the spontaneous, pulse-like but nevertheless coherent emission of radiation with high intensity. This is important for constructing pulsed lasers for frequencies where one has no reflecting materials (e.g. X-ray superradiance [16]). Another area where the theory of superradiance is important, is the physics of the free electron laser [17].

3 Material Mean-Field Systems and the Photon Weyl Algebra

3.1 Quantum-Mean-Field Systems

For the description of the material side of the model we use results established by Bóna [18] and Unnerstall [19, 20]. Since we make the idealization to regard the particles as two-level systems, we use $M_2(\mathbb{C})$, the complex 2×2 matrices, as the appropriate one particle algebra. The macroscopic system of particles (i.e. the emitting gas) is then described by the algebra $\mathcal{A} := \bigotimes_{n \in \mathbb{N}} M_2(\mathbb{C})$. Interested only in the collective structure of the system, we choose as a suitable set of states on \mathcal{A} the so called permutation invariant ones. These states are homogeneous over macroscopic dimensions. Let $\mathcal{S}^p(\mathcal{A})$ denote the set of all permutation invariant states and \mathcal{F}^p the corresponding folium.

With this setup we can define the representation (Π^p, \mathcal{H}^p) of \mathcal{A} :

$$(\Pi^p, \mathcal{H}^p) := \bigoplus_{\omega \in \mathcal{F}^p} (\Pi_\omega, \mathcal{H}_\omega) \quad (1)$$

where $(\Pi_\omega, \mathcal{H}_\omega)$ denotes the GNS representation connected with $\omega \in \mathcal{F}^p$. In the representation Π^p we construct the usual W^* -algebra $\mathcal{M}^p := \Pi^p(\mathcal{A})''$ and denote its center by $\mathcal{Z}^p := \mathcal{M}^p \cap (\mathcal{M}^p)'$. Let further \mathcal{G} be the set of all selfadjoint, traceless 2×2 matrices. On \mathcal{G} we introduce a basis (since \mathcal{G} is a 3 dimensional \mathbb{R} -vectorspace) $\{S^1, S^2, S^3\}$ such that:

$$\text{tr}[S^i S^k] = \delta_{ik}$$

It is obvious that in our case $S^k = \frac{1}{2}\sigma^k$ with σ^k being the Pauli matrices.

For every finite subset $\Lambda \in \mathcal{L} := \{\Lambda' \subset \mathbb{N} \mid |\Lambda'| < \infty\}$ we define the intensive operators ($S \in \mathcal{G}$):

$$S_\Lambda := \frac{1}{|\Lambda|} \sum_{n \in \Lambda} 1 \otimes \cdots \otimes 1 \otimes \underbrace{S}_{n-\text{th pos.}} \otimes 1 \otimes \cdots \otimes 1 \in \mathcal{A}_\Lambda = \bigotimes_{n \in \Lambda} M_2(\mathbb{C}) \hookrightarrow \mathcal{A}, \quad (2)$$

We do not use the more intuitive terminology *density operators* for the S_Λ 's to avoid confusion with the term *density matrix* !

The intensive operators S_Λ now converge in the representation Π^p to macroscopical *and* commuting (i.e. classical) observables by taking the limit overall finite subsets of \mathbb{N} [18], i.e.

$$S_\infty := \text{s-} \lim_{\Lambda \rightarrow \infty} \Pi^p(S_\Lambda) \quad \forall S \in \mathcal{G} \quad (3)$$

exists, where $S_\infty \in \mathcal{Z}^p$, which justifies the interpretation as classical observables. $\lim_{\Lambda \rightarrow \infty}$ should be read as inductive limit $\lim_{\Lambda \in \mathcal{L}}$ over the ordered set \mathcal{L} .

With the help of the SNAG-theorem [21] one gets a useful representation of the operators S_∞ . Since $\mathcal{G} \rightarrow \mathcal{Z}^p$, $S \mapsto \exp\{iS_\infty\}$ is a unitary representation of the additive group \mathcal{G} , we can write:

$$S_\infty = \int_{\mathcal{G}^*} f_S(S') d\mathcal{E}_G(S') \quad (4)$$

with a uniquely determined projection-valued measure \mathcal{E}_G on the Borel-algebra of \mathcal{G}^* . f_S is the canonical imbedding of S into the bidual, i.e. $f_S(S') := S'(S) \forall S \in \mathcal{G}, S' \in \mathcal{G}^*$.

If we define the smallest C^* -algebra containing $\Pi^p(\mathcal{A})$ and $\{S_\infty \mid S \in \mathcal{G}\}$:

$$\mathcal{C}_G := C^*-\text{hull}\{\Pi^p(\mathcal{A}), S_\infty\} \subset \mathcal{M}^p$$

it holds that [18]:

$$\mathcal{C}_G \cong \mathcal{A} \otimes C(E_G) \cong C(E_G, \mathcal{A}) \quad (5)$$

with $C(E_G)$ denoting the continuous functions on the compact subset $E_G := \text{supp}(\mathcal{E}_G)$ of \mathcal{G}^* . $C(E_G)$ is an abelian algebra and describes the classical substructure of the macroscopical quantum system.

With the basis $\{S^1, S^2, S^3\}$ of \mathcal{G} one can parameterize the state space $\mathcal{S}(\mathbb{M}_2(\mathbb{C}))$ via:

$$j : \mathcal{S}(\mathbb{M}_2(\mathbb{C})) \rightarrow \mathbb{R}^3, \varrho \mapsto (\varrho(S^1), \varrho(S^2), \varrho(S^3))$$

By using $\{S^{1*}, S^{2*}, S^{3*}\}$ as basis in \mathcal{G}^* (dual to $\{S^k\}$) and $\{e_1, e_2, e_3\}$ as the canonical basis in \mathbb{R}^3 , it is easy to show that we can identify $j(\mathcal{S}(\mathbb{M}_2(\mathbb{C})))$ with E_G under the map $e_k \mapsto S^{k*}$. Since $\mathcal{S}(\mathbb{M}_2(\mathbb{C})) \cong K_{1/2} := \{\vec{x} \in \mathbb{R}^3 \mid \|\vec{x}\| \leq \frac{1}{2}\}$ [22] (example 4.2.7) we have:

$$E_G \cong j(\mathcal{S}(\mathbb{M}_2(\mathbb{C}))) \cong \mathcal{S}(\mathbb{M}_2(\mathbb{C})) \cong K_{1/2} \quad (6)$$

With $\partial_e \mathcal{S}^p(\mathcal{A}) \ni \omega \iff \omega = \bigotimes_{n \in \mathbb{N}} \varrho$, $\varrho \in \mathcal{S}(\mathbb{M}_2(\mathbb{C}))$ [23] and the map $\bigotimes_{n \in \mathbb{N}} \varrho \mapsto \varrho$ one gets a 1-1-correspondence between $\partial_e \mathcal{S}^p(\mathcal{A})$ and $E_G = K_{1/2}$. Hence an extremal permutation invariant state can be fixed by choosing $\vec{x} \in K_{1/2}$ (see below). Since $\mathcal{S}^p(\mathcal{A})$ is a Bauer simplex [23] it follows that $\mathcal{S}^p(\mathcal{A}) \cong M_+^1(K_{1/2})$, the positive normed measures on $K_{1/2}$.

We now assume that the irreversible dynamics (on the matter side) of the finite system is given by a semigroup $T_t^\Lambda : \mathcal{A} \rightarrow \mathcal{A}$ with the Lindblad generator [11]:

$$\begin{aligned} L_\Lambda(\cdot) &:= L_\Lambda^I(\cdot) + L_\Lambda^{II}(\cdot) + L_\Lambda^{III}(\cdot) \\ &= i[H_M^\Lambda, \cdot] + |\Lambda| \{V_\Lambda^*[\cdot, V_\Lambda] + [V_\Lambda^*, \cdot]V_\Lambda\} + \sum_{n \in \Lambda} \underbrace{\{W_n^*[\cdot, W_n] + [W_n^*, \cdot]W_n\}}_{=:L_n(\cdot)} \end{aligned} \quad (7)$$

H_M^Λ is the Hamiltonian of the system ($|\Lambda|\varepsilon S_\Lambda^3 = \varepsilon \sum_{n \in \Lambda} s_n^3$ in our case) and describes the free evolution of the system, $V_\Lambda : \mathbb{R}^3 \rightarrow \mathbb{C}$ is a complex polynom (the index Λ indicates that one take the S_Λ^k 's as arguments, corresponding to the set Λ) which models the influence of a collective reservoir and $W_n \in \mathcal{A}_n = \mathbb{M}_2(\mathbb{C})$ which comes from the influence of an individual reservoir of every single particle. As we will see later these two different types of reservoirs have different influences on the collective structure of the macroscopic system.

It can be shown that the family of completely positive semigroups $\{T_t^\Lambda\}_{\Lambda \in \mathcal{L}}$ converges in the strong operator topology in the representation Π^p when $\Lambda \rightarrow \mathbb{N}$, and that the unique limit semigroup $T_t : \mathcal{C}_G \rightarrow \mathcal{C}_G$ bears the following properties:

Theorem 3.1:

1. T_t is completely positive, identity preserving and $T_0 = \text{id}$.
2. $T_t(C(K_{1/2})) \subseteq C(K_{1/2})$ ($C(K_{1/2}) = C(E_G) \cong \text{C}^*-\text{hull}\{S_\infty^1, S_\infty^2, S_\infty^3\}$), i.e. the classical observables are invariant under the dynamics.
3. T_t is strongly continuous in t .
4. $T_t(a \otimes f) = T_t(a \otimes 1) \cdot T_t(1 \otimes f) \quad \forall a \in \mathcal{A}, f \in C(K_{1/2})$.
5. $T_t|_{C(K_{1/2})}$ is given by a flow φ_t on $K_{1/2}$, i.e. $(T_t|_{C(K_{1/2})})(f)(\vec{x}) = f(\varphi_t \vec{x})$, which can be calculated explicitly using equation (7).

Proof: [18, 20]

Before we give details of the radiation field we want to specify the states $\omega \in \mathcal{F}^p$ in a more physical way. For this reason we define the following two parameters which partially determine the state ω . First the *degree of cooperation* μ_{co}^ω by setting:

$$\omega((S_\infty^1)^2 + (S_\infty^2)^2 + (S_\infty^3)^2) =: \left(\frac{\mu_{\text{co}}^\omega}{2}\right)^2 \quad (8)$$

and second the *degree of excitation* $\gamma_{\text{ex}}^\omega$ via:

$$\omega(S_\infty^3) =: \gamma_{\text{ex}}^\omega - \frac{1}{2} \quad (9)$$

(the constant factors in the eqns. (8) and (9) are conventionally chosen to be $\frac{1}{2}$). It is easy to show that $\mu_{\text{co}}^\omega, \gamma_{\text{ex}}^\omega \in [0, 1]$ and $|\gamma_{\text{ex}}^\omega - \frac{1}{2}| \leq \frac{\mu_{\text{co}}^\omega}{2}$. As we have learned above $\partial_e \mathcal{S}^p(\mathcal{A})$ is isomorphic to $K_{1/2} \subset \mathbb{R}^3$. For $\partial_e \mathcal{S}^p(\mathcal{A}) \ni \omega \cong \vec{x} \in K_{1/2}$ we have:

$$\left(\frac{\mu_{\text{co}}^\omega}{2}\right)^2 = x_1^2 + x_2^2 + x_3^2 \quad \text{and} \quad \gamma_{\text{ex}}^\omega - \frac{1}{2} = x_3$$

To illustrate these definitions we regard the finite system ($|\Lambda| < \infty$) as a spin system, i.e. generated by $|\Lambda|$ spin- $\frac{1}{2}$ -particles. We then have $(\gamma_{\text{ex}} - \frac{1}{2})|\Lambda|$ as spin in the x_3 -direction (system in the state ω) and $\frac{\mu_{\text{co}}\omega}{2}(\frac{\mu_{\text{co}}\omega}{2} + 1)$ as the square of the total spin (again system in the state ω). In this picture the terminology is obvious.

3.2 The Photonic Weyl Algebra

We now discuss the mathematics describing the radiation field. We start with defining the Weyl algebra $\mathcal{W}(E)$ over a symplectic vector space (E, σ) via:

$$\left. \begin{array}{l} 1.) \quad W(f)W(g) = \exp\{-\frac{i}{2}\sigma(f, g)\}W(f + g) \\ 2.) \quad W(f)^* = W(-f) \end{array} \right\} \quad \forall f, g \in E \quad (10)$$

(σ is the non-degenerate symplectic form on E) This algebra is unique up to $*$ -isomorphisms [22] (theorem 5.2.8). Since the model cannot be treated in the abstract algebra $\mathcal{W}(E)$ (unbounded creation- and destruction operators are needed in the interaction) we have to introduce a representation.

A representation Π of $\mathcal{W}(E)$ is called *regular* iff 1.) $\Pi(W(tf))$ is strongly continuous in t for all $f \in E$ and 2.) $\Pi(W(0)) = 1$. Hence, using Stones theorem, we can write:

$$\Pi(W(tf)) = \exp\{it\Phi(f)\} \quad (11)$$

Obviously $\Phi(f)$ depends on the choice of the representation Π but to simplify the notation we omit another index. The $\Phi(f)$'s are called *field operators* and obey the canonical commutation relations (ccr) on a dense subset of E . Using linear combinations of the $\Phi(f)$'s one can define $a(f)$ resp. $a^*(f)$, the destruction- resp. creation-operator, on a dense subset of E in the usual way [22] (section 5.2).

For the rest of this work we will use $E = L^2(\mathbb{R}^3)$ and the Fock representation $\Pi_{\mathcal{F}}$. This is the GNS-representation induced by the state $\omega_{\mathcal{F}}$:

$$\omega_{\mathcal{F}}(W(f)) := \exp\{-\frac{1}{4}\|f\|^2\} \quad \forall f \in E \quad (12)$$

There we have $\sigma(f, g) = \text{Im}\langle f|g \rangle_{L^2}$, and it is obvious that this is a regular representation.

The free time evolution on $\mathcal{W}(E)$ is given by the quasifree automorphism

$$\gamma_t(W(f)) := W(e^{itH_{\text{rad}}}f) \quad \forall f \in \mathcal{D}(H_{\text{rad}}) \quad (13)$$

with H_{rad} the “one photon” Hamiltonian on E . Because H_{rad} is usually unbounded (in our case $H_{\text{rad}} = \sqrt{-\Delta}$) we need to define the dynamics on the domain $\mathcal{D}(H_{\text{rad}})$ of H_{rad} .

Since $\omega_{\mathcal{F}}$ is invariant under the dynamics γ_t one can implement γ_t unitarily in the representation $\Pi_{\mathcal{F}}$, i.e.:

$$\Pi_{\mathcal{F}}(\gamma_t(W(f))) = e^{itd\Gamma(H_{\text{rad}})}\Pi_{\mathcal{F}}(W(f))e^{-itd\Gamma(H_{\text{rad}})} \quad \forall f \in \mathcal{D}(H_{\text{rad}}) \quad (14)$$

where $d\Gamma(H_{\text{rad}})$ denotes the second quantization of H_{rad} .

One of the key features of CSE is the coherent emission of light. So we now have to define what we mean by coherence.

Definition 3.2 *An analytical state ω on $\mathcal{W}(E)$ is called coherent of order $n \in \mathbb{N}$ iff there exists a real linearform $L : E \rightarrow \mathbb{C}$, such that*

$$\omega(a_\omega^*(f_1) \cdots a_\omega^*(f_m) a_\omega(g_1) \cdots a_\omega(g_m)) = L(f_1) \cdots L(f_m) \overline{L(g_1)} \cdots \overline{L(g_m)} \quad (15)$$

for all $f_i, g_i \in E$ and $\forall m \leq n$. ω is called fully coherent iff ω is coherent of order n , $\forall n \in \mathbb{N}$.

a_ω^* resp. a_ω is the creation- resp. destruction-operator in the GNS-representation induced by ω (compare [14, 24]).

In this description “coherent of 1.order” is equivalent to visibility 1 in an interference experiment (e.g. double slit); this is what we have to check here.

4 The Model for CSE

We base our model on the discussions [6] and [8, 9, 10] of the infinite Dicke model. They coupled the matter ($|\Lambda|$ two-level-atoms) and the radiation field with the interaction Hamiltonian H_I^Λ (in rotating-wave-approximation):

$$H_I^\Lambda := \lambda \left(S_\Lambda^- \otimes a^*(\phi) + S_\Lambda^+ \otimes a(\phi) \right) \quad (16)$$

identifying S_Λ^\pm with $\Pi^p(S_\Lambda^\pm)$. $\lambda \in \mathbb{R}$ is a coupling constant and $\phi \in E$ the coupling function. Alltogether they use the local Hamiltonian H^Λ :

$$H^\Lambda := H_M^\Lambda \otimes \mathbb{1} + \mathbb{1} \otimes d\Gamma(H_{\text{rad}}) + \lambda H_I^\Lambda \quad (17)$$

where the first two parts describe the two free evolutions (see above, remember the identification) of the matter resp. the field.

It can be shown that the limit $\Lambda \rightarrow \mathbb{N}$ exists [6, 9, 10] and one gets the following result for the limit dynamics τ_t on $\mathcal{M} := [(\Pi^p \otimes \Pi_{\mathcal{F}})(\mathcal{A} \otimes \mathcal{W}(E))]'' = \mathcal{M}^p \overline{\otimes} \mathcal{B}(\mathcal{F}(L^2(\mathbb{R}^3)))$ (α_t denotes the limit dynamics on the matter side without damping!):

$$\tau_t(Z) = Q(\psi_t)[\alpha_t \otimes \gamma_t(Z)]Q^*(\psi_t) \quad \forall t \in \mathbb{R}, Z \in \mathcal{M} \quad (18)$$

and $\tau_s \circ \tau_t = \tau_{s+t} \quad \forall s, t \in \mathbb{R}$, with:

$$\psi_t(\vec{x}) = \int_{r=0}^t e^{irH_{\text{rad}}} \tilde{\phi}(\varphi_r \vec{x}) dr \quad \forall \vec{x} \in K_{\frac{1}{2}} \quad (19)$$

$\tilde{\phi}(\vec{x}) := \sqrt{2}\lambda(x_1 - i x_2)\phi$, φ_t is the flow on the macroscopic parameters, H_{rad} the Hamiltonian of the field (see above) and with $\eta \in C(K_{\frac{1}{2}}, L^2(\mathbb{R}^3))$:

$$Q(\eta) := \int_{K_{\frac{1}{2}}} [\mathbb{1}_{\mathcal{A}} \otimes W_{\mathcal{F}}(\eta(\vec{x}))] d[\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes \mathbb{1}] \in \mathcal{Z}^p \otimes \mathcal{B}(\mathcal{F}(L^2(\mathbb{R}^3))) \quad (20)$$

(compare [25] (chapter 6) for the details concerning the generalized spectral integral).

We now define the emitted intensity when the material subsystem is in the state ω_m at $t = 0$ by setting:

$$I(t) := \frac{d}{dt} \left[(\omega_m \otimes \omega_{\mathcal{F}})(\tau_t(\mathbb{1}_{\mathcal{C}_G} \otimes \mathcal{N})) \right] \quad (21)$$

with \mathcal{N} the unbounded number operator in the Fock representation. Since $\mathcal{N} \notin \Pi_{\mathcal{F}}(\mathcal{W}(E))'' = \mathcal{B}(\mathcal{F}(L^2(\mathbb{R}^3)))$ we have to take care in calculating expectation values. But \mathcal{N} is affiliated with the algebra $\mathcal{B}(\mathcal{F}(L^2(\mathbb{R}^3)))$ and the expectation values are finite in our cases. If the radiation field consists of an infinite number of modes we get a constant intensity [6] (section 4).

Since we are looking for a pulse-like emission we have to modify this model by introducing a semigroup dynamics on the matter side. We assume no representation for $\mathcal{W}(E)$ at the moment, but E is equipped with an inner product $\langle \cdot | \cdot \rangle$ (i.e. the symplectic form σ is given via $\sigma(\cdot, \cdot) = \text{Im}(\langle \cdot | \cdot \rangle)$). Using the notations introduced above we have the following result:

Theorem 4.1 *Let $\psi : \mathbb{R} \times K_{1/2} \rightarrow E$; $(t, \vec{x}) \mapsto \psi_t(\vec{x})$ be a function, such that $\vec{x} \mapsto \text{Im}(\langle \psi_t(\vec{x}) | \eta \rangle)$ is continuous for all $t \in \mathbb{R}_+$ and every $\eta \in E$, then:*

$$(\tilde{\tau}_t Z)(\vec{x}) = [\mathbb{1}_{\mathcal{A}} \otimes W(\psi_t(\vec{x}))][T_t \otimes \gamma_t](Z)(\vec{x})[\mathbb{1}_{\mathcal{A}} \otimes W(\psi_t(\vec{x}))]^*$$

defines for every $t \in \mathbb{R}_+$ a completely positive, identity preserving map $\tilde{\tau}_t$ on $\mathcal{B} := \mathcal{A} \otimes C(K_{1/2}) \otimes \mathcal{W}(E) \cong C(K_{1/2} \mathcal{A} \otimes \mathcal{W}(E))$ with $\tilde{\tau}_0 = \text{id}$. Further, if $\psi_t(\vec{x})$ fullfills the cocycle equation, i.e.:

$$\psi_{s+t}(\vec{x}) = \psi_s(\vec{x}) + e^{isH_{\text{rad}}} \psi_t(\varphi_s \vec{x}) \quad \forall s, t \in \mathbb{R}_+, \quad (22)$$

it holds that: $\tilde{\tau}_s \circ \tilde{\tau}_t = \tilde{\tau}_{s+t} \quad \forall s, t \in \mathbb{R}_+$

$\psi_t(\vec{x})$ is given in equation (19), T_t and γ_t in Section 3.

Proof: Let $Z \in C(K_{1/2}, \mathcal{A} \otimes \mathcal{W}(E))$ and $t \in \mathbb{R}_+$. We define the map:

$$\chi_t(Z) : E_{\mathcal{G}} \rightarrow \mathcal{A} \otimes \mathcal{W}(E); \vec{x} \mapsto [\mathbb{1}_{\mathcal{A}} \otimes W(\psi_t(\vec{x}))]Z(\vec{x})[\mathbb{1}_{\mathcal{A}} \otimes W(\psi_t(\vec{x}))]^*$$

Since the Weyl operators are unitary we have:

$$\|\chi_t(Z)\|_{\infty} = \sup \{ \|\chi_t(Z)(\vec{x})\| \mid \vec{x} \in K_{1/2} \} = \|Z\|_{\infty}$$

First we set $Z = X \otimes W(\xi)$, $X \in C(K_{1/2}, \mathcal{A})$ and $\xi \in E$; with

$$W(\eta)W(\xi)W(-\eta) = \exp\{-i \text{Im}\langle \xi | \eta \rangle\}W(\xi)$$

we get $\chi_t(Z)(\vec{x}) = \exp\{-i \text{Im}\langle \psi_t(\vec{x}) | \xi \rangle\}X(\vec{x}) \otimes W(\xi)$ which is continuous because of the assumptions. For an arbitrary $Z \in C(K_{1/2}, \mathcal{A} \otimes \mathcal{W}(E))$ we take linear combinations Z_n of elements of the form $X \otimes W(\xi)$, such that $\lim_{n \rightarrow \infty} \|Z - Z_n\|_{\infty} = 0$. Thus $\|\chi_t(Z) - \chi_t(Z_n)\|_{\infty} = \|\chi_t(Z - Z_n)\|_{\infty} = \|Z - Z_n\|_{\infty} \rightarrow 0$ when $n \rightarrow \infty$. Hence, by choosing n such

that $\|Z_n - Z\|_\infty < \frac{\varepsilon}{3}$ and, since $\chi_t Z_n$ is continuous, pick $\delta > 0$ so that $\|\vec{x} - \vec{y}\|_{\mathbb{R}^3} < \delta$ implies $\|(\chi_t Z_n)(\vec{x}) - (\chi_t Z_n)(\vec{y})\|_\infty < \frac{\varepsilon}{3}$ we have:

$$\begin{aligned} \|(\chi_t Z)(\vec{x}) - (\chi_t Z)(\vec{y})\|_\infty &\leq \|(\chi_t Z)(\vec{x}) - (\chi_t Z_n)(\vec{x})\|_\infty + \\ &\quad + \|(\chi_t Z_n)(\vec{x}) - (\chi_t Z_n)(\vec{y})\|_\infty + \\ &\quad + \|(\chi_t Z_n)(\vec{y}) - (\chi_t Z)(\vec{y})\|_\infty \\ &\leq 2 \cdot \underbrace{\|Z - Z_n\|_\infty}_{< \frac{\varepsilon}{3}} + \underbrace{\|(\chi_t Z_n)(\vec{x}) - (\chi_t Z_n)(\vec{y})\|_\infty}_{< \frac{\varepsilon}{3}} \leq \varepsilon \end{aligned}$$

which implies that $\chi_t Z$ is continuous in \vec{x} . Since $T_t|_{C(K_{1/2})}$ is given by a flow on $K_{1/2}$, and $(\tilde{\tau}_t Z)(\vec{x}) = (\chi_t \circ (T_t \otimes \gamma_t))(\vec{x})$, it follows that $(\tilde{\tau}_t Z) \in C(K_{1/2}, \mathcal{A} \otimes \mathcal{W}(E))$.

With $W(\psi_0(\vec{x})) = W(0) = \mathbb{1}$ it is obvious that $\tilde{\tau}_0 = \text{id}$, and it is easy to show that $\tilde{\tau}_t(\mathbb{1}) = \mathbb{1}$. Since T_t and γ_t are completely positive, $T_t \otimes \gamma_t$ is completely positive after [26] (proposition IV.4.23) (the restriction on the minimal tensor product can be dropped here since both algebras are nuclear), and we have that $\tilde{\tau}_t$ is completely positive for every $t \in \mathbb{R}_+$, because $\tilde{\tau}_t = \chi_t \circ (T_t \otimes \gamma_t)$ and all maps of the type $a \mapsto bab^*$ are completely positive.

Now, by using theorem 3.1 (4.), we get for $A \otimes f \otimes Y \in \mathcal{A} \otimes C(E_\mathcal{G}) \otimes \mathcal{W}(E)$:

$$\begin{aligned} (T_s \otimes \gamma_s)(A \otimes f \otimes Y) &= (T_s \otimes \gamma_s)([A \otimes \mathbb{1}_{C(K_{1/2})} \otimes \mathbb{1}_{\mathcal{W}(E)}] \cdot [\mathbb{1}_\mathcal{A} \otimes f \otimes Y]) \\ &= [T_s([A \otimes \mathbb{1}_{C(K_{1/2})}] \cdot [\mathbb{1}_\mathcal{A} \otimes f])] \otimes [\gamma_s(\mathbb{1}_{\mathcal{W}(E)} \cdot Y)] \\ &\stackrel{3.1(4.)}{=} [T_s(A \otimes \mathbb{1}_{C(K_{1/2})})T_s(\mathbb{1}_\mathcal{A} \otimes f)] \otimes [\underbrace{\gamma_s(\mathbb{1}_{\mathcal{W}(E)})}_{=\mathbb{1}_{\mathcal{W}(E)}} \gamma_s(Y)] \\ &= [(T_s \otimes \gamma_s)(A \otimes \underbrace{\mathbb{1}_{C(K_{1/2})} \otimes \mathbb{1}_{\mathcal{W}(E)}}_{\cong \mathbb{1}_{C(K_{1/2}, \mathcal{W}(E))}})] \cdot [(T_s \otimes \gamma_s)(\mathbb{1}_\mathcal{A} \otimes f \otimes Y)] \end{aligned}$$

and hence with $C(K_{1/2}) \otimes \mathcal{W}(E) \ni f \otimes Y \cong Y(\vec{x}) \in C(K_{1/2}, \mathcal{W}(E))$:

$$\begin{aligned} (T_s \otimes \gamma_s)(\chi_t(A \otimes Y(\vec{x}))) &= (T_s \otimes \gamma_s)(\chi_t(A \otimes \mathbb{1}_{C(K_{1/2}, \mathcal{W}(E))}) \cdot \chi_t(\mathbb{1}_\mathcal{A} \otimes Y(\vec{x}))) \\ &= (T_s \otimes \gamma_s)([A \otimes \mathbb{1}_{C(K_{1/2}, \mathcal{W}(E))}] \times \\ &\quad \times [\mathbb{1}_\mathcal{A} \otimes W(\psi_t(\vec{x}))Y(\vec{x})W(-\psi_t(\vec{x}))]) \\ &= (T_s \otimes \gamma_s)(\chi_t(A \otimes \mathbb{1}_{C(K_{1/2}, \mathcal{W}(E))})) \times \\ &\quad \times (T_s \otimes \gamma_s)(\chi_t(\mathbb{1}_\mathcal{A} \otimes Y(\vec{x}))) \end{aligned}$$

and finally with equation (22):

$$(\tilde{\tau}_s \circ \tilde{\tau}_t)(Z)(\vec{x}) = [\chi_s \circ (T_s \otimes \gamma_s) \circ \chi_t \circ (T_t \otimes \gamma_t)](Z)(\vec{x})$$

$$\begin{aligned}
&= [\mathbb{1}_{\mathcal{A}} \otimes W(\psi_s(\vec{x}) + e^{isH_{\text{rad}}}\psi_t(\varphi_s\vec{x}))](T_{s+t} \otimes \gamma_{s+t})(Z)(\vec{x}) \times \\
&\quad \times [\mathbb{1}_{\mathcal{A}} \otimes W(-\psi_s(\vec{x}) - e^{isH_{\text{rad}}}\psi_t(\varphi_s\vec{x}))] \\
&\stackrel{\text{eq. (22)}}{=} \tilde{\tau}_{s+t}(Z)(\vec{x}) \quad \forall Z \in C(K_{1/2}, \mathcal{A} \otimes \mathcal{W}(E)), \forall s, t \in \mathbb{R}_+
\end{aligned}$$

□

Up to now we have described the dynamics in the algebra $\mathcal{C}_G \otimes \mathcal{W}(E)$. We now extend $\tilde{\tau}_t$ to the algebra $\mathcal{M} := \mathcal{M}^p \overline{\otimes} \mathcal{M}^F = [(\text{id}_{\mathcal{C}_G} \otimes \Pi_F)(\mathcal{C}_G \otimes \mathcal{W}(E))]''$.

Proposition 4.2 *For every $t \in \mathbb{R}_+$ we can extend $\tilde{\tau}_t$ (which is defined on $\mathcal{C}_G \otimes \mathcal{W}(E)$) in the representation $\text{id}_{\mathcal{C}_G} \otimes \Pi_F$ to a completely positive, identity preserving map τ_t on the W^* -algebra $\mathcal{M} := \mathcal{M}^p \overline{\otimes} \mathcal{M}^F = [(\text{id}_{\mathcal{C}_G} \otimes \Pi_F)(\mathcal{C}_G \otimes \mathcal{W}(E))]''$, and τ_t is given by:*

$$\tau_t(Z) = Q(\psi_t)(T_t \otimes \gamma_t)(Z)Q^*(\psi_t) \quad \forall Z \in \mathcal{M} \quad (23)$$

($Q(\psi_t)$ is defined in equation (20)). We again get $\tau_0 = \text{id}$ and $\tau_{s+t} = \tau_s \circ \tau_t \quad \forall s, t \in \mathbb{R}_+$ if $\psi_t(\vec{x})$ satisfies equation (22).

Proof: It is obvious that τ_t is a well defined map on \mathcal{M} . We now show the correspondence between $\tau_t \circ (\text{id}_{\mathcal{C}_G} \otimes \Pi_F)$ and $(\text{id}_{\mathcal{C}_G} \otimes \Pi_F) \circ \tilde{\tau}_t$. With $X \otimes w_\xi \in \mathcal{C}_G \otimes \mathcal{W}(E)$, $\Pi_F(w_\xi) = W_F(\xi)$ (by definition) and $v_t \xi := e^{itH_{\text{rad}}} \xi$ we get:

$$\tau_t(X \otimes W_F(\xi)) = \left[\int_{K_{1/2}} \exp\{-i \text{Im}\langle \psi_t(\vec{x}) | v_t \xi \rangle\} d[\mathcal{E}_G(\vec{x}) \otimes 1] \right] [(T_t \otimes \gamma_t)(X \otimes W_F(\xi))]$$

On the other hand, using theorem 4.1 and $w_\xi w_\eta = \exp\{-\frac{i}{2} \text{Im}\langle \xi | \eta \rangle\} w_{\xi+\eta}$ (eq. (10)), we get:

$$\begin{aligned}
\tilde{\tau}_t(X \otimes w_\xi)(\vec{x}) &= [\mathbb{1}_{\mathcal{A}} \otimes w_{\psi_t(\vec{x})}][T_t(X) \otimes w_{v_t \xi}][\mathbb{1}_{\mathcal{A}} \otimes w_{-\psi_t(\vec{x})}] \\
&= \exp\{-i \text{Im}\langle \psi_t(\vec{x}) | v_t \xi \rangle\} (T_t \otimes \gamma_t)(X \otimes w_\xi)(\vec{x})
\end{aligned}$$

Using the isomorphism between $C(K_{1/2})$ and C^* -hull $\{S_\infty \mid S \in \mathcal{G}\}$, given through the map $f \mapsto \int_{K_{1/2}} f d\mathcal{E}_G(\vec{x})$ (see equation(4)), we finally have:

$$(\text{id}_{\mathcal{C}_G} \otimes \Pi_F) \circ \tilde{\tau}_t(X \otimes w_\xi) = \tau_t(X \otimes W_F(\xi)) \quad (24)$$

The other results follow with equation (24) and theorem 4.1. □

From now on we will drop the index F because for the remaining part of the paper we will work in the Fock representation solely. Finally, we show that the above introduced dynamics is the right one, i.e. that it is the limiting dynamics of our system.

Proposition 4.3 *The dynamics τ_t defined in eq. (23) is the limiting dynamics of the damped two level system (eq. (7)) interacting via the Hamiltonian H_1^Λ (eq. (16)) with the radiation field, and its generator is of the form $L_{\text{mat}}(\cdot) + L_{\text{rad}}(\cdot) + i[H_{\text{int}}, \cdot]$.*

Proof: We prove, that the dynamics τ_t is generated by the limiting elements of the local generators. $\tau_0 = \text{id}$ and the strong continuity are obvious. Further we have ($Z = B \otimes W(g) \in \mathcal{M}$):

$$\begin{aligned} \frac{d}{dt} \Big|_{t=0} \tau_t(Z) &= \frac{d}{dt} \Big|_{t=0} Q(\psi_t) \cdot Z + Z \cdot \frac{d}{dt} \Big|_{t=0} Q^*(\psi_t) + \\ &+ \underbrace{\frac{d}{dt} \Big|_{t=0} T_t(B) \otimes \mathbb{1}}_{L_{\text{mat}}} + \underbrace{\mathbb{1} \otimes (i[d\Gamma(H_{\text{rad}}), W(g)])}_{L_{\text{rad}}} \end{aligned}$$

Using (a) [27] theorem A.5.1, (b) $\Phi(\phi) = 2^{-1/2}(a^*(\phi) + a(\phi))$ and (c) $\tilde{\phi}(\vec{x}) = \lambda\sqrt{2}(x_1 - ix_2)\phi$ we get:

$$\begin{aligned} \frac{d}{dt} \Big|_{t=0} Q(\psi_t) &= \frac{d}{dt} \Big|_{t=0} \int_{K_{\frac{1}{2}}} [\mathbb{1}_{\mathcal{A}} \otimes W(\psi_t)] d[\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes \mathbb{1}] \\ &\stackrel{(a)}{=} \int_{K_{\frac{1}{2}}} [\mathbb{1}_{\mathcal{A}} \otimes i\Phi(\tilde{\phi}(\vec{x}))] d[\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes \mathbb{1}] \\ &\stackrel{(b)}{=} i \int_{K_{\frac{1}{2}}} [\mathbb{1}_{\mathcal{A}} \otimes [2^{-1/2}a^*(\tilde{\phi}(\vec{x})) + 2^{-1/2}a(\tilde{\phi}(\vec{x}))]] d[\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes \mathbb{1}] \\ &\stackrel{(c)}{=} i\lambda \left[\underbrace{\int_{K_{\frac{1}{2}}} (x_1 - ix_2) d\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes a^*(\phi)}_{\lim_{\Lambda \rightarrow \infty} \Pi^p(S_{\Lambda}^-) = S_{\infty}^-} + \underbrace{\int_{K_{\frac{1}{2}}} (x_1 + ix_2) d\mathcal{E}_{\mathcal{G}}(\vec{x}) \otimes a(\phi)}_{\lim_{\Lambda \rightarrow \infty} \Pi^p(S_{\Lambda}^+) = S_{\infty}^+} \right] \end{aligned}$$

Finally, with $Q^*(\psi_t) = Q(-\psi_t) \Rightarrow \dot{Q}^* = -\dot{Q}$ and $H_{\text{int}} = \lambda(S_{\infty}^- \otimes a^*(\phi) + S_{\infty}^+ \otimes a(\phi))$, we have for the generator of τ_t : $L(\cdot) = L_{\text{mat}}(\cdot) + L_{\text{rad}}(\cdot) + i[H_{\text{int}}, \cdot]$. \square

5 Results

5.1 The Emitted Intensity

After having established the dynamics on \mathcal{M} we now calculate the emitted intensity for two different types of reservoirs. Looking at equation (7) we see, that one can distinguish between the influence of a collective reservoir, represented through V_{Λ} , and the influence of individual reservoirs, represented through the W_n 's. The individual reservoirs act independently from each other on the single atom ($W_n \in \mathcal{A}_n$). It is obvious that such reservoirs, since they act independently, have a disturbing influence on the collective structure. As we will see later one can avoid such a behaviour with the assumption of a collective reservoir.

5.1.1 Assuming the Action of Individual Reservoirs

We start with the intensity if we assume the influence of a set of reservoirs acting at each atom separately. This is done by setting $V_\Lambda = 0 \quad \forall \Lambda \in \mathcal{L}$ and choosing the W_n 's in equation (7). In our first explicit model we choose $W_n = W := cS^- = c(S^1 + iS^2)$, $c \in \mathbb{C}$ and interpret the action of W as transitions without radiation. We get:

$$L_n(S^k) = \sum_{i=1}^3 \alpha_{ki} S^i + a_k \mathbb{1}$$

with

$$A := (\alpha_{ki}) = \begin{pmatrix} -|c|^2 & 0 & 0 \\ 0 & -|c|^2 & 0 \\ 0 & 0 & -2|c|^2 \end{pmatrix} \quad \text{and} \quad \vec{a} = \begin{pmatrix} 0 \\ 0 \\ -|c|^2 \end{pmatrix}$$

We then can construct the flow φ_t , given in theorem 3.1 (5.), as follows. The vectorfield $\vec{\lambda}$, generating φ_t , is given through ($k \in \{1, 2, 3\}$) (see [20]):

$$\lambda_k(\vec{x}) := \underbrace{\sum_{j=1}^3 \frac{\partial Q}{\partial x_j} \sum_{l=1}^3 \varepsilon_{kjl} x_l}_{\lambda^I} + \underbrace{2 \sum_{j=1}^3 \left(V_1 \frac{\partial V_2}{\partial x_j} - V_2 \frac{\partial V_1}{\partial x_j} \right) \sum_{l=1}^3 \varepsilon_{kjl} x_l}_{\lambda^{II}} + \underbrace{(A\vec{x} + \vec{a})_k}_{\lambda^{III}} \quad (25)$$

where Q is the polynom corresponding to H_M^Λ , i.e. $H_M^\Lambda = |\Lambda|Q(S_\Lambda^1, S_\Lambda^2, S_\Lambda^3)$ (here $Q(\vec{x}) = \varepsilon x_3$, since $H_M^\Lambda = |\Lambda|\varepsilon S_\Lambda^3$), $V_1 := \text{Re } V_\Lambda^*$ and $V_2 := \text{Im } V_\Lambda^*$. λ^{II} is zero because $V_\Lambda = 0$. Integrating equation (25) we obtain the flow φ_t on $K_{1/2}$:

$$\varphi_t(\vec{x}) = \begin{pmatrix} e^{-|c|^2 t} (x_1 \cos(\varepsilon t) - x_2 \sin(\varepsilon t)) \\ e^{-|c|^2 t} (x_1 \sin(\varepsilon t) + x_2 \cos(\varepsilon t)) \\ e^{-2|c|^2 t} x_3 - \frac{1}{2}(1 - e^{-2|c|^2 t}) \end{pmatrix} \quad t \geq 0 \quad (26)$$

With this preparatory work we now can calculate the intensity. With $\mu(\vec{x}) := \omega_m(\mathcal{E}_G(\vec{x}))$ and $k := \|\vec{k}\|_{\mathbb{R}^3}$ we get (see eq. (21)):

$$\begin{aligned} I(t) &= \frac{d}{dt} \left[(\omega_m \otimes \omega_F)(\tau_t(\mathbb{1}_{\mathcal{C}_G} \otimes \mathcal{N})) \right] \\ &= \frac{d}{dt} \left[(\omega_m \otimes \omega_F) \left(\int_{K_{\frac{1}{2}}} \mathbb{1} \otimes \left[\frac{1}{2} \|\psi_t(\vec{x})\|_{L^2(\mathbb{R}^3)}^2 \cdot \mathbb{1} \right] d[\mathcal{E}_G(\vec{x}) \otimes \mathbb{1}] \right) \right] \\ &= \int_{K_{\frac{1}{2}}} (x_1^2 + x_2^2) d\mu(\vec{x}) \times \frac{d}{dt} \left[\int_{\mathbb{R}^3} |\hat{\phi}(\vec{k})|^2 \frac{e^{-2|c|^2 t} - 2e^{-|c|^2 t} \cos((k - \varepsilon)t) + 1}{(k - \varepsilon)^2 + |c|^4} d^3 \vec{k} \right] \end{aligned} \quad (27)$$

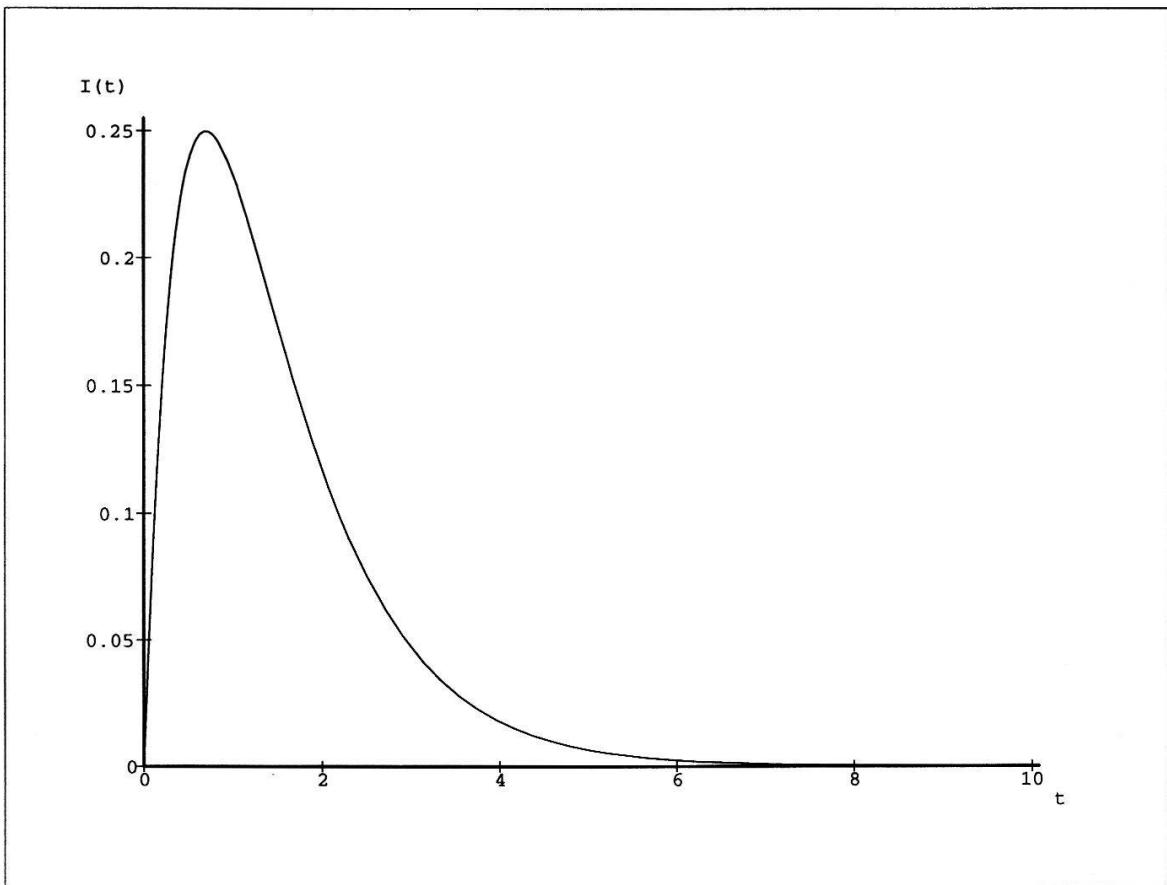


Figure 1: Emitted intensity according to equation (28).

where $\hat{\phi}(\vec{k})$ is the Fourier transform of the coupling function ϕ (equation (16)). $\phi(\vec{k})$ can be calculated explicitly if the two wave functions of the atomic levels are known; compare [9] (section 4).

For our purpose it is enough to illustrate the temporal behaviour of the intensity when we assume that the photons occupy a single mode with a fixed \vec{k} (this of course is only possible in a box, otherwise H_{rad} has a continuous spectrum). It is clear that the overall shape of the pulse would not change much if we calculate the intensity (eq. (27)) for fields with more than one mode, as long as $|\hat{\phi}(\vec{k})|^2$ is concentrated around $k = \varepsilon$ (which must be true if we chose the rotating-wave-approximation from the beginning (see above)). For a simple computation, we look only for the intensity which is emitted into the resonant mode. With this assumptions equation (27) simplifies to:

$$I(t) = \frac{4\lambda^2}{|c|^2} \left(e^{-|c|^2 t} - e^{-2|c|^2 t} \right) \int_{K_{\frac{1}{2}}} (x_1^2 + x_2^2) d\mu(\vec{x}) \quad (28)$$

Equation (28) is plotted in Figure 1 for $\lambda^2 = |c|^2 = 1$ and $\omega \in \partial_e \mathcal{S}^p(\mathcal{A})$ with $\mu_{\text{co}}^\omega = 1$ and $\gamma_{\text{ex}}^\omega = \frac{1}{2}$.

We remark two interesting features: first we cannot start with a state ω_m with $\mu_{\text{co}}^\omega = \gamma_{\text{ex}}^\omega = 1$ at $t = 0$ because this would imply $\int_{K_{1/2}} (x_1^2 + x_2^2) d\mu(\vec{x}) = 0$, i.e. no intensity at all (in [4] it is argued that one starts with a state with total population inversion, i.e. $\gamma_{\text{ex}}^\omega = 1$, and “...as the system radiates it passes ... to the superradiant region...”, where $\gamma_{\text{ex}} = 1/2$; compare [4], eqns. 29 & 30). Starting with $\gamma_{\text{ex}}^\omega = 1$ would be closer at the experimental situation, since the intermediate level is empty at $t = 0$ (compare Section 2).

Second we notice that the observable $(S^1)^2 + (S^2)^2 + (S^3)^2$ (which determines μ_{co}^ω) is not a conserved quantity, hence the degree of cooperation is time-dependent. This is the disturbing effect of the independently acting reservoirs we have mentioned above.

5.1.2 Assuming the Action of a Collective Reservoir

In the second model we want to describe a different physical situation. The first point is, that we want to start the process with a state ω_m with $\mu_{\text{co}}^\omega = \gamma_{\text{ex}}^\omega = 1$, i.e. we want to start with total population inversion, which seems to be closer to the experimental situation (see above). And second we demand the conservation of the degree of cooperation, a fact which might be of physical interest and which is given in the local models (finite number of atoms) mentioned at the begining and in the model without damping [6, 8, 9, 10].

For these two reasons we need a vector field $\vec{\lambda}$ in $K_{1/2}$ which has only components perpendicular to $\vec{x} \in K_{1/2}$. To obtain the desired result we choose a different way as above. This time we specify a flow $\varphi_t(\vec{x})$ such that it gives the two required details, and then show that it is generated by a collective reservoir. It is easy to see, that the flow (\vec{x} is given in polar coordinates (r, θ, ξ)):

$$\varphi_t(\vec{x}) = \begin{pmatrix} r_0 \sin(\pi - (\pi - \theta_0)e^{-ct}) \cos(\xi_0 + \varepsilon t) \\ r_0 \sin(\pi - (\pi - \theta_0)e^{-ct}) \sin(\xi_0 + \varepsilon t) \\ r_0 \cos(\pi - (\pi - \theta_0)e^{-ct}) \end{pmatrix} \quad t, c \in \mathbb{R}_+ \quad (29)$$

is a suitable choice. We again get $\varphi_s \circ \varphi_t = \varphi_{s+t} \quad \forall s, t \in \mathbb{R}_+$, thus $\psi_t(\vec{x})$ fullfills the cocycle equation (equation (22)).

If we set $\frac{d}{dt}[\varphi_t(\vec{x})]_{t=0} =: \vec{\lambda}(\vec{x})$ we see that $\lambda^{\text{III}} = 0$ (eq. (25)), i.e. there is no term which is linear in \vec{x} . It is therefore clear, that one cannot construct a flow, stated in equation (29), by assuming the action of a set of individual reservoirs (compare equation (7)). Although the choice of the flow is somehow arbitrary this argument is valid for all flows which describe motions on spheres in $K_{1/2}$.

Another point which should be mentioned here is, that such a flow cannot be generated using only polynomials for V_Λ (aswell independent from the exact choice of the flow; see above). For our model we need a larger class of generators, like the one treated in [28] (in [28] the class of generators is extended to C^2 functions. In our concrete model we need the square root on \mathbb{R}_+ , which of course is C^2). Using the notations introduced above we now get for the intensity:

$$I(t) := \frac{d}{dt} [(\omega_m \otimes \omega_F)(\tau_t(\mathbb{1}_{C_G} \otimes \mathcal{N}))]$$

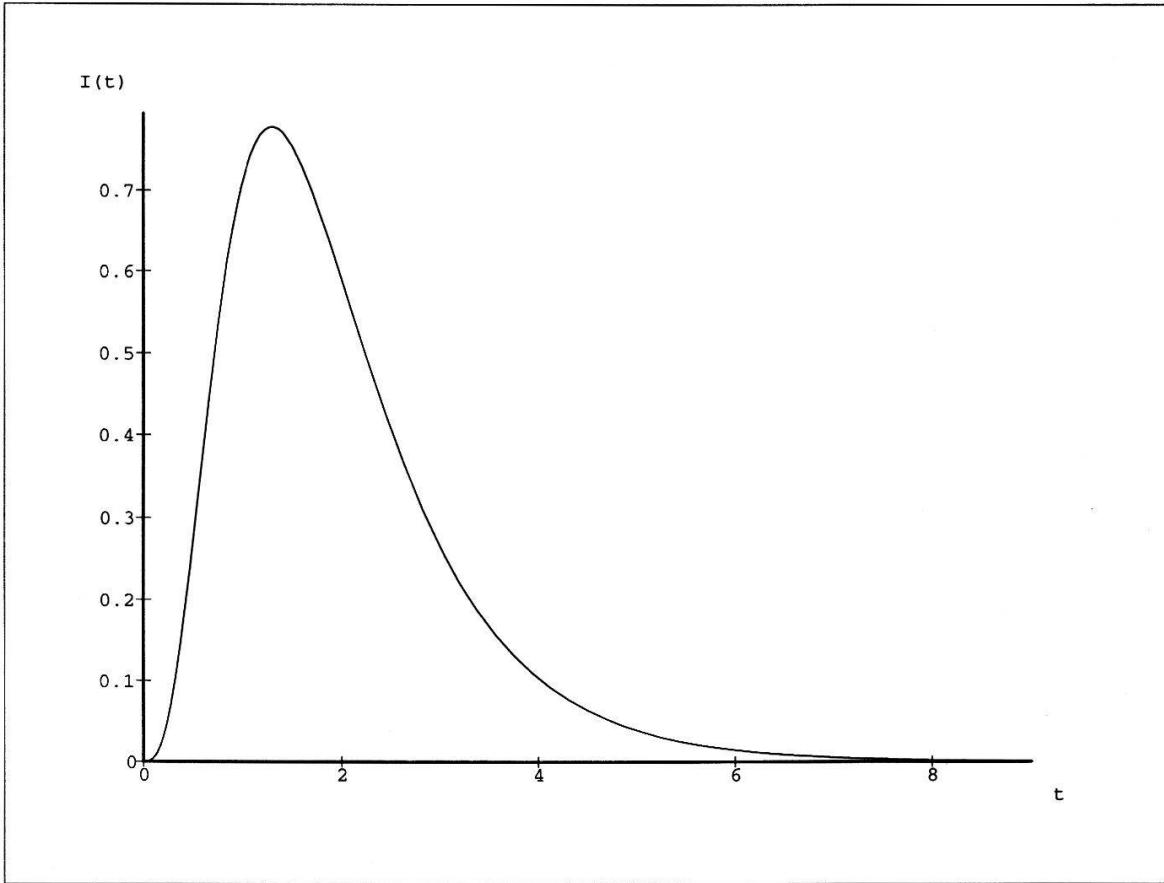


Figure 2: Emitted intensity according to equation (31).

$$\begin{aligned}
 &= \frac{d}{dt} \left[(\omega_m \otimes \omega_F) \left(\int_{K_{\frac{1}{2}}} \mathbb{1} \otimes [\lambda^2 \int_{\mathbb{R}^3} |\hat{\phi}(\vec{k})|^2 \int_{a=0}^t \int_{b=0}^t r_0^2 \sin(\theta_a) \times \right. \right. \\
 &\quad \times \sin(\theta_b) e^{i(b-a)(k-\varepsilon)} db da d^3 \vec{k} \cdot \mathbb{1} d[\mathcal{E}_G(\vec{x}) \otimes \mathbb{1}] \left. \right] \quad (30)
 \end{aligned}$$

where $\theta_t := (\pi - \theta_0)e^{-ct}$. If we again assume (just for illustration!) emission into the resonant mode only, we have $(r_0 \sin(\theta_t)) = \sqrt{x_1^2(t) + x_2^2(t)}$:

$$I(t) = 2\lambda^2 \sin(\theta_t) \int_{a=0}^t \sin(\theta_a) da \underbrace{\int_{K_{\frac{1}{2}}} \|\vec{x}\|^2 d\mu(\vec{x})}_{=(\mu_{co}^{\omega})^2/4} \quad (31)$$

Equation (31) is plotted in Figure 2 for $c = \lambda = \mu_{co}^{\omega} = \gamma_{ex}^{\omega}(0) = 1$.

The main difference between Figure 1 (individual reservoirs) and Figure 2 (collective reservoir) is, that in Figure 2 the curve starts with a horizontal tangent, whereas the overall shape of the pulses is nearly identical for both models.

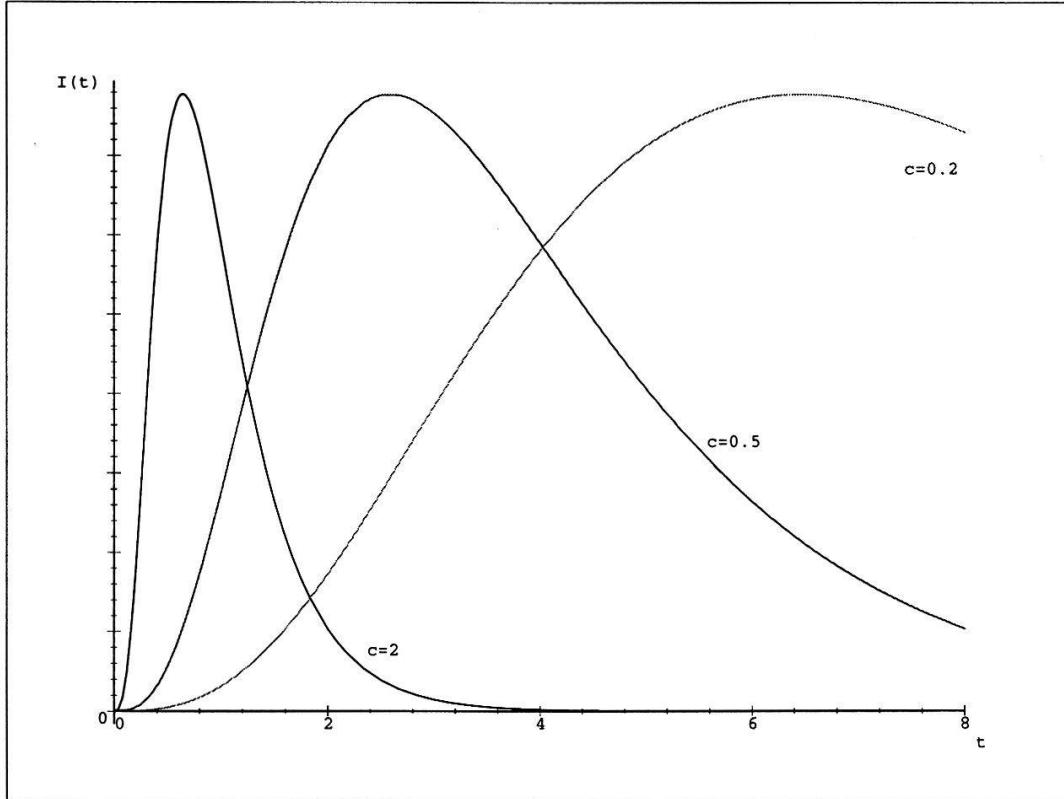


Figure 3: Normalized intensity according to eq. (31), with various c 's.

5.2 Coherence of the Emitted Radiation

Since we now have pulse-like emission, we are interested in the second question, whether the emitted radiation is coherent or not. If we denote the restricted dynamics on the states of $\mathcal{W}(E)$ with ν_t , i.e. $\nu_t(\omega)(Y) := (\omega_m \otimes \omega)(\tau_t(\mathbb{1} \otimes Y))$, $\forall Y \in \mathcal{W}(E)$, we have the following result:

Proposition 5.1 *In both cases, individual or collective reservoirs, $\nu_t(\omega_{\mathcal{F}})$ is a first order coherent state $\forall t \in \mathbb{R}_+$, i.e. $\nu_t(\omega_{\mathcal{F}})(a^*(f)a(g)) = L_t(f)\overline{L_t(g)}$ where (according to definition 3.2) the linear form (or smeared coherence function) L_t is given through:*

$$L_t : E \rightarrow \mathbb{C}; \quad f \mapsto L_t(f) := \lambda \sqrt{\omega_m((S_{\infty}^1)^2 + (S_{\infty}^2)^2)} \cdot \langle \tilde{\phi}_t | e^{itH_{\text{rad}}} f \rangle$$

with $\tilde{\phi}_t = \int_{a=0}^t \exp\{ia(H_{\text{rad}} - \varepsilon) - |c|^2 a\} \phi da$ for the individual reservoirs, and:

$$L_t : E \rightarrow \mathbb{C}; \quad f \mapsto L_t(f) := \frac{1}{2} \lambda \mu_{\text{co}}^{\omega_m} \left\langle \int_{a=0}^t \sin(\theta_a) e^{ia(H_{\text{rad}} - \varepsilon)} \phi da \middle| e^{itH_{\text{rad}}} f \right\rangle$$

when we assume the action of a collective reservoir.

Proof: Easy calculations. □

Remark *In the first case (individual reservoirs) it holds that*

$$\omega_m((S_\infty^1)^2 + (S_\infty^2)^2) = \left(\frac{\mu_{\text{co}}^\omega}{2}\right)^2 - (\gamma_{\text{ex}}^\omega - \frac{1}{2})^2$$

iff $\omega_m((S_\infty^3)^2) = \omega_m(S_\infty^2)^2$, i.e. when the material system is prepared in a state with sharp degree of excitation.

5.3 Interpretation and Conclusions

As a free parameter in both models we have c , the damping constant. If we look for example at equation (31) for various values of c , we get the behaviour shown in Figure 3. It is known from the experiment [1] that the pulses become narrower and appear earlier as the pressure in the gas increases. If we take c in Figure 3 to be the *stosszahl* (i.e. the mean number of collisions per second of a particle), we learn from kinetic gas theory that the *stosszahl* is proportional to the pressure (in the range of the ideal gas equation, a condition which is true in the experiment described above). Realistic values for c are in the range of $10^9\text{--}10^{10}\text{s}^{-1}$, which leads to time scales in the figures of 0.1–1ns. Hence, with this interpretation we have a good agreement with the experimental situation.

It is important to notice that our model is completely solved for the operators. Earlier models which follow the lines of the one treated in [2] (section 2; see there for references) pass over to dynamical equations for expectation values in an early stage of their discussion. To solve these equations they need assumptions that certain matter–radiation expectation values factorize (sometimes called *self consistent field approximation*), i.e. they assume that $I(t) \sim \langle R_n^+ R_n^- \rangle$ [2, eq. (2.4)] (in their notation $R_n^\pm = |\Lambda| S_\Lambda^\pm$, $n = |\Lambda|$, $\langle \cdot \rangle$ denoting the quantum mechanical expectation value and a is representing the radiation field). Using $I(t) = \frac{d}{dt} \langle a^* a \rangle$ and $[R_n^3 + a^* a, H^\Lambda] = 0$, H^Λ is similar to the Hamiltonian defined in eq. (17), the assumption is equivalent to $\frac{d}{dt} \langle R_n^3 \rangle \sim \langle R_n^+ R_n^- \rangle$. On the other hand, if we assume the action of individual reservoirs in our model we have:

$$\frac{d}{dt} S_\Lambda^3 = L_\Lambda(S_\Lambda^3) = 2|c|^2 (S^+ S^-)_\Lambda$$

which shows that the assumption above acts like a partial trace and thus introduces an irreversibility. This argument is substantiated by the fact that, if one solves the model (without damping) with a finite number of particles exactly [29], the eigenvalues of the Hamiltonian become equidistant, hence a periodic radiation pattern is predicted by the model.

A proper pulse–like solution is only possible when the unavoidable losses are incorporated via a damping (see above; compare [30], where irreversibility is obtained by letting the quantization volume of the field tend to infinity).

Another advantage of the approach we have used is that it is independent of the number of modes of the radiation field. Of course we have used a one–mode approximation for our explicit calculations here, but equation (27) resp. equation (30) give the emitted intensity for *any* number of modes!

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References

- [1] N. SKRIBANOWITZ, I.P. HERMAN, J.C. MACGILLIVRAY, M.S. FELD: Phys. Rev. Lett. **30** (1973) 309; and *Laser spectroscopy* Plenum, New York 1975, p.379.
- [2] A.V. ANDREEV, V.I. EMEL'YANOV, YU.A. IL'INSKII: Sov. Phys. Usp. **23** (1980) 493.
- [3] M. GROSS, S. HAROCHE: Phys. Reports **93** (1982) 301.
- [4] R.H. DICKE: Phys. Rev. **93** (1954) 99.
- [5] R. BONIFACIO, G. PREPARATA: Phys. Rev. **A2** (1970) 336.
- [6] E.B. DAVIES: Comm. Math. Phys. **33** (1973) 187.
- [7] E.B. DAVIES: Comm. Math. Phys. **34** (1973) 237.
- [8] R. HONEGGER, A. RIECKERS, TH. UNNERSTALL: Z. Naturforsch. **48a** (1993) 1043.
- [9] R. HONEGGER, A. RIECKERS: Publ. RIMS, Kyoto Univ. **30** (1994) 111.
- [10] R. HONEGGER: J. Math. Phys. **37** (1996) 263.
- [11] G. LINDBLAD: Comm. Math. Phys. **48** (1976) 119.
- [12] K. HEPP, E. H. LIEB: Helv. Phys. Acta **46** (1973) 573.
- [13] G. ALLI, G. L. SEWELL: J. Math. Phys. **36** (1995) 5598.
- [14] R.L. GLAUBER: Phys. Rev. **130** (1963) 2529.
- [15] A.T. ROSENBERGER, S.J. PETUCHOWSKI, T.A. DETEMPLE: in L. MANDEL, E. WOLF (EDS.): *Coherence and quantum optics IV (Rochester Conference 1977)*. Plenum, New York 1978.
- [16] A. MCPHERSON, K. BOYER, C. K. RHODES: J. Phys. B **27** (1994) L637.
- [17] N. S. GINZBURG, YU. V. NOVOZHILOVA, A. S. SERGEEV: Nucl. Inst. Methods Phys. Res. A **341** (1994) 230.
- [18] P. BÓNA: J. Math. Phys. **29** (1988) 2223.
- [19] TH. UNNERSTALL: *Makroskopische Quantenphänomene in Quantengittersystemen*. Ph.D. Thesis, Tübingen 1990.
- [20] TH. UNNERSTALL: Lett. Math. Phys. **20** (1990) 183.
- [21] A.O. BARUT, R. RĄCZKA: *Theory of group representations and applications*. World Scientific, Singapore 1986.

- [22] O. BRATTELI, D.W. ROBINSON: *Operator algebras and quantum statistical mechanics II*. Springer-Verlag, New York, Heidelberg, Berlin 1981.
- [23] E. STØRMER: J. Funct. Anal. **3** (1969) 48.
- [24] R. HONEGGER, A. RAPP: Physica A **167** (1990) 945.
- [25] W.O. AMREIN, J.M. JAUCH, K.B. SINHA: *Scattering theory in quantum mechanics* Benjamin, London, Amsterdam, Tokyo 1977.
- [26] M. TAKESAKI: *Theory of operator algebras I*. Springer-Verlag, Berlin, Heidelberg, New York 1979.
- [27] R. HONEGGER: *Globale Quantentheorie der Strahlung*. Ph.D. Thesis, Tübingen 1991.
- [28] N.G. DUFFIELD, R.F. WERNER: Helv. Phys. Acta **65** (1992) 1016.
- [29] G. SCHAFER: Helv. Phys. Acta **43** (1970) 806.
- [30] R. BONIFACIO, L.A. LUGIATO: Phys. Rev. **A11** (1975) 1507.