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Helv Phys Acta
Vol. 68 (1995)

0018-0238/95/030264-18$1.50+0.20/0

Algebraic Renormalization of $N = 2$ Supersymmetric Yang-Mills Chern-Simons Theory in the Wess-Zumino Gauge[1]

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Abstract. We consider a $N = 2$ supersymmetric Yang-Mills-Chern-Simons model, coupled to matter, in the Wess-Zumino gauge. The theory is characterized by a superalgebra which displays two kinds of obstructions to the closure on the translations: field dependent gauge transformations, which give rise to an infinite algebra, and equations of motion. The aim is to put the formalism in a closed form, off-shell, without introducing auxiliary fields. In order to perform that, we collect all the symmetries of the model into a unique nilpotent Slavnov-Taylor operator. Furthermore, we prove the renormalizability of the model through the analysis of the cohomology arising from the generalized Slavnov-Taylor operator. In particular, we show that the model is free of anomaly.

1 Introduction

In this paper, we consider a Yang-Mills-Chern-Simons model coupled to matter in a $N = 2$ supersymmetric extension [1, 2, 3, 4, 5, 6] of the usual Yang-Mills–Chern-Simons theory [7, 8, 9] Although a superfield version of the theory does exist [3, 5, 6], we shall work in components, moreover without auxiliary fields, and in the Wess-Zumino gauge (we recall that the Wess-Zumino gauge fixes all the supergauge of freedom except the

[1]Supported in part by the Swiss National Science Foundation.

ordinary one for the vector gauge field).

Because of the complications following from this choice, namely the nonlinearity of the supersymmetry transformations and the closure of the superalgebra only modulo field equations and field dependent gauge transformations [10, 11], it is convenient to follow the approach of [12], suitable to study more complicated situations, in which a simple superspace formalism, either is not available, as for instance in some extended supersymmetry theories, or does not bring simplifications like in case of broken supersymmetry [13].

However, mastering the main complication of the Wess-Zumino gauge, which is the infinite dimensional algebra spanned by the field dependent gauge transformations, turns out to be very difficult [11].

On the other hand, a satisfactory regularization procedure compatible with both BRS invariance and supersymmetry is lacking.

All of this makes much advisable the adoption of the algebraic method of renormalization [14], which is indeed regularization scheme independent, and powerful enough to overcome the intrinsic difficulties of the problem at hand.

Fixing the remaining gauge invariance gives rise to an additional minor problem: it is not possible to construct a gauge fixing term which is invariant under both BRS and supersymmetry.

The difficulties we have just mentioned and which will be described in Section 2, can simultaneously be solved by collecting all the symmetries into a unique nilpotent operator $\mathcal{D}$. In this way, the original algebra reduces to the simple nilpotency relation characterizing the generalized BRS operator. This is achieved in Section 3 and translated in the form of functional identities in Sections 4 and 5. Section 6 is devoted to the renormalization of the model, by finding the counterterms and proving that the model is free of anomalies.

2 The Model

The model is described by two supermultiplets of fields: the Chern-Simons supermultiplet (CSM), which belongs to the adjoint representation of a semi-simple gauge group, and the matter supermultiplet (MM), which is in an arbitrary representation of the gauge group.

Here are given the fields of both supermultiplets:

$$\begin{array}{lllll} \text{CSM:} & A^a_\mu & S^a & \lambda^a & \bar\lambda^a, \\ \text{MM:} & A_i & A^{*i} & \psi_i & \bar\psi^i, \end{array}$$

where A^a_μ is the gauge field, S^a is a real scalar field, λ^a, $\bar\lambda^a$, ψ_i and $\bar\psi^i$ are Dirac spinors,

A_i and A^{*i} are complex scalar fields.

The indices a,b,c run over the adjoint representation of the gauge group, whereas i,j concern the arbitrary representation in which the matter fields A_i, ψ_i live. The matrix generators $(T_a)^i_j$ are supposed to be antihermitian.

In the following, we shall adopt a vector space notation. For ψ, ψ' in an arbitrary representation: $(\psi^*, \psi') \equiv \psi^{*i}\psi'_i$. Moreover, for ϕ in the adjoint representation, we define $(\psi^*, \phi\psi') \equiv \phi^a(\psi^*, T_a\psi') \equiv \phi^a\psi^{*i}(T_a)^j_i\psi'_j$, where the T_a's are the generators of the representation where ψ, ψ' live. Finally, we use also the notation $\phi \equiv \phi^a\tau_a$, for $\phi = A_\mu, \lambda$ or S, the τ_a's being the generators of the group in the fundamental representation, with the normalization $\mathrm{Tr}\,(\tau^a\tau^b) = \delta^{ab}$.

The action is given by

$$\Sigma_{\mathrm{inv}} = \Sigma_{CS} + \Sigma_{VM} + \Sigma_{SM}, \tag{2.1}$$

where

$$\Sigma_{CS} = m\int d^3x \left(-\tfrac{1}{2}\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\,\left(F_{\mu\nu}A_\rho - \tfrac{2}{3}gA_\mu A_\nu A_\rho\right) - 2\,\mathrm{Tr}\,(\bar\lambda\lambda) - 2m\,\mathrm{Tr}\,S^2 + 4ig(A^*, SA)\right), \tag{2.2}$$

$$\Sigma_{VM} = \mathrm{Tr}\int d^3x \left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar\lambda \not{D}\lambda + \tfrac{1}{2}D_\mu S D^\mu S + 2ig(\bar\lambda\lambda)S\right), \tag{2.3}$$

and

$$\begin{aligned}\Sigma_{SM} \;=\;& \int d^3x \Big(2(D^\mu A^*, D_\mu A) + i(\bar\psi, \not{D}\psi) + ig(\bar\psi, S\psi) \\ &+ 2ig(A^*, \bar\lambda\psi) + 2ig(\bar\psi, \lambda A) + 2g^2(A^*, TA)^2 + 2g^2(SA^*, SA)\Big),\end{aligned} \tag{2.4}$$

where m is a dimensionful coupling constant and g is the gauge coupling constant. We have defined the conjugate spinors as usual by $\bar\psi = \psi^\dagger\gamma^0$. The γ-matrices satisfy a Clifford algebra and may be expressed in terms of the Pauli matrices as

$$\gamma^0 = \sigma^3, \quad \gamma^1 = i\sigma^2, \quad \gamma^2 = i\sigma^1. \tag{2.5}$$

The field strength and the covariant derivatives are defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu], \tag{2.6}$$

$$\begin{aligned} D_\mu\phi \;&=\; \partial_\mu\phi + g[A_\mu, \phi], \\ &\quad \text{for } \phi \text{ in the adjoint representation, and} \\ D_\mu\psi \;&=\; \partial_\mu\psi + gA^a_\mu T_a\psi, \\ &\quad \text{for } \psi \text{ in the matter field representation.} \end{aligned} \tag{2.7}$$

Besides being gauge invariant, the action (2.1) is left unchanged by the $N = 2$ supersymmetric transformations

$$
\begin{aligned}
\delta A_\mu &= \bar{\epsilon}\gamma_\mu\lambda + \bar{\lambda}\gamma_\mu\epsilon \\
\delta\lambda &= \tfrac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon - iD_\mu S\gamma^\mu\epsilon - 2mS\epsilon + 2ig(A^*, TA)\epsilon \\
\delta S &= \bar{\epsilon}\lambda + \bar{\lambda}\epsilon \\
\delta A &= \bar{\epsilon}\psi \\
\delta\psi &= -2iD_\mu A\gamma^\mu\epsilon + 2igSA\epsilon
\end{aligned}
\tag{2.8}
$$

where ϵ is an infinitesimal Dirac spinor parameter.

Notice that the infinitesimal supersymmetric variations of the spinor fields exhibit the nonlinearities arising from the adoption of the Wess-Zumino gauge.

The commutators of two subsequent infinitesimal supersymmetric variations of the fields are given by

$$
\begin{aligned}
[\delta_1, \delta_2]\Phi &= 2i\partial_\mu\Phi(\bar{\epsilon}_1\gamma^\mu\epsilon_2 - \bar{\epsilon}_2\gamma^\mu\epsilon_1) \\
&+ 2i\delta_g^{(\omega)}\Phi \\
&+ \text{equations of motion,}
\end{aligned}
\tag{2.9}
$$

where Φ stands for all the fields and ω is a field dependent parameter:

$$
\omega \equiv A_\mu(\bar{\epsilon}\gamma^\mu\epsilon) - S(\bar{\epsilon}\epsilon). \tag{2.10}
$$

3 Collecting Symmetries

We have seen in the previous section that the supersymmetry (2.8) was realized nonlinearly and, consequently, that the commutators did not close on the translations. The usual approach when dealing with algebraic structures like (2.9) is to add auxiliary fields in order to put the formalism off-shell, with possibly the appearance of a central charge in the algebra of the matter multiplet [10].

As well explained in [11], the algebra (2.9) is infinite dimensional, because of the field dependent gauge transformations in the result of the commutators. To control it, one should introduce an infinite number of external sources with increasing negative dimensions. This makes the renormalization problematic. Our aim, in view of the quantum extension of the theory, is to construct an operator $\mathcal{D}$ which contains all the relevant symmetries of the model. The algebra will be characterized in a closed way by demanding the nilpotency of the operator $\mathcal{D}$. With the help of this operator, it becomes very easy to calculate the counterterms as well as the possible anomalies. In order to construct such a nilpotent operator, all the symmetries are summed up: in addition to the supersymmetry and to the BRS symmetry, we must take into account also the translation invariance of the model. This method [12] allows one to put the formalism off-shell without the help of auxiliary fields, whose role is played, in this formalism, by the external sources coupled to the nonlinear variations of the quantum fields.

In view of fixing the gauge, we introduce a ghost c, an antighost $\bar{c}$ and a Lagrange multiplier field b implementing the gauge fixing condition

$$\partial_\mu A^\mu = 0. \tag{3.1}$$

Once the gauge is fixed, the gauge invariance evolves into the BRS invariance described by the following action on the quantum fields:

$$\begin{aligned} &sA_\mu = -(D_\mu c), \quad s\lambda = g[c,\lambda], \quad sS = g[c,S], \\ &sA = gcA, \quad s\psi = gc\psi, \\ &sc = gc^2, \quad s\bar{c} = b, \quad sb = 0. \end{aligned} \tag{3.2}$$

The s-operator is nilpotent

$$s^2 = 0. \tag{3.3}$$

We complete the BRS symmetry with the supersymmetry (2.8) and the translations, collecting these in a unique operator $\mathcal{D}$

$$\mathcal{D} = s + \delta + \xi^\mu\partial_\mu - 2i(\bar{\epsilon}\gamma^\mu\epsilon)\frac{\partial}{\partial\xi^\mu}, \tag{3.4}$$

where the infinitesimal parameters ϵ and ξ (for supersymmetry and translations, respectively) are promoted to the status of global ghosts, the spinor ϵ being now commuting and the vector ξ anticommuting. The last term in (3.4) then ensures the nilpotency of $\mathcal{D}$ – valid on-shell, i.e. modulo the equations of motion:

$$\mathcal{D}^2 = \text{equations of motion}. \tag{3.5}$$

The action of $\mathcal{D}$ is explicitly given by

$$
\begin{aligned}
\mathcal{D}A_\mu &= g[c,A_\mu] - \partial_\mu c + \bar\epsilon\gamma_\mu\lambda + \bar\lambda\gamma_\mu\epsilon + \xi\partial A_\mu \\
\mathcal{D}\lambda &= g[c,\lambda] + \tfrac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon - iD_\mu S\gamma^\mu\epsilon - 2mg^2 S\epsilon + 2ig(A^*,TA)\epsilon + \xi\partial\lambda \\
\mathcal{D}S &= g[c,S] + \bar\epsilon\lambda + \bar\lambda\epsilon + \xi\partial S \\
\mathcal{D}A &= gcA + \bar\epsilon\psi + \xi\partial A \\
\mathcal{D}\psi &= gc\psi - 2iD_\mu A\gamma^\mu\epsilon + 2igSA\epsilon + \xi\partial\psi \\
\mathcal{D}\epsilon &= 0 \\
\mathcal{D}\xi_\mu &= -2i(\bar\epsilon\gamma_\mu\epsilon) \\
\mathcal{D}c &= gc^2 - 2i\omega + \xi\partial c \\
\mathcal{D}\bar c &= b + \xi\partial\bar c \\
\mathcal{D}b &= 2i(\bar\epsilon\gamma\epsilon)\partial\bar c + \xi\partial b,
\end{aligned}
\tag{3.6}
$$

The quantity ω in the transformation law for c has been given by (2.10). The operator $\mathcal{D}$ thus defined turns out to be nilpotent on all fields except the spinors, for which one gets

$$
\begin{aligned}
\mathcal{D}^2\lambda &= \epsilon\left(\left(\frac{\delta\Sigma_{\rm inv}}{\delta\lambda}\epsilon\right) - \left(\bar\epsilon\frac{\delta\Sigma_{\rm inv}}{\delta\bar\lambda}\right)\right), \\
\mathcal{D}^2\psi &= \frac{\delta\Sigma_{\rm inv}}{\delta\bar\psi}(\bar\epsilon\epsilon) - \gamma_\mu\frac{\delta\Sigma_{\rm inv}}{\delta\bar\psi}(\bar\epsilon\gamma^\mu\epsilon).
\end{aligned}
\tag{3.7}
$$

The action (2.1) is $\mathcal{D}$-invariant:

$$
\mathcal{D}\Sigma_{\rm inv} = 0. \tag{3.8}
$$

The canonical dimensions and the Faddeev-Popov charges of the quantum fields and of the global ghosts are listed in Table 1.

	A_μ	λ	S	A	ψ	c	$\bar c$	b	ϵ	ξ
d	1/2	1	1/2	1/2	1	-1/2	3/2	3/2	-1/2	-1
$\Phi\Pi$	0	0	0	0	0	1	-1	0	1	1

Table 1: Dimensions d and ghost charges $\Phi\Pi$.

In conclusion, the dependence on the gauge transformations is now removed from the algebra (2.9). This results from the ω-dependent term in the $\mathcal{D}$-transformation of the ghost field c, as given in (3.6). Moreover, as anticipated in the Introduction, this goes not alone: at the same time, we have completed the construction of an on-shell nilpotent operator $\mathcal{D}$.

Finally, we increase the action by a $\mathcal{D}$-invariant gauge fixing term

$$\begin{aligned}\Sigma_{gf} &= \mathcal{D}\,\mathrm{Tr}\int d^3x\, \bar{c}\partial A \\ &= \mathrm{Tr}\int d^3x\, \left(b\partial^\mu A_\mu + \partial^\mu \bar{c} g[c, A_\mu] + \bar{c}\partial^2 c + \partial_\mu \bar{c}(\bar{\epsilon}\gamma^\mu\lambda + \bar{\lambda}\gamma^\mu\epsilon)\right),\end{aligned} \tag{3.9}$$

thus escaping from the impossibility of writing a supersymmetry invariant gauge fixing term.

Hence the total gauge-fixed action

$$\Sigma_0 = \Sigma_{\mathrm{inv}} + \Sigma_{gf} \tag{3.10}$$

is $\mathcal{D}$-invariant: $\mathcal{D}$ is a symmetry of the action.

4 Slavnov-Taylor Identity and Off–Shell Formulation

The problem of putting the formalism off-shell, namely of getting rid of the equations of motion in (3.7), is tightly related to the task of writing the Slavnov-Taylor identity associated to the $\mathcal{D}$-symmetry (3.6). In order to do that, it is necessary to couple external sources to the nonlinear $\mathcal{D}$-variations of the quantum fields. But this is not sufficient for putting the formalism off-shell. To obtain that, it is necessary to add to the action also a term quadratic in the external sources . The total action

Fields φ:	A_μ	λ	S	A	ψ	c
Sources K_φ:	Ω_μ	$\bar{\Lambda}$	M	U^*	$\bar{\Psi}$	L

Table 2: Notations for the sources.

$$\Sigma = \Sigma_0 + \Sigma_1 + \Sigma_2, \tag{4.1}$$

with Σ_0 defined in (3.10), and with (See Table 2 for the notations of the sources)

$$\Sigma_1 = \sum_\varphi \int d^3x\, K_\varphi \mathcal{D}\varphi, \tag{4.2}$$

$$\Sigma_2 = \mathrm{Tr}\int d^3x\, ((\bar{\Psi}\bar{\Psi})(\bar{\epsilon}\epsilon) - (\bar{\Psi}\gamma_\mu\bar{\Psi})(\bar{\epsilon}\gamma^\mu\epsilon) - (\bar{\epsilon}\Lambda)(\bar{\epsilon}\Lambda) - (\bar{\Lambda}\epsilon)(\bar{\Lambda}\epsilon) - (\bar{\Lambda}\epsilon)(\bar{\epsilon}\Lambda)), \tag{4.3}$$

satisfies the Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = \int d^3x\, \left(\sum_\varphi \frac{\delta\Sigma}{\delta K_\varphi}\frac{\delta\Sigma}{\delta\varphi} + \mathrm{Tr}\left(\mathcal{D}\bar{c}\frac{\delta\Sigma}{\delta\bar{c}} + \mathcal{D}b\frac{\delta\Sigma}{\delta b}\right)\right) + \mathcal{D}\xi\frac{\partial\Sigma}{\partial\xi} = 0. \tag{4.4}$$

The linearized Slavnov-Taylor operator given by

$$\mathcal{B}_\Sigma = \int d^3x \left(\sum_\varphi \left(\frac{\delta\Sigma}{\delta K_\varphi}\frac{\delta}{\delta\varphi} + \frac{\delta\Sigma}{\delta\varphi}\frac{\delta}{\delta K_\varphi} \right) + \mathrm{Tr}\left(\mathcal{D}\bar{c}\frac{\delta}{\delta\bar{c}} + \mathcal{D}b\frac{\delta}{\delta b} \right) \right) + \mathcal{D}\xi\frac{\partial}{\partial\xi} \tag{4.5}$$

is off-shell nilpotent

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0. \tag{4.6}$$

$\mathcal{B}_\Sigma$ acts on the nonspinorial quantum fields in the same way as $\mathcal{D}$, whereas it acts on the spinor fields as follows

$$\begin{aligned} \mathcal{B}_\Sigma\lambda &= \frac{\delta\Sigma}{\delta\bar{\Lambda}} = \mathcal{D}\lambda - 2\epsilon(\bar{\Lambda}\epsilon) - \epsilon(\bar{\epsilon}\Lambda), \\ \mathcal{B}_\Sigma\psi &= \frac{\delta\Sigma}{\delta\bar{\Psi}} = \mathcal{D}\psi + \Psi(\bar{\epsilon}\epsilon) - \gamma_\mu\Psi(\bar{\epsilon}\gamma^\mu\epsilon). \end{aligned} \tag{4.7}$$

The addition of the bilinear terms in the sources has the effect of modifying the transformation laws of the spinor fields in order to get an off-shell nilpotent operator. In this sense, the external sources play the same role as the auxiliary fields.

5 Constraint Equations

In addition to the Slavnov-Taylor identity (4.4), the model is defined by the following constraints:

1) the ξ-equation

$$\frac{\partial\Sigma}{\partial\xi^\mu} = \Delta_\mu, \tag{5.1}$$

where

$$\begin{aligned} \Delta_\mu &= -\int d^3x \Big(\mathrm{Tr}\left(\Omega^\nu\partial_\mu A_\nu + \bar{\Lambda}\partial_\mu\lambda + \partial_\mu\bar{\lambda}\Lambda + M\partial_\mu S - L\partial_\mu c \right) \\ &+ U\partial_\mu A^* + U^*\partial_\mu A + \bar{\Psi}\partial_\mu\psi + \partial_\mu\bar{\psi}\Psi \Big); \end{aligned} \tag{5.2}$$

2) the gauge fixing equation

$$\frac{\delta\Sigma}{\delta b} = \partial^\mu A_\mu; \tag{5.3}$$

3) the antighost equation

$$\bar{\mathcal{F}}\Sigma \equiv \frac{\delta\Sigma}{\delta\bar{c}} + \partial_\mu\frac{\delta\Sigma}{\delta\Omega_\mu} - \xi\partial\frac{\delta\Sigma}{\delta b} = 0, \tag{5.4}$$

which arises from the commutator of the linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$ with the gauge fixing equation (5.3);

4) the ghost equation

$$\mathcal{F}\Sigma \equiv \int d^3x \left(\frac{\delta\Sigma}{\delta c} + g[\bar{c}, \frac{\delta\Sigma}{\delta b}] \right) = g\Delta, \tag{5.5}$$

where

$$\begin{aligned} \Delta &= \int d^3x \Big([\eta, A] + [M, S] + [\bar{\Lambda}, \lambda] + [\bar{\lambda}, \Lambda] \\ &+ AU^* + UA^* + \bar{\Psi}\psi + \bar{\psi}\Psi + [L, c] \Big). \end{aligned} \tag{5.6}$$

Notice that all the above constraints but the antighost equation (5.4), have the form of symmetries broken by terms which are linear in the quantum fields, therefore not receiving radiative corrections.

6 Algebraic Renormalization

Calculations up to two loops [5, 6] have shown the vanishing of the β-functions associated to the coupling constants g^2 and m. From these results, a finiteness conjecture has been inferred, which is still far from having being transformed into a proof to all orders. The formulation presented in the previous sections is the only one suitable for a discussion of the renormalization to all orders performed in the Wess-Zumino gauge [12]. The reason for that is twofold. First of all, the lack of a coherent regularization scheme entails the algebraic approach. Secondly, the nonlinearities of the supersymmetry transformations (2.8), together with the consequent non-closure of the algebra, render problematic a separate analysis of the BRS symmetry and of the supersymmetry.

The algebraic study of the renormalizability of the model develops into two steps.

First, we will find the counterterms through the study of the stability of the classical action and we will show that the radiative corrections can be reabsorbed by a redefinition of the fields and of the coupling constants of the theory.

Next, we will compute the possible anomaly through a cohomological analysis of the nilpotent operator $\mathcal{B}_\Sigma$. We will use the filtration method developed by Dixon [17], which will be explained in more details later. It basically consists in making a judicious choice of a filtration operator, which will lead to a simplification of the cohomology problem.

6.1 Counterterms

In order to study the stability of the model under radiative corrections, we perturb the classical action Σ by a functional Σ_c

$$\Sigma \longrightarrow \Sigma + \eta\Sigma_c, \tag{6.1}$$

where η is an infinitesimal parameter. We then ask that the perturbed action satisfies all the symmetries and constraints defining the theory, which, at first order in η, imply the following conditions on Σ_c:

1) from the ξ-equation:

$$\frac{\partial \Sigma_c}{\partial \xi^\mu} = 0 \tag{6.2}$$

2) from the gauge condition:

$$\frac{\delta \Sigma_c}{\delta b} = 0 \tag{6.3}$$

3) from the antighost equation:

$$\frac{\delta \Sigma_c}{\delta \bar{c}} + \partial_\mu \frac{\delta \Sigma_c}{\delta \Omega_\mu} = 0 \tag{6.4}$$

4) from the ghost equation:

$$\int d^3x \, \frac{\delta \Sigma_c}{\delta c} = 0 \tag{6.5}$$

5) from the Slavnov-Taylor identity:

$$\mathcal{B}_\Sigma \Sigma_c = 0 \tag{6.6}$$

The conditions (6.2) and (6.3) mean that the counterterm Σ_c does depend neither on the global ghost ξ^μ nor on the Lagrange multiplier b. Moreover, the constraints (6.4) and (6.5) are satisfied by a functional depending on $\bar{c}$ and Ω^μ only through the combination

$$\eta^\mu \equiv \partial \bar{c} + \Omega^\mu \tag{6.7}$$

and depending on the ghost c only if differentiated

$$c_\mu \equiv \partial_\mu c. \tag{6.8}$$

The last condition (6.6), which derives from imposing the Slavnov-Taylor identity on the perturbed action, due to the nilpotency of the linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$, constitutes a cohomology problem whose solution must be found within the space of functionals having canonical dimensions up to three, vanishing Faddeev-Popov charge and satisfying all the previous constraints. Therefore, the most general solution is

$$\Sigma_c = \hat{\Sigma}_c + \mathcal{B}_\Sigma \tilde{\Sigma}_c. \tag{6.9}$$

The functional $\hat{\Sigma}_c$ is a nontrivial BRS cocycle, in the sense that it cannot be written as a $\mathcal{B}_\Sigma$-variation. It represents the cohomology of the $\mathcal{B}_\Sigma$ operator in the sector of zero $\Phi\Pi$ charge, and it corresponds to renormalizations of the physical parameters of the theory.

The trivial cocycle $\mathcal{B}_\Sigma\tilde{\Sigma}_c$, on the other hand, stands for (unphysical) field amplitude renormalizations. In order to explicitely find the possible renormalizations of the theory, we can use the result, explained in the Appendix, according to which the counterterm is at least of order g^2. This constitutes a considerable simplification because in practice one should write the most general functional with canonical dimensions two, vanishing $\Phi\Pi$-charge, which satisfies the constraints from 1) to 4). The Slavnov-Taylor condition (6.6), finally leads to the following simple expression for the most general counterterm:

$$\Sigma_c = Z_m \Sigma_{CS}, \tag{6.10}$$

where Z_m is an arbitrary constant proportional to g^2 and Σ_{CS} is given by (2.2). We see from (6.10), that *a priori* only the dimensionful constant m can get radiative corrections. This means that the beta function related to the gauge coupling constant g^2 is vanishing to all orders of perturbation theory, and the anomalous dimensions of the fields as well. We stress that this is a purely algebraic result, valid to all orders of perturbation theory, to be compared with the results given in [6], which led to the finiteness conjecture corresponding to a theory whose counterterm is a trivial BRS cocycle. To our knowledge, up to now, no algebraic proof has been given of this property. Here we can just state that the radiative corrections can be reabsorbed through a redefinition of the topological mass only, thus concluding the first part of the renormalizability proof.

6.2 Anomaly

To complete the proof of the renormalizability of the model, we have to show that all the symmetries defining the theory can be extended to the quantum level, or, in other words, we must prove that it is possible to define a quantum vertex functional

$$\Gamma = \Sigma + O(\hbar) \tag{6.11}$$

such that

$$\frac{\partial \Gamma}{\partial \xi^\mu} = \Delta_\mu, \quad \frac{\delta \Gamma}{\delta b} = \partial^\mu A_\mu, \quad \bar{\mathcal{F}}\Gamma = 0, \quad \mathcal{F}\Gamma = g\Delta, \tag{6.12}$$

$$\mathcal{P}_\mu \Gamma = \mathcal{W}_{rig}\Gamma = 0 \tag{6.13}$$

$$\mathcal{S}(\Gamma) = 0, \tag{6.14}$$

where $\bar{\mathcal{F}}$, $\mathcal{F}$ are defined by (5.4), (5.5), and $\mathcal{P}_\mu$ and $\mathcal{W}_{rig}$ are the Ward operators for translations and rigid gauge transformations respectively.

On the one hand, the extension of the ξ-equation (5.1), the gauge condition (5.3), the antighost equation (5.4) and the ghost equation (5.5) to their quantum counterparts is trivial and we refer to [14] for the details of the proof. The quantum implementation (6.14) of the classical Slavnov-Taylor identity (4.4), on the other hand, requires some care. The rest of this section will be devoted to this end.

According to the quantum action principle [14, 15], the Slavnov-Taylor identity gets a quantum breaking

$$\mathcal{S}(\Gamma) = \Delta \cdot \Gamma \tag{6.15}$$

which, at lowest order in $\hbar$, is a local integrated functional with canonical dimension three and Faddeev-Popov charge one

$$\Delta \cdot \Gamma = \Delta + O(\hbar\Delta). \tag{6.16}$$

The fact that Γ satisfies the identities (6.12) to (6.13) and the following algebraic relations – which a direct check shows to be valid for any functional γ:

$$\frac{\delta}{\delta b}\mathcal{S}(\gamma) - \mathcal{B}_\gamma\left(\frac{\delta\gamma}{\delta b} - \partial A\right) = \bar{\mathcal{F}}\gamma \tag{6.17}$$

$$\frac{\partial}{\partial\xi^\mu}\mathcal{S}(\gamma) + \mathcal{B}_\gamma\left(\frac{\partial\gamma}{\partial\xi^\mu} - \Delta_\mu\right) = \mathcal{P}_\mu\gamma \tag{6.18}$$

$$\bar{\mathcal{F}}\mathcal{S}(\gamma) + \mathcal{B}_\gamma\bar{\mathcal{F}}\gamma = 0 \tag{6.19}$$

$$\mathcal{F}\mathcal{S}(\gamma) + \mathcal{B}_\gamma(\mathcal{F}\gamma - \Delta) = \mathcal{W}_{rig}\gamma \tag{6.20}$$

$$\mathcal{B}_\gamma\mathcal{S}(\gamma) = 0 \tag{6.21}$$

implies the following consistency conditions on the breaking Δ:

$$\frac{\delta\Delta}{\delta b} = 0, \quad \frac{\partial\Delta}{\partial\xi^\mu} = 0, \quad \bar{\mathcal{F}}\Delta = 0, \quad \mathcal{F}\Delta = 0 \tag{6.22}$$

$$\mathcal{B}_\Sigma\Delta = 0 \tag{6.23}$$

Notice that the consistency conditions (6.22) and (6.23) formally coincide with the relations determining the counterterm. The difference is that now the solution must belong to the space of functionals having Faddeev-Popov charge one instead of zero. Therefore, the first four conditions tell us that the breaking Δ does depend neither on ξ^μ nor on b, that $\bar{c}$ and Ω^μ appear only in the combination η^μ (6.7) and that the ghost must always be differentiated.

The last consistency condition (6.23) is often called the Wess-Zumino consistency condition. Like the corresponding one in the zero-$\Phi\Pi$ sector (6.6), solving it is a cohomology problem.

We will show that the cohomology of the $\mathcal{B}_\Sigma$ operator in the $\Phi\Pi$ charge one sector is empty, namely that the more general solution of (6.23) is

$$\Delta = \mathcal{B}_\Sigma\hat{\Delta}, \tag{6.24}$$

with $\hat{\Delta}$ obeying the constraints (6.2) to (6.5). This will entail the possibility of absorbing the breaking Δ as a counterterm $-\hat{\Delta}$, leading thus to the desired conclusion concerning the absence of anomalies.

To analyze the cohomology of the $\mathcal{B}_\Sigma$ operator (4.5), we adopt the strategy of [16] and [17], which consists into passing from functionals to functions. This corresponds in practice into translating the functional operator $\mathcal{B}_\Sigma$, which acts on the space of local functionals satisfying (6.22), into an ordinary differential operator B_Σ, which acts on a space of functions $\Delta(x)$ with dimension three, $\Phi\Pi$ charge one and also restricted to be invariant under (6.23). Consequently, the cohomology problem (6.23) can be cast into the following local identity:

$$B_\Sigma \Delta(x) + d\Delta'(x) = 0, \tag{6.25}$$

where d is the exterior derivative: $d^2 = 0$, and $\Delta(x)$ is a 3-form defined by $\Delta = \int \Delta(x)$.

Given the functional operator $\mathcal{B}_\Sigma$, the form of the corresponding differential operator B_Σ is straightforward, provided we consider as independent the fields and their derivatives. The results of [17] insure that the cohomology of B_Σ is isomorphic to a subspace of the cohomology of $B_\Sigma^{(0)}$. $B_\Sigma^{(0)}$ is obtained from B_Σ by making an arbitrary filtration on the fields by means of an operator $\mathcal{N}$, and taking the lowest order

$$B_\Sigma = \sum_{n=0}^{N} B_\Sigma^{(n)}. \tag{6.26}$$

Let us analyze the identity (6.25), written for $B_\Sigma^{(0)}$:

$$B_\Sigma^{(0)} \Delta_3^1 + d\Delta_2^2 = 0, \tag{6.27}$$

where, as usual, Δ_p^q denotes a p-form with ghost number q. Due to the anticommutation relation $[B_\Sigma^{(0)}, d] = 0$ and to the vanishing of the cohomology of d [14], the equation (6.27) gives rise to a ladder of descent equations

$$B_\Sigma^{(0)} \Delta_2^2 + d\Delta_1^3 = 0 \tag{6.28}$$

$$B_\Sigma^{(0)} \Delta_1^3 + d\Delta_0^4 = 0 \tag{6.29}$$

$$B_\Sigma^{(0)} \Delta_0^4 = 0. \tag{6.30}$$

It is evident that a good choice of the filtration leads to an operator $B_\Sigma^{(0)}$ whose cohomology space is easy to find. The filtration we adopt is associated to the operator $\mathcal{N}$ which assigns the weights displayed[2] in Table 3, where $S_{\mu\nu} \equiv \partial_\mu A_\nu + \partial_\nu A_\mu$, $G_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $\eta_{\mu\nu} \equiv \epsilon_{\mu\nu\rho}\eta^\rho$, $\eta_{\mu\nu\rho} \equiv \partial_\mu \eta_{\nu\rho} + \partial_\nu \eta_{\rho\mu} + \partial_\rho \eta_{\mu\nu}$, and $L_{\mu\nu\rho} \equiv \epsilon_{\mu\nu\rho} L$.

[2]Due to the power counting restrictions, only the (differentiated) fields up to a certain dimension are considered.

ϵ	A_μ	$S_{\mu\nu}$	$G_{\mu\nu}$	$\partial_\mu S_{\nu\rho}$	$\partial_\mu G_{\nu\rho}$	S	$\partial_\mu S$	$\partial_\mu\partial_\nu S$	λ	$\partial_\mu\lambda$
2	2	2	1	2	2	1	2	2	1	2
$\partial_\mu\partial_\nu\lambda$	A	$\partial_\mu A$	$\partial_\mu\partial_\nu A$	ψ	$\partial_\mu\psi$	$\partial_\mu\partial_\nu\psi$	$\eta_{\mu\nu}$	$\eta_{\mu\nu\rho}$	Λ	$\partial_\mu\Lambda$
3	3	3	1	1	1	1	1	1	1	2
M	$\partial_\mu M$	U	$\partial_\mu U$	Ψ	$\partial_\mu\Psi$	$L_{\mu\nu\rho}$	c	c_μ	$\partial_\mu c_\nu$	$\partial_\mu\partial_\nu c_\rho$
1	2	1	1	1	1	1	1	2	2	2

Table 3: Weights.

The nonvanishing $B_\Sigma^{(0)}$-transformations of the fields are

$$
\begin{array}{rclrclrcl}
B_\Sigma^{(0)} A_\mu &=& -c_\mu, & B_\Sigma^{(0)} S_{\mu\nu} &=& -2\partial_\mu c_\nu, & B_\Sigma^{(0)}\partial_\mu S_{\nu\rho} &=& -2\partial_\mu\partial_\nu c_\rho, \\
B_\Sigma^{(0)} A &=& \bar\epsilon\psi, & B_\Sigma^{(0)}\partial_\mu A &=& \bar\epsilon\partial_\mu\psi, & B_\Sigma^{(0)}\eta_{\mu\nu} &=& -mG_{\mu\nu}, \\
B_\Sigma^{(0)} M &=& -4m^2 S, & B_\Sigma^{(0)}\partial_\mu M &=& -4m^2\partial_\mu S, & B_\Sigma^{(0)}\Lambda &=& -2m\lambda, \\
B_\Sigma^{(0)}\partial_\mu\Lambda &=& -2m\partial_\mu\lambda, & B_\Sigma^{(0)} U &=& -2\partial^2 A, & B_\Sigma^{(0)}\partial_\mu U &=& -2\partial_\mu\partial^2 A, \\
B_\Sigma^{(0)}\Psi &=& i\gamma^\mu\partial_\mu\psi, & B_\Sigma^{(0)}\partial_\mu\Psi &=& i\gamma^\nu\partial_\nu\partial_\mu\psi, & B_\Sigma^{(0)} L_{\mu\nu\rho} &=& -\eta_{\mu\nu\rho}.
\end{array}
\tag{6.31}
$$

As a general property, $B_\Sigma^{(0)}$ is nilpotent.

The criterion in choosing the filtration displaied in Table 3 has been making the cohomology calculations as simpler as possible. In order to do so, the weights have been assigned in order that most of the fields forming the basis for the space $\Delta(x)$ transform as BRS doublets ($B_\Sigma^{(0)} u = v$, $B_\Sigma^{(0)} v = 0$), and therefore do not appear in the local cohomology of $B_\Sigma^{(0)}$. We can then restrict the computation of the cohomology to the space spanned by the polynomials of the fields c, $\partial_\mu(\partial_\nu A_\rho - \partial_\rho A_\nu)$, ψ, $\partial_\mu\psi$, $\partial_\mu\partial_\nu\psi$, A, $\partial_\mu A$, $\partial_\mu\partial_\nu A$, ϵ and their complex conjugates (the number of derivatives being limited by the powercounting bound on the dimension of Δ).

The last of the descent equations (6.30) is a problem of local cohomology, for which we can therefore use the result just stated. The most general scalar with dimension bounded by zero and $\Phi\Pi$ charge four, candidate for the nontrivial solution of (6.30), is

$$
\begin{array}{l}
a_1 g^2 (T^{abcd})^i_j c^a c^b c^c c^d A_i A^{*j} + a_2 g^2 (\bar\epsilon\epsilon)(T^{ab})^i_j c^a c^b A_i A^{*j} \\
+ a_3 g^2 (\bar\epsilon\epsilon)^2 A_i A^{*i} + a_4 g^4 T^{abcd} c^a c^b c^c c^d + \text{c.c.}
\end{array}
\tag{6.32}
$$

where T^{abcd}, $(T^{abcd})^i_j$ and $(T^{ab})^i_j$ are invariant tensors, and a_i are arbitrary coefficients. The invariance under $B_\Sigma^{(0)}$ imposes that all the a_i vanish, but a_4. No invariant tensor of degree four exists, which is completely antisymmetric in its indices, therefore the only solution is the trivial one

$$
\Delta_0^4 = B_\Sigma^{(0)} \Delta_0^3. \tag{6.33}
$$

Substituting the expression for Δ_0^4 in (6.29) transforms the problem of local cohomology modulo d in a pure local one

$$B_\Sigma^{(0)}(\Delta_1^3 - d\Delta_0^3) \equiv B_\Sigma^{(0)}\hat{\Delta}_1^3 = 0. \tag{6.34}$$

With the aforementioned fields spanning the cohomology space of $B_\Sigma^{(0)}$, it is possible to form the following vector, with dimension bounded by one and $\Phi\Pi$ charge three:

$$a_1 g^2 (T^{ab})^i_j c^a c^b A_i(\bar{\psi}^j \gamma_\mu \epsilon) + a_2 g^2 (\bar{\epsilon}\gamma_\mu\epsilon)(\bar{\epsilon}\psi_i)A^{*i} + a_3 g^3 (T^a)^i_j (\bar{\epsilon}\gamma_\mu\epsilon)c^a A_i A^{*j} + c.c. \tag{6.35}$$

For such term to be invariant, it must be $a_1 = a_3 = 0$, and the term left is a trivial cocycle. This implies that, again, the most general solution of (6.34) is

$$\hat{\Delta}_1^3 = B_\Sigma^{(0)}\Delta_1^2. \tag{6.36}$$

The equation (6.28) becomes a problem of local cohomology as well

$$B_\Sigma^{(0)}\Delta_2^2 + d(d\Delta_0^3 + B_\Sigma^{(0)}\Delta_1^2) = B_\Sigma^{(0)}(\Delta_2^2 - d\Delta_1^2) \equiv B_\Sigma^{(0)}\hat{\Delta}_2^2 = 0. \tag{6.37}$$

The most general 2-form with dimension bounded by two and $\Phi\Pi$ charge two, candidate for being a nontrivial solution of (6.37) is

$$\begin{aligned}&a_1 g^2 (T^{ab})^i_j \epsilon_{\mu\nu\rho} c^a c^b A_i (\partial^\rho A^{*j}) + a_2 g^2 \epsilon_{\mu\nu\rho}(\bar{\epsilon}\gamma^\rho\psi_i)(\bar{\psi}^i\epsilon) + a_3 g^2 \epsilon_{\mu\nu\rho}(\bar{\epsilon}\epsilon)A_i(\partial^\rho A^{*i})\\ &+a_4 g^2(\bar{\epsilon}\gamma_\nu\epsilon)A_i(\partial_\nu A^{*i}) + a_5 g^2 (T)^{ij}_{kl}\epsilon_{\mu\nu\rho}(\bar{\epsilon}\gamma^\rho\epsilon)A_iA_jA^{*k}A^{*l} + a_6 g^3 (T^a)^i_j \epsilon_{\mu\nu\rho}c^a A_i(\bar{\psi}^j\gamma^\rho\epsilon)\\ &+a_7 g^3 \epsilon_{\mu\nu\rho}(\bar{\epsilon}\gamma^\rho\epsilon)A_iA^{*i} + c.c.\end{aligned} \tag{6.38}$$

It is easily seen that only the second term is invariant under $B_\Sigma^{(0)}$. Since it can be written as a trivial cocycle, the equation (6.27) also becomes a problem of local cohomology:

$$B_\Sigma^{(0)}\Delta_3^1 + d(d\Delta_1^2 + B_\Sigma^{(0)}\Delta_2^1) = B_\Sigma^{(0)}(\Delta_3^1 - d\Delta_2^1) \equiv B_\Sigma^{(0)}\hat{\Delta}_3^1 = 0. \tag{6.39}$$

The scalar candidate for belonging to the local cohomology in the $\Phi\Pi$ sector of charge one, with dimension bounded by three is

$$\begin{aligned}&a_1 g^2 (\bar{\epsilon}\gamma^\mu\partial_\mu\psi_i)A^{*i} + a_2 g^2 (\bar{\epsilon}\gamma^\mu\psi_i)(\partial_\mu A^{*i}) + a_3 g^2 (T)^{ik}_{jl}(\bar{\epsilon}\psi_i)A^{*j}A_kA^{*l}\\ &+a_4 g^3 (T^a)^{ij}_{kl} c^a A_iA_jA^{*k}A^{*l} + a_5 g^4 (\bar{\epsilon}\psi_i)A^{*i} + a_6 g^5 (T^a)^i_j c^a A_i A^{*j} + c.c.\end{aligned} \tag{6.40}$$

After imposing the $B_\Sigma^{(0)}$ invariance, what remains is $B_\Sigma^{(0)}(a_5 g^4 A_i A^{*i})$. This evidently implies that the local cohomology modulo d of $B_\Sigma^{(0)}$ in the sector of $\Phi\Pi$-charge one is empty, which in turn entails the vanishing of the cohomology of the functional operator $\mathcal{B}_\Sigma$. We therefore conclude that it is possible to construct a quantum vertex functional Γ satisfying the Slavnov-Taylor identity (6.14) to all orders of perturbation theory, without the presence of any anomaly.

7 Conclusion

We were able to unify into a single nilpotent generalized BRS operator $\mathcal{D}$ the gauge invariance of the $2+1$-dimensional, $N=2$ supersymmetric Yang-Mills–Chern-Simons model in the Wess-Zumino gauge, without auxiliary fields, together with supersymmetry and translation invariance. As a main result, this led to a finite algebra, closed off-shell after the introduction of the external fields associated to the nonlinear symmetry transformations, and particularly after the introduction of quadratic terms in these external fields.

The use of the algebraic method of renormalization together with the study of the cohomology of the operator $\mathcal{D}$, has allowed us to show that the model is perturbatively renormalizable to all orders. First, anomalies have been proven to be absent. Next, the study of the possible counterterms has led to the conclusion that the theory is multiplicatively renormalizable, namely that the counterterms can be reabsorbed by a redefinition of the topological mass only. We were able to prove in a very simple way, avoiding any Feynman graph computation, that, to all orders of perturbation theory, the gauge coupling constant does not get radiative corrections and that the fields have no anomalous dimensions. This latter result, on the one hand, is weaker than the conjectured ultraviolet finiteness [5, 6] of the model, i.e. the nonrenormalization of its coupling constant *and* mass. On the other hand, our approach leads to a statement valid to all orders, whereas the finiteness, at the best of our knowledge, has been completely proven up to the two loops order only[3]. We stress that our algebraic analysis does not involve any regularization scheme, nor consequently any diagramatic calculation. Moreover, its general character makes convenient to apply this method to all those theories for which adopting a particular regularization procedure and making explicit loop calculations is too laborious.

Our work extends the renormalizability of the Yang-Mills–Chern-Simons theory – which was in fact shown to be ultraviolet finite [9] – to its $N=2$ supersymmetric generalization.

Acknowledgments Sylvain Wolf and Guy Bonneau are gratefully acknowledged for their constructive observations and for a critical reading of the manuscript.

Appendix: Power Counting

The degree of divergence of a 1-particle irreducible Feynman graph γ is given by

$$d(\gamma) = 3 - \sum_{\varphi} d_{\varphi} N_{\varphi} - \frac{1}{2} N_g . \tag{A.1}$$

[3]Actually, the two-loop finiteness and the superrenormalizability expressed by the power counting formula (A.1), (A.2) imply the finiteness to all orders.

Here N_φ is the number of external lines of γ corresponding to the field φ (the global ghost ϵ being considered as a field, too), d_φ is the dimension of φ as given in Table 1, and N_g is the power of the coupling constant g in the integral corresponding to the diagram γ.

Let us recall that a nonnegative value of $d(\gamma)$ corresponds to ultraviolet divergence. The dependence on the coupling constant, i.e. on the perturbative order, is characteristic of a superrenormalizable theory. The equivalent expression

$$d(\gamma) = 4 - \sum_\varphi \left(d_\varphi + \frac{1}{2}\right) N_\varphi - L, \tag{A.2}$$

where L is the number of loops of the diagram, shows that only graphs up to the too-loop order are divergent.

In order to apply the known results on the quantum action principle [14, 15] to the present situation, one may consider g as an external field of dimension 1/2. Including it in the summation under φ, (A.1) gets the same form as in a strictly renormalizable theory:

$$d(\gamma) = 3 - \sum_\varphi d_\varphi N_\varphi. \tag{A.3}$$

Thus, including the dimension of g into the calculation, we may state that the dimension of the counterterms of the action is bounded by 3. But, since they are generated by loop graphs, they are of order 2 in g at least. This means that, not taking now into account the dimension of g, we can conclude that their real dimension is bounded by 2.

In the same way, we arrive at the result that the Slavnov-Taylor breaking Δ defined in (6.15) has a dimension bounded by 3 (counting the dimension of g), bounded by 2 if we don't count the dimension of g, since, being produced by the radiative corrections, it is of order g^2 at least, too.

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