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Local Gravitational Supersymmetry of Chern-Simons Theory in the Vielbein Formalism

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Abstract. We discuss the Chern-Simons theory in three-dimensional curved space-time in the vielbein formalism. Due to the additional presence of the local Lorentz symmetry, beside the diffeomorphisms, we will include a local gravitational supersymmetry (superdiffeomorphisms and super-Lorentz transformations), which allows us to show the perturbative finiteness at all orders.

1 Introduction

Topological gauge theories[4], at least in a Landau type gauge[5], possess an interesting kind of symmetry, namely the linear vector supersymmetry [5]. This feature has been extensively discussed by several groups for a whole class of topological field theories, including the BF models [6, 7, 8, 9, 10], the bosonic and the fermionic string, respectively in the Beltrami and super-Beltrami parametrization [11, 12, 13, 14], four-dimensional topological Yang-Mills theory [15, 16], etc.

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[4]See [1] for a general review.

[5]Some non-covariant gauges, like the axial gauge, are also possible [2, 3, 4].

Another interesting topological example is the Chern-Simons theory [1, 17, 18, 19, 20] in three-dimensional flat space-time formulated in the Landau gauge. Indeed, it is invariant under a set of supersymmetry transformations whose generators form a Lorentz three-vector [21, 22, 23, 24]. There, the generators of BRS transformations, supersymmetry and translations obey a graded algebra of the Wess-Zumino type, which closes on the translations. With the help of this supersymmetric structure one is able to show the perturbative finiteness of such a theory [24, 25].

On an arbitrary curved space-time three-manifold, a local version of this supersymmetry for the Chern-Simons theory in the Landau gauge was derived in [23] and it has also been shown that, in the perturbative approach, this local supersymmetry is anomaly-free and that the theory is UV finite. In [23], the Einstein formalism was used to describe the explicit metric dependence of the gauge-fixing term[6]. Starting from the requirement of invariance under diffeomorphisms, a local supersymmetry for diffeomorphisms has been constructed, which together with the BRS transformations form a closed graded algebra.

In the present paper we extend the discussion of [23] in such a way that we are using the vielbein formalism to describe the local version of the Chern-Simons theory on a three-manifold. This allows us to incorporate also torsion. Then, instead of only imposing diffeomorphism invariance, we demand invariance of the action under gravity transformations, which include beside the diffeomorphisms also local Lorentz rotations. This generalization of the approach leads to a local gravitational supersymmetry, which contains both superdiffeomorphisms and super-Lorentz transformations.

This work is organized as follows. In section 2 we briefly introduce the vielbein formalism and describe the geometry with the Cartan structure equations. An important fact is the independence of the model with respect to the affine spin connection and the torsion. We will also formulate the local theory by demanding the invariance under local gravity transformations. It has been shown that the "physical" content of the theory is metric independent [18, 26, 27, 28, 29, 30], because the vielbein (or the metric) plays the role of a gauge parameter. In order to control the vielbein dependence, which follows from the Landau gauge fixing, we extend the BRS transformations [23, 31] and introduce the vielbein in a BRS doublet, which guarantees its non-physical meaning.

The discussion of the symmetry content of that local theory will follow in section 3. As it will be shown, beside the superdiffeomorphisms, a further symmetry exists, which will be called super-Lorentz transformations, and both can be combined to give a local gravitational supersymmetry, which breaks the gauge-fixed action at the functional level. Indeed, the corresponding Ward identity contains a "hard" breaking term, *i.e.* a non-linear term in the quantum fields. Fortunately, we are able to control this breaking by means of a standard procedure [32] using the fact, that the breaking is a BRS-exact term. Thus, we enlarge the BRS transformations by introducing a set of external fields grouped in a BRS doublet. This allows us to present the local gravitational supersymmetry in terms of unbroken Ward

[6]Remark that although the invariant Chern-Simons action is topological, the gauge-fixing is not metric independent.

identities. The Ward operator of this supersymmetry, together with the Ward operator of gravity and the linearized Slavnov operator, form then a closed linear graded algebra.

Section 4 is devoted to the study of the stability and the finiteness of the local Chern-Simons theory. The whole set of constraints (gauge condition, ghost equation, antighost equation, Slavnov identity, Ward identities for gravity and gravitational supersymmetry), which are fulfilled by the classical action, completely fixes the total action, *i.e.* it forbids any deformations of it. Furthermore, it will be shown that all the symmetries are free of anomalies. The stability of the classical theory, together with the possibility of extending the classical constraints to all orders of perturbation theory, ensures UV finiteness of the quantum theory [23].

In the appendix one finds an analysis with respect to the trivial counterterms and their parameter independence.

2 Local Chern-Simons Theory in Curved Space-Time at the Classical Level

The classical invariant Chern-Simons action Σ_{CS} can be written in the space of forms according to[7]

$$\Sigma_{CS} = -\frac{1}{2}\int_{\mathcal{M}}\left(A^A dA^A + \frac{\lambda}{3} f^{ABC} A^A A^B A^C\right) , \tag{2.1}$$

where the gauge field $A^A = A^A_\mu dx^\mu$ has form degree one and $d = dx^\mu \partial_\mu$ denotes the exterior derivative. The coupling constant λ is related to the parameter k by $\lambda^2 = 2\pi/k$ (see [18, 19, 23]). We also assume that the gauge group is compact and that all fields belong to its adjoint representation with f^{ABC} as structure constants[8].

As usual, one has to fix the gauge. We choose a Landau type gauge fixing procedure. For the curved space case we consider here, this gauge fixing term will be implemented by means of the well-known Cartan approach. This formalism allows us to describe a general Riemannian manifold $\mathcal{M}$ with the help of the vielbein $e^a_\mu(x)$ and the affine spin connection ${\omega^{ab}}_\mu(x)$. The endowed metric $g_{\mu\nu}(x)$ on $\mathcal{M}$ in local coordinates can be decomposed as

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab} , \tag{2.2}$$

where η_{ab} denotes the Euclidean metric in the tangent space[9]. In order to describe the inverse metric $g^{\mu\nu}(x)$ one needs the inverse vielbein $E^\mu_a(x)$ which is related to the vielbein by

$$e^a_\mu E^\mu_b = \delta^a_b \quad , \quad e^a_\mu E^\nu_a = \delta^\nu_\mu . \tag{2.3}$$

[7]The wedge product $\wedge$ has to be understood in the space of forms.

[8]Gauge group indices are denoted by capital Latin letters $(A, B, C, ...)$ and refer to the adjoint representation, $[G^A, G^B] = f^{ABC} G^C$, $Tr(G^A G^B) = \delta^{AB}$.

[9]The tangent space indices $(a, b, c, ...)$ are referred to the group $SO(3)$.

Therefore, $g^{\mu\nu}(x)$ takes the form

$$g^{\mu\nu} = E^\mu_a E^\nu_b \eta^{ab} \ , \tag{2.4}$$

In order to describe symmetry transformations, one introduces also the affine spin connection ${\omega^{ab}}_\mu$, which is antisymmetric in the indices (ab). Both, the vielbein and the affine spin connection can be written in terms of forms according to

$$e^a = e^a_\mu dx^\mu \quad , \quad \omega^{ab} = {\omega^{ab}}_\mu dx^\mu = {\omega^{ab}}_m e^m \ . \tag{2.5}$$

Finally, we are able to write down the Cartan structure equations which are defined as

$$T^a = de^a + {\omega^a}_b e^b = \frac{1}{2} T^a_{\mu\nu} dx^\mu dx^\nu = \frac{1}{2} T^a_{mn} e^m e^n \ , \tag{2.6}$$

$${R^a}_b = d{\omega^a}_b + {\omega^a}_c {\omega^c}_b = \frac{1}{2} {R^a}_{b\mu\nu} dx^\mu dx^\nu = \frac{1}{2} {R^a}_{bmn} e^m e^n \ , \tag{2.7}$$

with the torsion 2-form T^a and the curvature 2-form ${R^a}_b$.

The Landau type gauge-fixing takes the following form

$$\Sigma_{gf} = -s \int d^3x \ \left[e E^\mu_a E^{\nu a} (\partial_\nu \bar{c}^A) A^A_\mu \right] \ , \tag{2.8}$$

where $e = det(e^a_\mu)$ and $E^\mu_a = E^\mu_a(e^b_\nu)$. With s as the nilpotent BRS operator, the BRS transformations are given by

$$sA^A = Dc^A = dc^A + \lambda f^{ABC} A^B c^C \ , \tag{2.9}$$

$$sc^A = \frac{\lambda}{2} f^{ABC} c^B c^C \ , \tag{2.10}$$

$$s\bar{c}^A = B^A \ , \tag{2.11}$$

$$sB^A = 0 \ . \tag{2.12}$$

One observes that the gauge-fixing term depends explicitly on the vielbein field and therefore represents an unavoidable non-topological contribution. An important fact is, that the vielbein e^a plays the role of a gauge parameter. In order to guarantee its non-physical meaning, we let it transform as a BRS doublet [23, 31]:

$$se^a = \hat{e}^a \ , \tag{2.13}$$

$$s\hat{e}^a = 0 \ . \tag{2.14}$$

At this point let us remark that the vielbein, the affine spin connection and the torsion, allow for a complete tangent space formulation of the Chern-Simons action which reads

$$\Sigma_{CS} = -\frac{1}{2} \int d^3x \ \varepsilon^{mnk} \left[A^A_m \partial_n A^A_k + A^A_m A^A_l \left(\frac{1}{2} T^l_{nk} - {\omega^l}_{kn} \right) + \frac{\lambda}{3} f^{ABC} A^A_m A^B_n A^C_k \right] , \tag{2.15}$$

where ε^{mnk} denotes the totally antisymmetric tensor with indices in the tangent space[10]. On the other hand, by introducing the covariant derivative ∇_μ with respect to the affine connection $\Gamma^\rho_{\mu\nu}$, which acts on an arbitrary covariant vector field in the usual way [33]

$$\nabla_\mu X_\nu = e^a_\nu \mathcal{D}_\mu (E^\rho_a X_\rho) = \partial_\mu X_\nu - \Gamma^\rho_{\mu\nu} X_\rho \ , \tag{2.16}$$

the invariant Chern-Simons action (2.1) can be rewritten as follows:

$$\Sigma_{CS} = -\frac{1}{2}\int d^3x \ \varepsilon^{\mu\nu\rho}\left(A^A_\mu \nabla_\nu A^A_\rho + \frac{1}{2}A^A_\mu A^A_\lambda E^\lambda_l T^l_{\nu\rho} + \frac{\lambda}{3} f^{ABC} A^A_\mu A^B_\nu A^C_\rho\right) . \tag{2.17}$$

Due to the presence of the totally antisymmetric contravariant tensor density $\varepsilon^{\mu\nu\rho}$ with weight 1, the symmetric part of the affine connection vanishes and the antisymmetric part of it will be canceled by the torsion term, *i.e.* (2.17) reduces to

$$\Sigma_{CS} = -\frac{1}{2}\int d^3x \ \varepsilon^{\mu\nu\rho}\left(A^A_\mu \partial_\nu A^A_\rho + \frac{\lambda}{3} f^{ABC} A^A_\mu A^B_\nu A^C_\rho\right) . \tag{2.18}$$

All three formulations (2.15), (2.17) and (2.18) are completely equivalent to the original Chern-Simons action (2.1), which implies that the model is independent of the affine spin-connection ω and the torsion T.

In order to translate the BRS invariance of the gauge-fixed action into a Slavnov identity, one has to couple the non-linear parts of the BRS transformations (2.9) and (2.10) to external sources $\gamma^{\mu A}$ and τ^A. In terms of forms, these two external sources correspond to the dual 2-form $\tilde{\gamma}^A$ and the dual 3-form $\tilde{\tau}^A$

$$\tilde{\gamma}^A = \frac{1}{2}\varepsilon_{\mu\nu\rho}\gamma^{\rho A} dx^\mu dx^\nu \ , \tag{2.19}$$

$$\tilde{\tau}^A = \frac{1}{6}\varepsilon_{\mu\nu\rho}\tau^A dx^\mu dx^\nu dx^\rho \ , \tag{2.20}$$

where $\varepsilon_{\mu\nu\rho}$ is the totally antisymmetric covariant tensor density with weight -1. This implies that $\gamma^{\mu A}$ is a contravariant vector density and τ^A is a scalar density, both with weights 1. The contribution from the external sources to the complete action hence writes in terms of forms

$$\Sigma_{ext} = \int_{\mathcal{M}} \left(\tilde{\gamma}^A Dc^A + \frac{\lambda}{2} f^{ABC} \tilde{\tau}^A c^B c^C\right) , \tag{2.21}$$

or in components

$$\Sigma_{ext} = \int d^3x \ \left(-\gamma^{\mu A} D_\mu c^A + \frac{\lambda}{2} f^{ABC} \tau^A c^B c^C\right) , \tag{2.22}$$

with the BRS transformations

$$s\tilde{\gamma}^A = s\tilde{\tau}^A = 0 \ . \tag{2.23}$$

[10]Through our definition $\varepsilon^{mnk} = e^m_\mu e^n_\nu e^k_\rho \varepsilon^{\mu\nu\rho}$, with $\varepsilon^{\mu\nu\rho}$ as the totally antisymmetric contravariant tensor density with weight 1, ε^{mnk} is a scalar density with weight 1 under diffeomorphisms.

So far, the total Chern-Simons action in the Landau gauge with external sources

$$\Sigma = \Sigma_{CS} + \Sigma_{gf} + \Sigma_{ext} \tag{2.24}$$

is invariant under the BRS transformations[11].

At the functional level, this invariance is implemented by means of the Slavnov identity

$$\mathcal{S}(\Sigma) = \int d^3x \left(\frac{\delta\Sigma}{\delta\gamma^{\mu A}} \frac{\delta\Sigma}{\delta A^A_\mu} + \frac{\delta\Sigma}{\delta\tau^A} \frac{\delta\Sigma}{\delta c^A} + B^A \frac{\delta\Sigma}{\delta\bar{c}^A} + \hat{e}^a_\mu \frac{\delta\Sigma}{\delta e^a_\mu} \right) = 0 \,, \tag{2.25}$$

and the corresponding linearized Slavnov operator $\mathcal{S}_\Sigma$ writes

$$\mathcal{S}_\Sigma = \int d^3x \left(\frac{\delta\Sigma}{\delta\gamma^{\mu A}} \frac{\delta}{\delta A^A_\mu} + \frac{\delta\Sigma}{\delta A^A_\mu} \frac{\delta}{\delta\gamma^{\mu A}} + \frac{\delta\Sigma}{\delta\tau^A} \frac{\delta}{\delta c^A} + \frac{\delta\Sigma}{\delta c^A} \frac{\delta}{\delta\tau^A} + B^A \frac{\delta}{\delta\bar{c}^A} + \hat{e}^a_\mu \frac{\delta}{\delta e^a_\mu} \right) . \tag{2.26}$$

Moreover, the classical action Σ (2.24) fulfills the gauge condition

$$\frac{\delta\Sigma}{\delta B^A} = \partial_\mu (e E^\mu_a E^{\nu a} A^A_\nu) \,, \tag{2.27}$$

and the ghost equation of motion

$$\left(\frac{\delta}{\delta\bar{c}^A} + \partial_\mu (e E^\mu_a E^{\nu a} \frac{\delta}{\delta\gamma^{\nu A}}) \right) \Sigma = -\partial_\mu \left(s(e E^\mu_a E^{\nu a}) A^A_\nu \right) \,, \tag{2.28}$$

which one easily finds by (anti-)commuting the gauge condition with the Slavnov identity [24]. The homogenous ghost equation implies that the external field $\gamma^{\mu A}$ and the antighost $\bar{c}^A$ can only appear through the combination $\chi^{\mu A} = \gamma^{\mu A} + e E^\mu_a E^{\nu a} \partial_\nu \bar{c}^A$, which when written as a dual 2-form is given by

$$\tilde{\chi}^A = \tilde{\gamma}^A + \frac{1}{2} \varepsilon_{\mu\nu\rho} e E^\rho_a E^{\sigma a} (\partial_\sigma \bar{c}^A) dx^\mu dx^\nu \,. \tag{2.29}$$

In addition, the action (2.24) fulfills a further global constraint, namely the antighost equation [34]

$$\int d^3x \left(\frac{\delta\Sigma}{\delta c^A} + \lambda f^{ABC} \bar{c}^B \frac{\delta\Sigma}{\delta B^C} \right) = \int_{\mathcal{M}} \lambda f^{ABC} \left(\tilde{\gamma}^B A^C + \tilde{\tau}^B c^C \right) \,. \tag{2.30}$$

Furthermore, (2.24) possesses other symmetries of the gravitational type which will be discussed in the next section.

[11]The total action (2.24) is by construction invariant under diffeomorphisms and local Lorentz transformations.

3 Gravitational Type Symmetries of the Classical Action

In the last paragraph we have stated the complete action (2.24) for the Chern-Simons model in the vielbein formalism. We showed that such a formulation allows us to consider the general case for a Riemannian Manifold $\mathcal{M}$. We also exhibit its BRS invariance. Moreover, this model contains a larger class of invariances. Indeed, in the Einstein formalism, Σ is invariant under the diffeomorphisms and the superdiffeomorphisms [23]. In the Cartan framework, one has in addition also the local Lorentz symmetry which describes the invariance of the theory under local rotations. At this point, it is legitim to ask if there exists another symmetry beside the superdiffeomorphisms. As one will see, such a symmetry exists and will be called local super-Lorentz transformations throughout this paper.

In what follows, diffeomorphisms and local Lorentz transformations will be combined in the so-called local gravity transformations. Analogously, the superdiffeomorphisms and the super-Lorentz symmetry will generate a set of local gravitational supersymmetry transformations[12].

The gravity transformations of the elementary fields $\varphi = A, c, \bar{c}, B, \gamma, \tau, e, \hat{e}$ are given by:

$$\delta^{gr}_{(\varepsilon,\Omega)}\varphi = \mathcal{L}_\varepsilon \varphi + \delta_\Omega \varphi \ , \tag{3.1}$$

where ε^μ and Ω^{ab} are the infinitesimal parameters of diffeomorphisms and local Lorentz transformations respectively, both with ghost number $+1$. The symbol $\mathcal{L}_\varepsilon$ denotes the usual Lie derivative in the direction of ε^μ and δ_Ω describes the Lorentz rotation operator. At the functional level, the invariance of the classical action (2.24) under gravity transformations is expressed by a Ward identity

$$\mathcal{W}^{gr}_{(\varepsilon,\Omega)}\Sigma = 0 \ , \tag{3.2}$$

where $\mathcal{W}^{gr}_{(\varepsilon,\Omega)}$ denotes the Ward operator

$$\mathcal{W}^{gr}_{(\varepsilon,\Omega)} = \int d^3x \ \sum_\varphi \left(\delta^{gr}_{(\varepsilon,\Omega)}\varphi\right) \frac{\delta}{\delta\varphi} \ . \tag{3.3}$$

Concerning the aforementioned local gravitational supersymmetry, let us consider the following infinitesimal transformations:

[12] This local gravitational supersymmetry has nothing to do with the ordinary Wess-Zumino type supersymmetry [35].

$$
\begin{aligned}
&\delta^S_{(\xi,\theta)} c^A = -\xi^\mu A^A_\mu \ , && \delta^S_{(\xi,\theta)} A^A_\mu = \varepsilon_{\mu\nu\rho}\xi^\nu \chi^{\rho A} \ , \\
&\delta^S_{(\xi,\theta)} \chi^{\mu A} = \delta^S_{(\xi,\theta)} \gamma^{\mu A} = -\xi^\mu \tau^A \ , && \delta^S_{(\xi,\theta)} \tau^A = 0 \ , \\
&\delta^S_{(\xi,\theta)} B^A = \delta^{gr}_{(\xi,\theta)} \bar{c}^A \ , && \delta^S_{(\xi,\theta)} \bar{c}^A = 0 \ , \\
&\delta^S_{(\xi,\theta)} \hat{e}^a_\mu = \delta^{gr}_{(\xi,\theta)} e^a_\mu \ , && \delta^S_{(\xi,\theta)} e^a_\mu = 0 \ ,
\end{aligned}
\tag{3.4}
$$

where ξ^μ and θ^{ab} are the infinitesimal parameters of superdiffeomorphisms and super-Lorentz transformations carrying ghost number +2.

Analogously to (3.3), the Ward operator related to (3.4) is given by

$$
\mathcal{W}^S_{(\xi,\theta)} = \int d^3x \ \sum_\varphi \left(\delta^S_{(\xi,\theta)}\varphi\right) \frac{\delta}{\delta\varphi} \ . \tag{3.5}
$$

After some calculations the corresponding Ward identity takes the form

$$
\mathcal{W}^S_{(\xi,\theta)}\Sigma = \Delta^{cl}_{(\xi)} + \int d^3x \ \xi^\lambda s(e^a_\lambda e_{\mu a}\Xi^\mu) \ , \tag{3.6}
$$

where the classical breaking writes[13]

$$
\Delta^{cl}_{(\xi)} = \int d^3x \ \left[-\gamma^{\mu A}\mathcal{L}_\xi A^A_\mu + \tau^A \mathcal{L}_\xi c^A - \varepsilon_{\mu\nu\rho}\xi^\mu \gamma^{\nu A} s(eE^\rho_a E^{\sigma a}\partial_\sigma \bar{c}^A)\right] \ , \tag{3.7}
$$

and the field polynomial Ξ^μ stands for

$$
\Xi^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho}(\partial_\nu \bar{c}^A)(\partial_\rho \bar{c}^A) \ . \tag{3.8}
$$

This means that the local gravitational supersymmetry is broken by two kinds of terms. One linear in the quantum fields and a second which is quadratic and corresponds to the hard breaking. In the renormalization procedure, the latter needs more attention. It can be absorbed in the original action (2.24) by coupling it to two further auxiliary fields, namely M_μ and L_μ, forming a BRS doublet [32]

$$
sM_\mu = L_\mu \ , \quad sL_\mu = 0 \ . \tag{3.9}
$$

The corresponding action term writes

$$
\Sigma_{L,M} = -\int d^3x \ (L_\mu \Xi^\mu - M_\mu s\Xi^\mu) \ . \tag{3.10}
$$

This leads to the following modified total action

$$
\Sigma = \Sigma_{CS} + \Sigma_{ext} + \Sigma_{gf} + \Sigma_{L,M} \ , \tag{3.11}
$$

[13]Let us remark, that the breaking does not depend on the super-Lorentz parameter θ altough it is present in (3.4).

which fulfills the Ward identity

$$\mathcal{W}^S_{(\xi,\theta)}\Sigma = \Delta^{cl}_{(\xi)} \ . \tag{3.12}$$

The hard breaking of (3.6) is now controlled by the auxiliary fields M_μ and L_μ.

The local gravitational supersymmetry transformations are given by

$$\begin{aligned} \delta^S_{(\xi,\theta)} L_\mu &= \mathcal{L}_\xi M_\mu + \hat{e}^a_\mu e_{\nu a}\xi^\nu + e^a_\mu \hat{e}_{\nu a}\xi^\nu \ , \\ \delta^S_{(\xi,\theta)} M_\mu &= -e^a_\mu e_{\nu a}\xi^\nu \ . \end{aligned} \tag{3.13}$$

The contributions of these new fields L_μ and M_μ have to be incorporated in the Ward operators of gravity and gravitational supersymmetry in such a way that the summation over the fields φ in (3.3) and (3.5) includes L and M.

Moreover, the Slavnov identity (2.25) reads now:

$$\mathcal{S}(\Sigma) = \int d^3x \left(\frac{\delta\Sigma}{\delta\gamma^{\mu A}}\frac{\delta\Sigma}{\delta A^A_\mu} + \frac{\delta\Sigma}{\delta\tau^A}\frac{\delta\Sigma}{\delta c^A} + B^A\frac{\delta\Sigma}{\delta\bar{c}^A} + \hat{e}^a_\mu\frac{\delta\Sigma}{\delta e^a_\mu} + L_\mu\frac{\delta\Sigma}{\delta M_\mu}\right) = 0 \ . \tag{3.14}$$

We display now the resultant linear algebra obeyed by the three operators introduced before, namely the linearized Slavnov operator $\mathcal{S}_\Sigma$, the Ward operators of gravity $\mathcal{W}^{gr}$ and of gravitational supersymmetry $\mathcal{W}^S$:

$$\begin{aligned} \mathcal{S}_\Sigma\mathcal{S}_\Sigma &= 0 \ , \\ \left\{\mathcal{S}_\Sigma, \mathcal{W}^{gr}_{(\varepsilon,\Omega)}\right\} &= 0 \ , \\ \left\{\mathcal{W}^{gr}_{(\varepsilon,\Omega)}, \mathcal{W}^{gr}_{(\varepsilon',\Omega')}\right\} &= -\mathcal{W}^{gr}_{(\{(\varepsilon,\Omega),(\varepsilon',\Omega')\})} \ , \\ \left\{\mathcal{S}_\Sigma, \mathcal{W}^S_{(\xi,\theta)}\right\} &= \mathcal{W}^{gr}_{(\xi,\theta)} \ , \\ \left\{\mathcal{W}^{gr}_{(\varepsilon,\Omega)}, \mathcal{W}^S_{(\xi,\theta)}\right\} &= -\mathcal{W}^S_{([(\varepsilon,\Omega),(\xi,\theta)])} \ , \\ \left\{\mathcal{W}^S_{(\xi,\theta)}, \mathcal{W}^S_{(\xi',\theta')}\right\} &= 0 \ , \end{aligned} \tag{3.15}$$

where the new Ward operators on the r.h.s. are explicitly given by[14]

$$\mathcal{W}^{gr}_{(\{(\varepsilon,\Omega),(\varepsilon',\Omega')\})} = \int d^3x \ \sum_\varphi \left\{\delta^{gr}_{(\varepsilon,\Omega)}, \delta^{gr}_{(\varepsilon',\Omega')}\right\}\varphi\frac{\delta}{\delta\varphi} \ , \tag{3.16}$$

$$\mathcal{W}^S_{([(\varepsilon,\Omega),(\xi,\theta)])} = \int d^3x \left[\varepsilon_{\mu\nu\rho}(\mathcal{L}_\varepsilon\xi^\nu)\chi^{\rho A}\frac{\delta}{\delta A^A_\mu} - (\mathcal{L}_\varepsilon\xi^\mu)A^A_\mu\frac{\delta}{\delta c^A} + [\mathcal{L}_\varepsilon, \mathcal{L}_\xi]\bar{c}^A\frac{\delta}{\delta B^A}\right.$$

[14]Remark that $\mathcal{L}_\varepsilon\varepsilon'^\nu = \{\varepsilon,\varepsilon'\}^\nu$ and $\mathcal{L}_\varepsilon\xi^\nu = [\varepsilon,\xi]^\nu$.

$$-(\mathcal{L}_\varepsilon\xi^\mu)\tau^A\frac{\delta}{\delta\gamma^{\mu A}} + [\delta^{gr}_{(\varepsilon,\Omega)},\delta^{gr}_{(\xi,\theta)}]e^a_\mu\frac{\delta}{\delta\hat{e}^a_\mu} - e^a_\mu e_{\nu a}(\mathcal{L}_\varepsilon\xi^\nu)\frac{\delta}{\delta M_\mu}$$

$$+\Big([\mathcal{L}_\varepsilon,\mathcal{L}_\xi]M_\mu + (\hat{e}^a_\mu e_{\nu a} + e^a_\mu\hat{e}_{\nu a})(\mathcal{L}_\varepsilon\xi^\nu)\Big)\frac{\delta}{\delta L_\mu}\Bigg] \ . \qquad (3.17)$$

Starting with the gravitational invariances of the total action, we have constructed a local gravitational supersymmetry, which together with the Slavnov operator form a closed linear graded algebra.

4 Stability and Finiteness

We are now in position to address the problem of quantizing the theory. It is well-known, that this procedure can be achieved by solving the following independent problems. The stability problem of the theory leads to the discussion of the most general deformation of the classical action induced by quantum corrections and the anomaly problem with respect to a generalized nilpotent symmetry operator, which is related to the validity of the classical symmetries at the quantum level.

Let us hence start with the classical action

$$\Sigma = \Sigma_{CS} + \Sigma_{ext} + \Sigma_{gf} + \Sigma_{L,M} \ ,$$

we have constructed in the previous sections as a solution of:

1. the gauge condition[15] (2.27) ,
2. the ghost equation[16] (2.28) ,
3. the Slavnov identity (3.14) ,
4. the Ward identity for gravity (3.2) ,
5. the Ward identity for gravitational supersymmetry (3.12) ,

[15]Through the presence of L and M the gauge condition is modified into:

$$\frac{\delta\Sigma}{\delta B^A} = \partial_\mu(eE^\mu_a E^{\nu a}A^A_\nu) + \varepsilon^{\mu\nu\rho}(\partial_\mu M_\nu)\partial_\rho\bar{c}^A \ . \qquad (4.1)$$

[16]Also for the ghost equation one has now:

$$\left(\frac{\delta}{\delta\bar{c}^A} + \partial_\mu(eE^\mu_a E^{\nu a}\frac{\delta}{\delta\gamma^{\nu A}})\right)\Sigma = -\partial_\mu\left(s(eE^\mu_a E^{\nu a})A^A_\nu\right) + \varepsilon^{\mu\nu\rho}\partial_\mu(M_\nu\partial_\rho B^A - L_\nu\partial_\rho\bar{c}^A) \ . \qquad (4.2)$$

6. the antighost equation (2.30) .

In order to study the influence of quantum corrections, we now look at the most general action Σ', which fulfills the same set of constraints. More precisely, we consider

$$\Sigma' = \Sigma + \Delta \ ,$$

where the perturbation Δ is an integrated local field polynomial of dimension zero and ghost number zero. The latter is constrained by:

$$\frac{\delta\Delta}{\delta B^A} = 0 \ , \tag{4.3}$$

$$\left(\frac{\delta}{\delta\bar{c}^A} + \partial_\mu (e E^\mu_a E^{\nu a} \frac{\delta}{\delta\gamma^{\nu A}})\right) \Delta = 0 \ , \tag{4.4}$$

$$\mathcal{S}_\Sigma \Delta = 0 \ , \tag{4.5}$$

$$\mathcal{W}^{gr}_{(\varepsilon,\Omega)}\Delta = 0 \ , \tag{4.6}$$

$$\mathcal{W}^{S}_{(\xi,\theta)}\Delta = 0 \ , \tag{4.7}$$

$$\int d^3x \left(\frac{\delta\Delta}{\delta c^A} + \lambda f^{ABC}\bar{c}^B \frac{\delta\Delta}{\delta B^C}\right) = 0 \ . \tag{4.8}$$

At this point we can notice that, due to the first two equations (4.3) and (4.4), the perturbation Δ is independent of B, and that it depends on the antighost $\bar{c}$ and the external field γ^μ through the combination χ^μ (2.29), *i.e.* $\Delta = \Delta(A, c, \chi, \tau, e^a, \hat{e}^a, M, L)$. Therefore, the last equation (4.8) reduces to

$$\int d^3x \ \frac{\delta\Delta}{\delta c^A} = 0 \ .$$

The remaining three equations can be condensed into a single cohomology problem

$$\delta\Delta = 0 \ , \tag{4.9}$$

where[17]

$$\delta = \mathcal{S}_\Sigma + \mathcal{W}^{gr}_{(\varepsilon,\Omega)} + \mathcal{W}^{S}_{(\xi,\theta)} \ . \tag{4.10}$$

Indeed, δ can be transformed into a coboundary operator ($\delta^2 = 0$) if we let it act on the infinitesimal parameters $(\varepsilon, \xi, \theta, \Omega)$ in the following way:

$$\delta\varepsilon^\mu = \frac{1}{2}\{\varepsilon, \varepsilon\}^\mu - \xi^\mu \ , \tag{4.11}$$

$$\delta\xi^\mu = [\varepsilon, \xi]^\mu \ , \tag{4.12}$$

$$\delta\Omega^a{}_b = \Omega^a{}_c\Omega^c{}_b - \theta^a{}_b + \mathcal{L}_\varepsilon\Omega^a{}_b \ , \tag{4.13}$$

$$\delta\theta^a{}_b = -\theta^a{}_c\Omega^c{}_b + \Omega^a{}_c\theta^c{}_b + \mathcal{L}_\varepsilon\theta^a{}_b - \mathcal{L}_\xi\Omega^a{}_b \ . \tag{4.14}$$

The full list of ghost numbers and form degrees is given in Table 1.

[17] Let us remind that the two Ward operators $\mathcal{W}^{gr}_{(\varepsilon,\Omega)}$ and $\mathcal{W}^{S}_{(\xi,\theta)}$ carry ghost number one, just as the linearized Slavnov operator $\mathcal{S}_\Sigma$.

	A	c	$\tilde{\chi}$	$\tilde{\tau}$	M	L	e	$\hat{e}$	d	ε	ξ	Ω	θ
Dimension	1	0	2	3	-1	-1	0	0	1	-1	-1	0	0
ΦΠ	0	1	-1	-2	1	2	0	1	0	1	2	1	2
Form degree	1	0	2	3	1	1	1	1	1	0	0	0	0

Table 1: Dimensions, ghost numbers and form degrees.

In order to solve the cohomology problem, we will use the following general result: the cohomology of a general non-homogenous[18] coboundary operator δ is isomorphic to a subspace of the cohomology of its homogenous part [24, 36, 37, 38]. Therefore, we first split the operator δ as

$$\delta = \delta_0 + \delta_1 \ ,$$

where δ_0 is the part of δ which does not increase the homogeneity degree whereas δ_1 contains the remaining part. One can easily check that both δ_0 and δ_1 are nilpotent

$$\delta_0^2 = \{\delta_0, \delta_1\} = \delta_1^2 = 0 \ .$$

In a first step, we look for the solution of the following simplified cohomology problem:

$$\delta_0 \Delta(A, c, \chi, \tau, e^a, \hat{e}^a, M, L) = 0 \ , \tag{4.15}$$

where the δ_0 transformations are just the homogeneity preserving part of the transformation δ, that is

$$\begin{aligned}
\delta_0 A_\mu^A &= -\partial_\mu c^A \ , \\
\delta_0 c^A &= 0 \ , \\
\delta_0 \chi^{\mu A} &= -\varepsilon^{\mu\nu\rho} \partial_\nu A_\rho^A \ , \\
\delta_0 \tau^A &= -\partial_\mu \chi^{\mu A} \ , \\
\delta_0 e_\mu^a &= \hat{e}_\mu^a \ , \qquad & \delta_0 \hat{e}_\mu^a &= 0 \ , \\
\delta_0 M_\mu &= L_\mu \ , \qquad & \delta_0 L_\mu &= 0 \ , \\
\delta_0 \varepsilon^\mu &= -\xi^\mu \ , \qquad & \delta_0 \xi^\mu &= 0 \ , \\
\delta_0 \Omega^a{}_b &= -\theta^a{}_b \ , \qquad & \delta_0 \theta^a{}_b &= 0 \ ,
\end{aligned} \tag{4.16}$$

where all the fields which appear in doublets are out of the cohomology [24]. The transformation laws for the relevant fields, in the space of forms, are

$$\begin{aligned}
\delta_0 A^A &= dc^A \ , \qquad & \delta_0 c^A &= 0 \ , \\
\delta_0 \tilde{\chi}^A &= -dA^A \ , \qquad & \delta_0 \tilde{\tau}^A &= d\tilde{\chi}^A \ .
\end{aligned} \tag{4.17}$$

Thus, we have to look at the integrated polynomial $\Delta(A, c, \tilde{\chi}, \tilde{\tau})$ with dimension 0 and ghost number 0:

$$\Delta = \int_{\mathcal{M}} f_3^0 \ , \tag{4.18}$$

[18]We speak here about homogeneity in the field, a degree 1 is attributed to all the fields of the theory including the parameters.

where we use the notation f_p^q for a local polynomial of form degree p and ghost number q. The cohomology problem (4.15) is equivalent to

$$\delta_0 f_3^0 = -df_2^1 \ . \tag{4.19}$$

At this point, we use the facts that the operator δ_0 is nilpotent, that it anticommutes with the external derivative d and that the cohomology of d is trivial in the space of local field polynomials [24, 39]. This allows us to write the following tower of descent equations

$$\begin{aligned} \delta_0 f_2^1 &= -df_1^2 \ , \\ \delta_0 f_1^2 &= -df_0^3 \ , \\ \delta_0 f_0^3 &= 0 \ . \end{aligned} \tag{4.20}$$

The general solution for f_0^3 in (4.20) is a multiple of $f^{ABC}c^Ac^Bc^C$ which is the only polynomial in the field which has form degree 0 and ghost number 3. Then, at the level of f_3^0, this gives the following local solution

$$f_3^0 = \alpha f^{ABC}\left(-A^AA^BA^C + 6A^A\tilde{\chi}^Bc^C + 3\tilde{\tau}^Ac^Bc^C\right) \ ,$$

where trivial terms of the form $(\delta_0 l_3^{-1} + dl_2^0)$ have been droped out and α is some constant.

Therefore, at the integrated level, the solution of the homogenous cohomology problem takes the form

$$\Delta = \alpha\Delta_c + \delta_0\tilde{\Delta} \ , \tag{4.21}$$

with

$$\Delta_c = \int_{\mathcal{M}} f^{ABC}\left(-A^AA^BA^C + 6A^A\tilde{\chi}^Bc^C + 3\tilde{\tau}^Ac^Bc^C\right) \ . \tag{4.22}$$

At this point, we have to extend our result to the full cohomology operator (4.10). It is straightforward to check that Δ_c also satisfies $\delta\Delta_c = 0$. Therefore, the solution of the full δ cohomology problem takes the form

$$\Delta = \alpha\Delta_c + \bar{\Delta} \ , \tag{4.23}$$

with $\bar{\Delta} = \delta\hat{\Delta}$.

At this step, it is more simpler to discuss the problem at the level of $\bar{\Delta}$ than the one of $\hat{\Delta}$. Indeed, we know that $\bar{\Delta}$ directly satisfies the constraints (4.3)-(4.8). The same restrictions for $\hat{\Delta}$ have to be check case by case, because they strongly depend on the coboundary operator δ. Therefore, let us first look at all the possible terms with ghost number 0 and form degree 3 one can built with the set of fields in Table 1. These terms are[19]:

$$f^{ABC}A^AA^BA^C \ ,$$

$$f^{ABC}A^Ac^B\tilde{\chi}^C \ ,$$

[19] Actually, there are further terms depending on the parameters of the transformations which may be possible. Their irrelevance is analyzed in the appendix.

$$f^{ABC} c^A c^B \tilde{\tau}^C \ , \tag{4.24}$$
$$A^A dA^A \ ,$$
$$c^A d\tilde{\chi}^A \ .$$

The first three terms are the non-trivial ones, their invariant combination being Δ_c. Therefore, the most general solution for $\bar{\Delta}$ is given by

$$\bar{\Delta} = \int_{\mathcal{M}} \left[\beta_1 \left(A^A dA^A \right) + \beta_2 \left(c^A d\tilde{\chi}^A \right) \right] \ . \tag{4.25}$$

The parameters β_1 and β_2 have to be fixed to zero by imposing the invariance of (4.23):

$$\delta \left(\alpha \Delta_c + \bar{\Delta} \right) = \delta \int_{\mathcal{M}} \left[\beta_1 \left(A^A dA^A \right) + \beta_2 \left(c^A d\tilde{\chi}^A \right) \right] = 0 \ . \tag{4.26}$$

Then, the most general deformation of the classical action is just $\alpha \Delta_c$ which would correspond to a continuous renormalization of the parameter λ of the model. But such a term is forbidden by the antighost equation (4.8). Therefore, the classical action admits no deformation.

The remaining part to examine concerns the anomalies. It is well-known, that anomalies would correspond to a term of ghost number one. Thus, in our case, it would be of the form $\Delta = \int_{\mathcal{M}} f_3^1$. In order to determine such a term, let us use the same method as before. That is, to look first at the non-trivial solution of the following descent equations:

$$\delta_0 f_3^1 = -df_2^2 \ , \tag{4.27}$$
$$\delta_0 f_2^2 = -df_1^3 \ , \tag{4.28}$$
$$\delta_0 f_1^3 = -df_0^4 \ , \tag{4.29}$$
$$\delta_0 f_0^4 = 0 \ . \tag{4.30}$$

The only possible zero form for f_0^4 is

$$f_0^4 = t^{[ABCD]} c^A c^B c^C c^D \ ,$$

which is zero due to the absence of such a four rank, totally antisymmetric, invariant tensor. Therefore, the triviality of the δ_0 cohomology implies that of the full δ one and this concludes the proof of the absence of anomalies.

The consequences of the previous analysis are the following. The absence of counterterms is responsible for the stability of the extended classical action, *i.e.* the action which satisfies the constraints listed at the beginning of this section. This guarantees that the number of parameters of the theory remains fixed. In such a case, the renormalizability of the theory is proved by showing that the set of constraints are also valid at the quantum level. It is well-known, that the gauge condition (2.27), the ghost equation (2.28) and the antighost equation (2.30) fulfill this requirement [34, 40]. Nevertheless, in our case, we see that the hard breaking of (3.6) enforces us to extend our model by including the field doublet (L, M). Thus, the gauge condition and the ghost equation are also extended to (4.1) and (4.2) in order to be consistent. These modifications are linear in the quantum fields. Therefore the resultant conditions remain renormalizable.

The remaining constraints, *i.e.* the Slavnov and Ward identities, due to the absence of anomalies, also survive at the quantum level. Thus, the theory is UV finite.

Conclusions

The Chern-Simons theory in curved space-time [23] was reanalyzed in the vielbein formalism. We have seen the explicit independence of the model with respect to the affine spin connection and the torsion. The main extension lies in the additional local Lorentz symmetry and its corresponding local supersymmetry. The resultant graded algebra generated by the Ward operators has the same structure, but now with the local gravity transformations and the local gravitational supersymmetry instead of diffeomorphisms and superdiffeomorphisms respectively.

Although the additional exact symmetries require two further parameters, they do not change the stability property. They are also free of anomalies.

Then the UV finiteness follows, provided the existence of a consistent substraction scheme. This require that the manifold must be topologically equivalent to a flat space and possess an asymptotically flat metric.

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Appendix: Analysis of the Trivial Part of the Counterterms

The most general deformation of the classical action is given by

$$\Delta = \alpha\Delta_c + \bar{\Delta} \quad , \quad \bar{\Delta} = \delta\hat{\Delta} \ , \tag{A.1}$$

where Δ is an integrated local polynomial in the fields with dimension 0 and ghost number 0, which is the solution of the full cohomology problem for the operator δ defined in (4.10):

$$\delta\Delta = 0 \ . \tag{A.2}$$

The non-trivial part Δ_c has already been determined (4.22). The perturbation Δ and therefore also $\bar{\Delta}$ cannot depend on the fields (ε, Ω) and (ξ, θ), since they are nothing else than infinitesimal parameters of the corresponding field transformations. Furthermore, both have to fulfill the constraints (4.3) and (4.4). This means that $\bar{\Delta} = \bar{\Delta}(A, c, \chi, \tau, e^a, \hat{e}^a, M, L)$. But, due to the fact that the full cohomology operator δ (4.10) is nilpotent only if it acts on the parameters, it could be possible that $\hat{\Delta}$ depends on the parameter fields.

In the following, we want to discuss the trivial part of the solution of the cohomology problem at the level of $\hat{\Delta}$. To do this, we are looking first to the validity of the constraints (4.3) and (4.4) for $\hat{\Delta}$.

Let us introduce the bosonic filtering operator

$$\mathcal{N} = \int d^3x \left(B^A \frac{\delta}{\delta B^A} + \bar{c}^A \frac{\delta}{\delta \bar{c}^A} \right) . \tag{A.3}$$

Due to the independence of Δ with respect to B (4.3) and $\bar{c}$ (4.4), this implies that

$$\mathcal{N}\Delta = 0 . \tag{A.4}$$

If we define now the fermionic operator

$$\mathcal{R} = \int d^3x \left(\bar{c}^A \frac{\delta}{\delta B^A} \right) , \tag{A.5}$$

it follows from (4.3) that $\mathcal{R}\Delta = 0$. Therefore, the anticommutator between δ and $\mathcal{R}$ acting on Δ yields

$$\{\delta, \mathcal{R}\} \Delta = \mathcal{N}\Delta . \tag{A.6}$$

Here, it is important to consider the counterterm Δ and not the full action Σ. Indeed, if we act with the same combination on Σ, we would get some contributions from the breaking in (3.12), but for the counterterm Δ such breaking is not present. Therefore, using the Jacobi identity, we get the vanishing commutator

$$[\mathcal{N}, \delta] \Delta = 0 , \tag{A.7}$$

which is precisely the starting point for the general procedure. Following [39], it is clear that the expansion of Δ in terms of eigenvectors of $\mathcal{N}$

$$\Delta = \sum_{n=0}^{\bar{n}} \Delta^{(n)} \quad , \quad \mathcal{N}\Delta^{(n)} = n\Delta^{(n)} , \tag{A.8}$$

leads to

$$\delta\Delta^{(n)} = 0 . \tag{A.9}$$

With the equations (A.1), (A.4) and (A.7) one finds

$$\mathcal{N}\bar{\Delta} = 0 = \mathcal{N}\delta\hat{\Delta} = \delta\mathcal{N}\hat{\Delta} = \delta \sum_{n=1}^{\bar{n}} n\hat{\Delta}^{(n)} , \tag{A.10}$$

which implies

$$\delta\hat{\Delta}^{(n)} = 0 \qquad \forall n \geq 1 . \tag{A.11}$$

Therefore, the only contribution to the counterterm has the form

$$\bar{\Delta} = \delta\hat{\Delta}^{(0)} \,, \tag{A.12}$$

with $\mathcal{N}\hat{\Delta}^{(0)} = 0$. This means that also $\hat{\Delta}$ is independent of the fields B and $\bar{c}$, but they can depend on the parameter fields, i.e. $\hat{\Delta} = \hat{\Delta}(A, c, \chi, \tau, e^a, \hat{e}^a, M, L, \varepsilon, \Omega, \xi, \theta)$.

Indeed, for the complete set of possible terms with form degree 3 and ghost number 1 to $\hat{\Delta}$ one has:

$$\begin{aligned}
\Lambda_1 &= A^A \tilde{\chi}^A \,, \\
\Lambda_2 &= c^A \tilde{\tau}^A \,, \\
\Lambda_3 &= i_\varepsilon A^A \tilde{\tau}^A \,, \\
\Lambda_4 &= i_\varepsilon \tilde{\chi}^A \tilde{\chi}^A \,, \\
\Lambda_5 &= i_\varepsilon i_\varepsilon \tilde{\chi}^A \tilde{\tau}^A \,, \\
\Lambda_6 &= i_\varepsilon i_\varepsilon i_\varepsilon \tilde{\tau}^A \tilde{\tau}^A \,,
\end{aligned} \tag{A.13}$$

where i_ε denotes the inner derivative with respect to ε (e.g. $i_\varepsilon A^A = \varepsilon^\mu A^A_\mu$).

At the integrated level

$$\hat{\Delta}_i = \int_{\mathcal{M}} \Lambda_i \,, \qquad i = 1, ..., 6 \tag{A.14}$$

the action of the operator δ yields

$$\delta\hat{\Delta}_1 = \int_{\mathcal{M}} \left[-A^A dA^A + (dc^A)\tilde{\chi}^A + (i_\xi A^A)\tilde{\tau}^A + (i_\xi \tilde{\chi}^A)\tilde{\chi}^A \right] \,, \tag{A.15}$$

$$\delta\hat{\Delta}_2 = \int_{\mathcal{M}} \left[-(i_\xi A^A)\tilde{\tau}^A - (dc^A)\tilde{\chi}^A \right] \,, \tag{A.16}$$

$$\delta\hat{\Delta}_3 = \int_{\mathcal{M}} \left[-(i_\xi A^A)\tilde{\tau}^A + (i_\varepsilon dc^A)\tilde{\tau}^A + (i_\varepsilon i_\xi \tilde{\chi}^A)\tilde{\tau}^A - (i_\varepsilon A^A) d\tilde{\chi}^A \right] \,, \tag{A.17}$$

$$\delta\hat{\Delta}_4 = \int_{\mathcal{M}} \left[-(i_\xi \tilde{\chi}^A)\tilde{\chi}^A + 2(i_\varepsilon i_\xi \tilde{\chi}^A)\tilde{\tau}^A - 2(i_\varepsilon dA^A)\tilde{\chi}^A \right] \,, \tag{A.18}$$

$$\delta\hat{\Delta}_5 = \int_{\mathcal{M}} \left[-2(i_\varepsilon i_\xi \tilde{\chi}^A)\tilde{\tau}^A - (i_\varepsilon i_\varepsilon i_\xi \tilde{\tau}^A)\tilde{\tau}^A - (i_\varepsilon i_\varepsilon \tilde{\chi}^A) d\tilde{\chi}^A - (i_\varepsilon i_\varepsilon dA^A)\tilde{\tau}^A \right] \,, \tag{A.19}$$

$$\delta\hat{\Delta}_6 = \int_{\mathcal{M}} \left[-3(i_\varepsilon i_\varepsilon i_\xi \tilde{\tau}^A)\tilde{\tau}^A - 2(i_\varepsilon i_\varepsilon i_\varepsilon \tilde{\tau}^A) d\tilde{\chi}^A \right] \,. \tag{A.20}$$

The last expression $\delta\hat{\Delta}_6$ cannot contribute to $\bar{\Delta}$, because it has no corresponding term, which could cancel the parameter dependence of the second term on the r.h.s. of (A.20). Now this implies that also $\delta\hat{\Delta}_5$ is forbidden due to the second term in (A.19). Analogously, owing to the second term in (A.17), we have to reject $\delta\hat{\Delta}_3$, which demands the non-consideration of $\delta\hat{\Delta}_4$ caused by the second term. Finally, from the fourth term on the r.h.s. of (A.15) follows that one has to drop $\delta\hat{\Delta}_1$ and therefore also the last expression $\delta\hat{\Delta}_2$ does not contribute to $\bar{\Delta}$. Thus, we have shown the irrelevance of the terms (A.15)-(A.20).

At the end, let us remark that in our case the discussion of possible counterterms at the level of $\bar{\Delta} = \delta\hat{\Delta}$, as done in section 4, is much simpler than at the one of $\hat{\Delta}$.

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