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Autor: Bonneau, Guy

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B.R.S. Renormalization of Some On-Shell Closed Algebras of Symmetry transformations. The Example of Supersymmetric Non-Linear σ Models: 1) the N=1 Case

By Guy Bonneau

Laboratoire de Physique Théorique et des Hautes Energies,
Unité associée au CNRS URA 280, Université Paris 7,
2 Place Jussieu, 75251 Paris Cedex 05.

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Abstract. In order to study in a regularization free manner the renormalizability of d=2 supersymmetric non-linear σ models, one has to use the algebraic BRS methods ; moreover, in the absence of an off-shell formulation, one often has to deal with open algebras. We then recall in a pedagogical and non technical manner the standard methods used to handle these questions and illustrate them on N=1 supersymmetric non-linear σ model in component fields, giving the first rigorous proof of their renormalizability. In the special case of compact homogeneous manifolds (non-linear σ model on a coset space G/H), we obtain the supersymmetric extension of the analysis done some years ago in the bosonic case.

A further publication will be devoted to extended supersymmetry.

1 Introduction

The quantization of extended supersymmetry raises the difficulty of an "on-shell" formalism. Indeed, if one leaves aside harmonic superspace [1] where firm rules for quantization [1] are not at hand [2] on the contrary of ordinary superspace [3], one has to deal with (super)symmetry

[1] *i.e.* a subtraction algorithm insuring the locality of the counterterms.

transformations that are non-linear and close only on-shell. This problem was adressed in [4] by O. Piguet and K. Sibold for the Wess-Zumino model as a "toy-model" and, in a still uncomplete way, by P. Breitenlohner and D. Maison [5] for supersymmetric Yang-Mills in the Wess-Zumino gauge.

In this series, as new steps, we analyse the d=2, N=1 supersymmetric non-linear σ model without auxiliary fields and the N=2 and 4 cases in N=1 superfields.

On the other hand, and as should be by now well known [6], there is no consistent regularization that respects non-linear supersymmetry (dimensional reduction being mathematically inconsistent [7]). Then, if one wants to settle on a firm basis e.g. the finiteness of N=4 supersymmetric non-linear σ models, one needs a regulator free treatment : we shall then use the B.R.S. cohomological methods ([8],[9]).

The first task is to write a Slavnov identity. In the presence of on-shell closed algebras, this has first been studied by Kallosh [10], de Wit and van Holten [11] and later on systematised by Batalin and Vilkovisky [12] ; as in this work we intend to offer a pedagogical and self-contained point of view, we prefer to give here a concrete method for the construction of the effective classical action and of the Slavnov identity, for a special class of on-shell closed algebras (subsection 2.1). We illustrate this method with the examples of the d=2, N=1 supersymmetric non-linear σ model without auxiliary fields (subsection 2.2) and the N= 4 case in N=1 superfields (subsection 2.3).

In the same pedagogical and completeness purposes, we then recall in section 3 the essential cohomological tools of the algebraic approach to the renormalizability proof *à la* B.R.S., and explain with some details the power of the "filtration" method for the cohomological analyses of highly non-linear nilpotent linearized Slavnov operators S_L (the so-called spectral sequence method [13]). Some grading being introduced ("filtration" operator), S_L^0, the lowest order of the Slavnov operator is still nilpotent and its cohomology is simpler to find. In subsection 3.2, we then sketch the proof of the "filtration" theorem which asserts the isomorphism between this cohomology space and the one of the full Slavnov operator, in the special case where the cohomology of S_L^0 is empty in the Faddeev-Popov charged sectors [8]. An appendix gives a sketch of the modifications of the "filtration" theorem when the cohomology of S_L^0 is not empty in the Faddeev-Popov charged sectors ([8],[14]). We emphasize that in our work, we shall consider integrated cohomology (in the space of local functionals in the fields, sources, ghosts and their derivatives) and not local cohomology (in the Fock space of local polynomials in the fields, sources, ghosts and their derivatives).

The method is then exemplified in section 4 on the d=2, N=1 supersymmetric non-linear σ model in component fields (without auxiliary fields) leaving the N= 4 case in N=1 superfields (and as an intermediary step the N= 2 supersymmetry) to next publications [15]. Another interesting case would be super Yang-Mills theories in 4 space-time dimensions.

Of course, we are here mainly interested in the renormalization of the supersymmetry transformations : as discussed by Friedan [16], the action of a non-linear σ model may be identified with a distance on a Riemannian manifold $\mathcal{M}$, the metric depending *a priori* on an infinite number of parameters. One then speaks of "renormalizability in the space of metrics" or "*à la* Friedan". When there exist extra isometries, for example in the case of the non-linear σ models on coset spaces (homogeneous manifolds), the number of such physical parameters becomes finite and in ref. [8] we have proved the U.V. renormalizability of these isometries in the purely bosonic case. The present work gives the necessary supersymmetric extension in section 5. On the other hand, in the absence of isometries, our aim is the proof that no extra difficulty occurs in the supersymmetric extension of these non-linear σ models. We shall then prove in section 4 that the cohomology of S_L^0 is empty in the Faddeev-Popov charged sectors, which ensures the renormalizability of the theory " in the space of metrics".

Moreover, we only consider here the ultraviolet renormalizability of the d=2 supersymmetric non-linear σ models : of course, one has also to deal with infrared divergences. This would require the addition of an infrared regulator which of course breaks the symmetries, but only softly, and then does not affect our results on "hard" divergences and absence of an anomaly for the supersymmetry in 2 dimensions. In the special case of the non-linear σ models on coset spaces (homogeneous manifolds) studied in Section 5, the method of ref. [8], which uses a definite covariance under the isometries of the infrared regulating mass-term, will ensure the infrared finiteness of observables.

2 Slavnov identity for on-shell algebras

2.1 General method

Let $\Phi^a(x)$ and $A[\Phi]$ respectively be a set of fields (bosonic or fermionic) and the classical action of a theory. We want to analyse a global, non-linear symmetry transformation

$$\delta\Phi^a(x) = \epsilon^i W_i \Phi^a(x) \tag{2.1}$$

whose generators satisfy the following (anti)commutation relations :

$$[W_i, W_j]\Phi^a(x) = f_{ij}^k W_k \Phi^a(x) + X_{ij}^{ab} \frac{\delta A[\Phi]}{\delta\Phi^b(x)} \tag{2.2}$$

and where X_{ij}^{ab} is *a priori* a field dependent quantity [2]. This situation is a special case among the ones studied by Batalin and Vilkovisky [12], but, as we want to be as pedagogical as possible, we prefer to exemplify the method on a particular class of on-shell algebras. Of course, on mass-shell, the "algebra" (2.2) takes a canonical form.

[2] To fix the notations, we consider bosonic fields and transformations : then X_{ij}^{ab} will be antisymmetric in (i,j). Notice also that the integration over x of (2.2) multiplied by $\frac{\delta A[\Phi]}{\delta\Phi^a(x)}$ shows that if X_{ij}^{ab} was symmetric in (a,b), the algebra (2.2) could be rewritten as a closed one, with field dependent structure constants, a case that we exclude from our analysis : therefore, in the following, we restrict ourselves to the cases where X_{ij}^{ab} will be antisymmetric with respect to (a,b).
The necessary modifications for e.g. supersymmetry transformations are obvious.

As usual [9], in view of quantization one takes as classical effective action

$$\Gamma^{class.} = A + \int dx\, \gamma_a(x)[W_i\Phi^a(x)]C^i \tag{2.3}$$

where C^i and $\gamma_a(x)$ are respectively constant anticommuting Faddeev-Popov parameters and external anticommuting [3] sources for the B.R.S. variation of the fields $\Phi^a(x)$.

Then, although the W_i's transformations are generally non-linear, the variation of $\Gamma^{class.}$ will produce the commutator of two transformations and then will be expressible as a functional derivative with respect to the source γ_a. More precisely :

$$\begin{aligned}
S\Gamma &\equiv \int dx \frac{\delta\Gamma}{\delta\gamma_a(x)}\frac{\delta\Gamma}{\delta\Phi^a(x)} \\
&= \int dx[W_i\Phi^a(x)]C^i\frac{\delta A}{\delta\Phi^a(x)} + \int dx[W_i\Phi^a(x)]C^i\frac{\delta\left[\int dy\gamma_b(y)W_j\Phi^b(y)\right]}{\delta\Phi^a(x)}C^j \\
= \delta A &+ \frac{1}{2}C^iC^j\left[f_{ij}^k\frac{\delta\Gamma}{\delta C^k} - \int dx\gamma_b(x)X_{ij}^{bc}\left(\frac{\delta\Gamma}{\delta\Phi^c(x)} - \frac{\delta\left[\int dy\gamma_a(y)W_k\Phi^a(y)\right]}{\delta\Phi^c(x)}C^k\right)\right]
\end{aligned} \tag{2.4}$$

At this point we suppose the invariance of the classical action $A = A^{inv.}$. Then :

$$S\Gamma - \frac{1}{2}C^iC^jf_{ij}^k\frac{\delta\Gamma}{\delta C^k} = -\frac{1}{2}C^iC^j\int dxX_{ij}^{ab}\gamma_a(x)\frac{\delta\Gamma}{\delta\Phi^b(x)} + [3-\text{ghosts terms}]\ .$$

In order to suppress the 2-ghost terms, we modify the classical action (2.3)

$$\Gamma^{tot.} = A^{inv.} + \int dx\,\gamma_a(x)[W_i\Phi^a(x)]C^i - \frac{1}{4}\int dx\,\gamma_a(x)X_{ij}^{ab}\gamma_b(x)C^iC^j. \tag{2.5}$$

Then [4]

$$\begin{aligned}
S\Gamma^{tot.} - \frac{1}{2}C^iC^jf_{ij}^k\frac{\delta\Gamma^{tot.}}{\delta C^k} &= -\frac{1}{2}C^iC^j\int dx\left[X_{ij}^{ab} + X_{ij}^{ba}\right]\gamma_b(x)\frac{\delta\Gamma^{tot.}}{\delta\Phi^a(x)} \\
+\frac{1}{2}C^iC^jC^k\int dx&\left[X_{ij}^{bc}\gamma_b(x)\left(\gamma_a(x)W_k\Phi^a(x)\right)_{,c} - \frac{1}{2}(-f_{ij}^lX_{lk}^{ba} + W_kX_{ij}^{ba})\gamma_b(x)\gamma_a(x)\right] \\
&+\frac{1}{8}C^iC^jC^kC^l\int dx\gamma_b(x)X_{ij}^{ab}\left(X_{kl}^{cd}\gamma_c(x)\gamma_d(x)\right)_{,a}
\end{aligned} \tag{2.6}$$

The 2-ghost terms cancel due to the antisymmetry of X_{ij}^{ab} in the interchange a $\leftrightarrow$ b . The 3-ghost ones will be analysed through the Jacobi identity

$$C^iC^jC^k[W_k,[W_i,W_j]] = 0$$

giving on-shell :

$$C^iC^jC^kf_{ij}^lf_{lkn} = 0 \tag{2.7}$$

[3] See the last line of footnote 2.

[4] $(G[\Phi(x)])_{,c}$ means $\frac{\delta}{\delta\Phi^c(x)}\int dy(G[\Phi(y)])$

and then :

$$C^iC^jC^k\left(\left[-f^l_{ij}X^{ba}_{lk}+W_kX^{ba}_{ij}-X^{ca}_{ij}\left(W_k\Phi^b(x)\right)_{,c}\right]\frac{\delta A}{\delta\Phi^a(x)}+X^{ba}_{ij}W_k\left(\frac{\delta A}{\delta\Phi^a(x)}\right)\right)=0 \quad (2.8)$$

The invariance of the classical action is then used to transform equation (2.8) into :

$$C^iC^jC^k\left(\left[-f^l_{ij}X^{ba}_{lk}+W_kX^{ba}_{ij}-X^{ca}_{ij}(W_k\Phi^b(x))_{,c}\right]\frac{\delta A^{inv.}}{\delta\Phi^a(x)}\right.$$
$$\left.-X^{bc}_{ij}\int dy\frac{\delta(W_k\Phi^a(y))}{\delta\Phi^c(x)}\frac{\delta A^{inv.}}{\delta\Phi^a(y)}\right)=0. \quad (2.9)$$

Notice that when the transformation $W_k(\Phi^a(y))$ involves only the fields and not their derivatives, equation (2.9) may be "divided" by $\frac{\delta A^{inv.}}{\delta\Phi^a(x)}$ and, as a consequence, the 3-ghosts term of (2.6) vanishes.

This gives the spirit of the construction of the Slavnov identity for on-shell closed algebras; in the generic case, the total action (2.5) has to be modified by addition of 3-[and even more] ghost terms in order to obtain from equation (2.9) [and similarly obtained ones at higher order in the number of ghosts] the looked-for vanishing of the 3-[and higher order] ghost terms and the final Slavnov identity :

$$S\Gamma^{tot.}\equiv\int dx\frac{\delta\Gamma^{tot.}}{\delta\gamma_a(x)}\frac{\delta\Gamma^{tot.}}{\delta\Phi^a(x)}=\frac{1}{2}C^iC^jf^k_{ij}\frac{\delta\Gamma^{tot.}}{\delta C^k}\ . \quad (2.10)$$

In the present work, we shall show that in some interesting examples this is not necessary and that equation (2.10) will hold true with a $\Gamma^{tot.}$ given by (2.5). Moreover, due to the algebra (2.2), one expects nilpotency for the linearized Slavnov operator S_L defined through

$$S(\Gamma+\epsilon\Gamma^1)\equiv S\Gamma+\epsilon S_L\Gamma^1+\mathcal{O}(\epsilon^2)\ .$$

2.2 N=1 supersymmetric non-linear σ models in component fields

Let us exemplify this method on d=2, N=1 supersymmetric non-linear σ models in component fields $\phi^i(x),\psi^j_\pm(x)$ (i,j,.. = 1,2,..n). In light-cone coordinates and *in the absence of torsion*, the non-linear N=1 supersymmetry transformations and the invariant action respectively write

$$\begin{aligned}\delta\phi^i &= \epsilon^+\psi^i_+ + \epsilon^-\psi^i_-\\ \delta\psi^i_\pm &= i\epsilon^\pm\partial_\pm\phi^i+\epsilon^\mp\Gamma^i_{jk}\psi^j_\pm\psi^k_\mp\end{aligned} \quad (2.11)$$

$$A^{inv.}=\int d^2x\{g_{ij}(\phi)[\partial_+\phi^i\partial_-\phi^j+i(\psi^i_+D_-\psi^j_++\psi^i_-D_+\psi^j_-)]+\frac{1}{2}R_{ijkl}\psi^i_+\psi^j_+\psi^k_-\psi^l_-\} \quad (2.12)$$

where the covariant derivatives are

$$D_\pm\psi^i_\mp=\partial_\pm\psi^i_\mp+\Gamma^i_{jk}\partial_\pm\phi^j\psi^k_\mp$$

and where Γ^i_{jk} and R_{ijkl} are respectively the (symmetric) connexion and Rieman tensor associated to the target space metric $g_{ij}[\phi]$. The properties of the theory will be more transparent in tangent space where the metric is the Khrönecker δ_{ab}, the spin connexion ω^a_{bc}, the Riemann tensor R_{abcd} and the tangent space fermions $\psi^a_\pm$ are related to world space ones $\psi^i_\pm$ through the vielbeins $e^a_i[\phi]$:

$$g_{ij} = \delta_{ab} e^a_i e^b_j \ ; \ \psi^a_\pm = \psi^i_\pm e^a_i$$

$$\omega^a_{bc} = e^a_i e^j_b e^k_c \Gamma^i_{jk} - e^j_b e^i_c \partial_j e^a_i \ ; \ R_{abcd} = R_{ijkl} e^i_a e^j_b e^k_c e^l_d \text{ where } e^a_i e^i_b = \delta^a_b \ .$$

With $e^a_\pm = e^a_i \partial_\pm \phi^i$, the covariant derivatives and the invariant action are respectively :

$$D_\pm \psi^a_\mp = \partial_\pm \psi^a_\mp + \omega^a_{bc} e^b_\pm \psi^c_\mp \tag{2.13}$$

$$A^{inv.} = \int d^2x \{\delta_{ab}[e^a_+ e^b_- + i(\psi^a_+ D_- \psi^b_+ + \psi^a_- D_+ \psi^b_-)] + \frac{1}{2} R_{abcd} \psi^a_+ \psi^b_+ \psi^c_- \psi^d_-\} \ . \tag{2.14}$$

Then, the (highly non-linear) supersymmetry transformations are :

$$\begin{aligned} \delta\phi^i &= e^i_a(\epsilon^+ \psi^a_+ + \epsilon^- \psi^a_-) \\ \delta\psi^a_\pm &= i\epsilon^\pm e^a_\pm - \omega^a_{bc} \psi^c_\pm (\epsilon^+ \psi^b_+ + \epsilon^- \psi^b_-) \end{aligned} \tag{2.15}$$

and the algebra of equ.(2.2) specifies to :

$$\begin{aligned} \{W_\pm, W_\pm\}\phi^i &= 2i\partial_\pm \phi^i \\ \{W_+, W_-\}\phi^i &= 0 \\ \{W_\pm, W_\pm\}\psi^a_\pm &= 2i\partial_\pm \psi^a_\pm \\ \{W_\pm, W_\pm\}\psi^a_\mp &= 2i\partial_\pm \psi^a_\mp - \delta^{ab}\frac{\delta A^{inv.}}{\delta \psi^b_\mp} \\ \{W_+, W_-\}\psi^a_\pm &= \frac{1}{2}\delta^{ab}\frac{\delta A^{inv.}}{\delta \psi^b_\mp} \ . \end{aligned} \tag{2.16}$$

As we shall be only concerned by integrated local functionals (i.e. trivially translation invariant ones), we forget about the linear translation operators $P_\pm \equiv i\partial_\pm$, to which anti-commuting Faddeev-Popov parameters $p^\pm$ should be associated, and do not add in $\Gamma^{class.}$ of equ.(2.3) the effect of translations on the fields ϕ^i and $\psi^a_\pm$. Then the total effective action of equ.(2.5) writes:

$$\begin{aligned} \Gamma^{tot.} &= A^{inv.} + \int d^2x \{\gamma^+_a(x)[iC^+ e^a_+ + \omega^a_{bc} \psi^c_+ (C^+ \psi^b_+ + C^- \psi^b_-)] \\ &+ \gamma^-_a(x)[iC^- e^a_- + \omega^a_{bc} \psi^c_- (C^+ \psi^b_+ + C^- \psi^b_-)] + \eta_i(x)[C^+ e^i_a \psi^a_+ + C^- e^i_a \psi^a_-] \\ &+ \frac{1}{4}\delta^{ab}[\gamma^+_a(x)\gamma^+_b(x)(C^-)^2 + \gamma^-_a(x)\gamma^-_b(x)(C^+)^2 - 2\gamma^+_a(x)\gamma^-_b(x)(C^+C^-)]\} \end{aligned} \tag{2.17}$$

where $C^\pm$, $\gamma^\pm_a(x)$ and $\eta_i(x)$ are respectively commuting Faddeev-Popov parameters, commuting fermionic and anticommuting bosonic sources. Due to the simplicity of the algebra

(2.16)[5], the Slavnov identity of equ.(2.10) holds and writes :

$$\begin{aligned} S\Gamma^{tot.} &\equiv \int d^2x\{\frac{\delta\Gamma^{tot.}}{\delta\gamma_a^+}\frac{\delta\Gamma^{tot.}}{\delta\psi_+^a} + \frac{\delta\Gamma^{tot.}}{\delta\gamma_a^-}\frac{\delta\Gamma^{tot.}}{\delta\psi_-^a} + \frac{\delta\Gamma^{tot.}}{\delta\eta_i}\frac{\delta\Gamma^{tot.}}{\delta\phi^i}\} \\ &= \int d^2x\{(C^+)^2(\eta_k i\partial_+\phi^k + \gamma_a^+ i\partial_+\psi_+^a) + (C^-)^2(\eta_k i\partial_-\phi^k + \gamma_a^- i\partial_-\psi_-^a)\} \ . \end{aligned} \tag{2.18}$$

Moreover, one can also check that the linearized Slavnov operator :

$$\begin{aligned} S_L \equiv \int \ d^2x \ &\{\left(\frac{\delta\Gamma^{tot.}}{\delta\gamma_a^+}\right)\frac{\delta}{\delta\psi_+^a} + \left(\frac{\delta\Gamma^{tot.}}{\delta\gamma_a^-}\right)\frac{\delta}{\delta\psi_-^a} + \left(\frac{\delta\Gamma^{tot.}}{\delta\psi_+^a}\right)\frac{\delta}{\delta\gamma_a^+} + \left(\frac{\delta\Gamma^{tot.}}{\delta\psi_-^a}\right)\frac{\delta}{\delta\gamma_a^-} \\ &+ \left(\frac{\delta\Gamma^{tot.}}{\delta\eta_i}\right)\frac{\delta}{\delta\phi^i} + \left(\frac{\delta\Gamma^{tot.}}{\delta\phi^i}\right)\frac{\delta}{\delta\eta_i}\} \end{aligned} \tag{2.19}$$

is nilpotent : $(S_L)^2 = 0$, when acting in the space of integrated local functionals. The quantization of this theory will be studied in section 4.

2.3 N=4 supersymmetric non-linear σ models in N=1 superfields

Consider now d=2, N=4 supersymmetric non-linear σ models in N=1 superfields $\Phi^i(x,\theta)$ (i, j,.. = 1,2,..4n). In light-cone coordinates and in the absence of torsion, the non-linear N=4 supersymmetry transformations and the invariant action respectively write :

$$\begin{aligned} \delta\Phi^i &= J^i_{a\,j}(\Phi)[\epsilon_a^+ D_+\Phi^j + \epsilon_a^- D_-\Phi^j] \ , \quad a = 1,2,3. \\ A^{inv.} &= \int d^2x d^2\theta g_{ij}(\Phi)[D_+\Phi^i D_-\Phi^j] \end{aligned} \tag{2.20}$$

where the covariant derivatives $D_\pm = \frac{\partial}{\partial\theta^\pm} + i\theta^\pm\partial_\pm$ satisfy

$$\{D_\pm, D_\pm\} = 2i\partial_\pm \quad \{D_+, D_-\} = 0 \ . \tag{2.21}$$

As is well known (see for example ref.[17]), N=4 supersymmetry needs the $J^i_{a\,j}(\Phi)$ to be a set[6] of anticommuting integrable complex structures according to :

$$J^i_{a\,j}(\Phi)J^j_{b\,k}(\Phi) = -\delta_{ab}\delta^i_k + \epsilon_{abc}J^i_{c\,k}(\Phi) \ ,$$

and the invariance of the action needs the target space to be hyperkähler :

* the metric is hermitian with respect to each complex structure

$$J_a^{ij} \equiv J^i_{a\,k}g^{kj} = -J_a^{ji}$$

[5] Notice that in this N=1 supersymmetric case, an off-shell formalism actually exists.

[6] As a matter of facts, it is sufficient to have 2 anticommuting complex structures : then the product $J^i_{3\,k} \equiv J^i_{1\,j}J^j_{2\,k}$ offers a third complex structure .

* the $J^i_{a\,j}$ are covariantly constant with respect to the metric g_{ij}

$$D_k J^i_{a\,j} \equiv \partial_k J^i_{a\,j} + \Gamma^i_{kl} J^l_{a\,j} - \Gamma^l_{kj} J^i_{a\,l} = 0$$

where $\Gamma^n_{kl}[g_{ij}]$ is the (symmetric) connexion.

Then the algebra of equ.(2.2) specifies to :

$$\begin{aligned} \{W_{a\pm}, W_{b\pm}\}\Phi^i &= 2i\delta_{ab}\partial_\pm\Phi^i \\ \{W_{a+}, W_{b-}\}\Phi^i &= \epsilon_{abc} J^{ij}_c \frac{\delta A^{inv.}}{\delta\Phi^j} \end{aligned} \tag{2.22}$$

i.e. $X^{ab}_{ij} \equiv 2\epsilon_{abc}J^{ij}_c$. As in the previous section, we forget about the linear translation operators $P_\pm \equiv i\partial_\pm$, to which anticommuting Faddeev-Popov parameters $p^\pm$ should be associated, and do not add in $\Gamma^{class.}$ of equ.(2.3) the effect of translations on the fields Φ^i . Then the total effective action of equ.(2.5) writes [7] :

$$\Gamma^{tot.} = A^{inv.} + \int d^2x d^2\theta \{\eta_i J^i_{a\,j}(\Phi)[d^+_a D_+\Phi^j + d^-_a D_-\Phi^j] - \frac{1}{2}\epsilon_{abc}\eta_i\eta_j J^{ij}_c(\Phi) d^+_a d^-_b\} \tag{2.23}$$

where $d^\pm_a$ and $\eta_i(x)$ are respectively commuting Faddeev-Popov parameters and anticommuting bosonic sources. Here also, the Slavnov identity of equ.(2.10) holds and writes :

$$S\Gamma^{tot.} \equiv \int d^2x d^2\theta \frac{\delta\Gamma^{tot.}}{\delta\eta_k}\frac{\delta\Gamma^{tot.}}{\delta\Phi^k} = \int d^2x d^2\theta[(d^+_a)^2(\eta_k i\partial_+\Phi^k) + (d^-_a)^2(\eta_k i\partial_-\Phi^k)]\ . \tag{2.24}$$

Moreover, one can also check that the linearized Slavnov operator :

$$S_L \equiv \int d^2x d^2\theta \{\frac{\delta\Gamma^{tot.}}{\delta\eta_k}\frac{\delta}{\delta\Phi^k} + \frac{\delta\Gamma^{tot.}}{\delta\Phi^k}\frac{\delta}{\delta\eta_k}\} \tag{2.25}$$

is nilpotent : $(S_L)^2 = 0$, when acting in the space of integrated local functionals. These are not trivial results as in that N = 4 case, no finite set of auxiliary fields exists.

Notice that in the N=2 case - where there is only one complex structure -, there are no bilinear terms in the sources η_i in equation (2.23) : as a matter of fact, in that case the supersymmetry algebra on N=1 superfields closes off-shell.

The quantization of these theories will be studied in a second paper of this series [15].

[7] As previously mentioned, we consider only torsionless metrics and, as a consequence, there is a "parity" invariance : in the interchange $(+ \leftrightarrow -)$, $d^2\theta$ and η_i get a minus sign whether Φ^i is unchanged. Under this hypothesis, *there will be no room for a chiral anomaly.*

3 Agebraic approach to the renormalizability proof

3.1 Generalities

As recalled in refs.([8],[6]), the renormalization program consists in solving two main problems. These are :

i) all possible breakings which might affect the Ward identities, order by order in the radiative corrections, should be reabsorbed by a suitable choice of ("finite") counterterms [absence of anomalies in the theory],

ii) Ward identities being so ensured at a given order, the ("infinite") counterterms needed for the finiteness of the renormalized Green functions should be uniquely identified (up to a field redefinition) by the parameters characterizing the classical action [stability condition].

If the Ward, or rather Slavnov, identity writes

$$S\Gamma = 0 \text{ (or a classical quantity as in equs.(2.18, 2.24))} \tag{3.1}$$

the Quantum Action Principle ensures that, up to the first non-trivial order, the breaking of the Slavnov identity (3.1) corresponds to the insertion of a local functional of Faddeev-Popov charge +1, $\Delta_{[+1]}$:

$$S\Gamma = (\hbar)^p \Delta_{[+1]} + \text{higher orders}$$

and S_L being the linearized, nilpotent Slavnov operator, one gets

$$S_L \Delta_{[+1]} = 0 \ .$$

Any trivial cohomology $\Delta_{[+1]} = S_L \Delta_{[0]}$ corresponds to the looked-for "finite" counterterms as

$$S(\Gamma - (\hbar)^p \Delta_{[0]}) = 0 + \text{higher orders} \ .$$

In other words, this means that point i) supposes the vanishing of the cohomology of S_L in the Faddeev-Popov charge one sector.

As regards the stability condition

$$S(\Gamma^{class.} + \hbar \Gamma_{[0]}) = 0 \ ,$$

$\Gamma_{[0]}$, a pertubation of the classical action, is a Faddeev-Popov neutral local functional which must satisfy :

$$S_L \Gamma_{[0]} = 0 \ .$$

Any trivial cohomology $\Delta_{[0]} = S_L \Delta_{[-1]}$ may be shown to correspond to non physical field and source redefinitions. In other words, this means that point ii) supposes that the dimensionality of the cohomology space of this S_L operator in the neutral Faddeev-Popov sector is equal to the number of "physical" parameters of the classical action.

In the presence of highly non-linear Slavnov operators such as those of equations (2.19,2.25), it is technically useful to "aproximate" the complete S_L operator by a simpler one $S_L^{(0)}$ through a suitably chosen "filtration" (counting operation) [13] such as the cohomology spaces of S_L and $S_L^{(0)}$ are isomorphic. This relies on a theorem proved in ref. [8] which asserts that :

If the cohomology of $S_L^{(0)}$ is trivial in the Faddeev-Popov charged sectors, then the same is true for S_L and their cohomology spaces in the neutral sector are isomorphic.

In order to make this work as self-contained as possible, we sketch the proof in the next subsection, leaving to the appendix its extension to the case where $S_L^{(0)}$ has some non-trivial cohomology in the Faddeev-Popov charged sectors, a situation that will occur in the N=2 and 4 cases [15].

3.2 The filtration theorem

Let S_L be a nilpotent operator which acts in the linear space V of translation invariant functionals [8] of the fields, sources and their derivatives. We suppose that we have a counting operator N, with non-negative eigenvalues $\nu = 0, 1, 2, \ldots$, commuting with the Faddeev-Popov charge operator and that decomposes the linear space V in sectors $V^{(\nu)}$ and the operator S_L in $S_L^{(\nu)}$:

$$S_L = \sum_{\nu=0}^{\infty} S_L^{(\nu)} \quad \text{such that} \quad [N, S_L^{(\nu)}] = \nu S_L^{(\nu)} \tag{3.2}$$

The nilpotency of S_L induces on the $S_L^{(\nu)}$ operators the relations :

$$\sum_{\mu=0}^{\nu} S_L^{(\mu)} S_L^{(\nu-\mu)} = 0 \quad \nu = 0, 1, 2, .. \tag{3.3}$$

hence $S_L^{(0)}$ is still a nilpotent operator.

Let us first analyze the **Faddeev-Popov charged sectors** with the hypothesis :

$$S_L^{(0)}\Gamma = 0 \Rightarrow \Gamma = S_L^{(0)}\Delta \ . \tag{3.4}$$

Then, the filtration of equation $S_L\Gamma = 0$ first gives $S_L^{(0)}\Gamma^{(0)} = 0$ and from hypothesis (3.4) it results that $\Gamma^{(0)} = S_L^{(0)}\Delta^{(0)}$. The next step is :

$$S_L^{(0)}\Gamma^{(1)} + S_L^{(1)}\Gamma^{(0)} = 0 \quad \Rightarrow \quad S_L^{(0)}\Gamma^{(1)} + S_L^{(1)}S_L^{(0)}\Delta^{(0)} = 0$$

[8] The analysis is made still easier when one can adapt the formalism to "local" cohomology where the linear space is a space of functions of the fields, sources and their derivatives, taken as independent variables. In such a case, a Fock space formulation is at hand, and one can define the adjoint $S_L^{\dagger}$ of the operator S_L as well as the Laplace-Beltrami operator $\{S_L\, , S_L^{\dagger}\}$, the kernel of which gives the cohomology space of S_L [18].

and, due to equ. (3.3), this implies $S_L^{(0)}[\Gamma^{(1)} - S_L^{(1)}\Delta^{(0)}] = 0$.
$\Gamma^{(1)} - S_L^{(1)}\Delta^{(0)}$ being in a Faddeev-Popov charged sector, one gets from (3.4) $\Gamma^{(1)} - S_L^{(1)}\Delta^{(0)} = S_L^{(0)}\Delta^{(1)}$. At this point, one has :

$$\Gamma|_1 = \Gamma^{(0)} + \Gamma^{(1)} = [S_L^{(0)} + S_L^{(1)}]\Delta^{(0)} + S_L^{(0)}\Delta^{(1)} = [S_L\Delta]|_1 \quad .... \text{ e.t.c.} \tag{3.5}$$

Then we have the first part of the theorem : the cohomology of S_L vanishes in the Faddeev-Popov charged sectors.

Let us now analyze the **neutral sector** with the hypothesis :

$$S_L^{(0)}\Gamma = 0 \Rightarrow \Gamma = S_L^{(0)}\Delta + \tilde{\Delta} \tag{3.6}$$

Then, the filtration of equation $S_L\Gamma = 0$ first gives $S_L^{(0)}\Gamma^{(0)} = 0$ and from hypothesis (3.6) it results that $\Gamma^{(0)} = S_L^{(0)}\Delta^{(0)} + \tilde{\Delta}^{(0)}$. The next step is :

$$S_L^{(0)}\Gamma^{(1)} + S_L^{(1)}\Gamma^{(0)} = 0 \quad \Rightarrow \quad S_L^{(0)}\Gamma^{(1)} + S_L^{(1)}S_L^{(0)}\Delta^{(0)} + S_L^{(1)}\tilde{\Delta}^{(0)} = 0$$

and, due to equ. (3.3), this implies $S_L^{(0)}[\Gamma^{(1)} - S_L^{(1)}\Delta^{(0)}] = -S_L^{(1)}\tilde{\Delta}^{(0)}$. As a consequence of the nilpotency of $S_L^{(0)}$, this gives $S_L^{(0)}[S_L^{(1)}\tilde{\Delta}^{(0)}] = 0$.
The Faddeev-Popov charge of $S_L^{(1)}\tilde{\Delta}^{(0)}$ being +1, one gets from (3.4) :

$$S_L^{(1)}\tilde{\Delta}^{(0)} = S_L^{(0)}\bar{\Delta}^{(1)}. \tag{3.7}$$

At this point, one has : $S_L^{(0)}[\Gamma^{(1)} - S_L^{(1)}\Delta^{(0)} + \bar{\Delta}^{(1)}] = 0$, which, according to (3.6) solves to : $\Gamma^{(1)} = S_L^{(1)}\Delta^{(0)} - \bar{\Delta}^{(1)} + S_L^{(0)}\Delta^{(1)} + \tilde{\Delta}^{(1)}$.

This finaly gives :

$$\begin{aligned}\Gamma|_1 &= \Gamma^{(0)} + \Gamma^{(1)} = [S_L^{(0)} + S_L^{(1)}]\Delta^{(0)} + S_L^{(0)}\Delta^{(1)} + \tilde{\Delta}^{(0)} + \tilde{\Delta}^{(1)} - \bar{\Delta}^{(1)} \\ &= [S_L\Delta]|_1 + \tilde{\Delta}|_1 - \bar{\Delta}|_1 \quad ....\text{e.t.c.}\end{aligned} \tag{3.8}$$

where $\bar{\Delta}^{(1)}$, being determined by (3.7) up to $S_L^{(0)}\bar{\bar{\Delta}}^{(1)}$ which would unessentially modify Δ, adds **no new parameter** to the cohomology $\tilde{\Delta}$ of $S_L^{(0)}$.

What remains to be shown is that the cohomology space of S_L is not smaller that the one of $S_L^{(0)}$ in that neutral sector. Supposes that there exists some Γ belonging to the cohomology space of $S_L^{(0)}$:

$$S_L^{(0)}\Gamma = 0 \quad \text{with } \Gamma \neq S_L^{(0)}\tilde{\Delta} .$$

The previous demonstration then built Γ', a cocycle of S_L and let us suppose that it is a coboundary of S_L :

$$S_L\Gamma' = 0 \quad \text{and} \quad \Gamma' = S_L\tilde{\Delta}' . \tag{3.9}$$

At the lowest order this would give a contradiction as $\Gamma^{(0)} \equiv \Gamma'^{(0)}$. Then $\Gamma^{(0)} = 0 = S_L^{(0)}\tilde{\Delta}'^{(0)}$.
At the next order - in fact the first non-vanishing one -, one gets :

$$\Gamma^{(1)} \equiv \Gamma'^{(1)} = S_L^{(0)}\tilde{\Delta}'^{(1)} + S_L^{(1)}\tilde{\Delta}'^{(0)} \tag{3.10}$$

But $S_L^{(0)}\tilde{\Delta}'^{(0)} = 0$ where $\tilde{\Delta}'^{(0)}$ belongs to the Faddeev-Popov charge -1 sector [9] . From the hypothesis of the theorem, this gives $\tilde{\Delta}'^{(0)} = S_L^{(0)}\tilde{\tilde{\Delta}}'^{(0)}$ and equation (3.10) leads to :

$$\Gamma^{(1)} \equiv \Gamma'^{(1)} = S_L^{(0)}[\tilde{\Delta}'^{(1)} - S_L^{(1)}\tilde{\tilde{\Delta}}'^{(0)}] \, .$$

This means that Γ would be a coboundary of $S_L^{(0)}$ (at this order $\Gamma \equiv \Gamma^{(1)}$). The contradiction then gives again $\Gamma^{(1)} = 0$ ....e.t.c. Then we have the announced isomorphism between the cohomology spaces of S_L and $S_L^{(0)}$ in the Faddeev-Popov neutral sector.

This schematic proof illustrates the fact that a non vanishing cohomology for $S_L^{(0)}$ in a given Faddeev-Popov charge ν sector *a priori* obstructs the construction of the cocycles of S_L in the Faddeev-Popov charge ν-1 and transforms into coboundaries of S_L some of the cohomology elements of S_L^0 in the Faddeev-Popov charge ν+1 (see the appendix for some details).

This ends the proof of the filtration theorem (for a complete proof, see [13],[8]).

We shall now apply our methods to N=1 supersymmetric non-linear σ models in components fields.

4 N=1 supersymmetric non-linear σ models

Equations (2.17,2.19) respectively define the classical action and the nilpotent linearized Slavnov operator. N_{field} being the operator counting the number of fields (and of their derivatives) :

$$N_{field}\Delta \equiv \left[\int d^2x \phi(x)\frac{\delta}{\delta\phi(x)}\right] \Delta \ , \tag{4.1}$$

we will take as ghost number preserving counting (filtration) operator :

$$\begin{aligned} N &= N_{fields,sources} + \\ &+ \sum_{fields,\ sources,\ ghosts} [\text{spin of the field , source or ghost}] \, \mathrm{x} N_{field,\ source,\ ghost} \\ &\equiv \int d^2x \{\phi^i\frac{\delta}{\delta\phi^i} + \frac{3}{2}\psi_+^a\frac{\delta}{\delta\psi_+^a} + \frac{3}{2}\psi_-^a\frac{\delta}{\delta\psi_-^a} + \eta_i\frac{\delta}{\delta\eta_i} + \frac{3}{2}\gamma_a^+\frac{\delta}{\delta\gamma_a^+} + \frac{3}{2}\gamma_a^-\frac{\delta}{\delta\gamma_a^-}\} \\ &+ \frac{1}{2}(C^+\frac{\delta}{\delta C^+} + C^-\frac{\delta}{\delta C^-}) \ . \end{aligned} \tag{4.2}$$

[9] This gives a hint : if the cohomology of $S_L^{(0)}$ is not empty in the Faddeev-Popov charge -1 sector, and only in that case, the cohomology spaces of S_L in the Faddeev-Popov neutral sector may be smaller than the one of $S_L^{(0)}$. This is for example the case when extra isometries exist.

$e_i^a(0)$ being the vielbein $e_a^i(\phi)$ at $\phi \equiv 0$, S_L^0 is readily obtained as :

$$
\begin{aligned}
S_L^0 \equiv & \int d^2x\{\left[iC^+ e_i^a(0)\partial_+\phi^i \frac{\delta}{\delta\psi_+^a} + [2i\delta_{ab}\partial_-\psi_+^b - C^+ e_a^i(0)\eta_i]\frac{\delta}{\delta\gamma_a^+} - g_{ij}(0)\partial_{+-}^2\phi^j\frac{\delta}{\delta\eta_i}\right] \\
+ & \left[iC^- e_i^a(0)\partial_-\phi^i \frac{\delta}{\delta\psi_-^a} + [2i\delta_{ab}\partial_+\psi_-^b - C^- e_a^i(0)\eta_i]\frac{\delta}{\delta\gamma_a^-} - g_{ij}(0)\partial_{+-}^2\phi^j\frac{\delta}{\delta\eta_i}\right]\}
\end{aligned}
\tag{4.3}
$$

4.1 The cohomology of S_L^0

The most general functional in the fields, sources, ghosts and their derivatives, of a given Faddeev-Popov charge, is built using Lorentz and parity invariance (see footnote 7) and power counting [10].

4.1.1 The Faddeev-Popov negatively charged sectors

The most general functional in the fields, sources, ghosts and their derivatives, of Faddeev-Popov charge -1 is [11] :

$$\Delta_{[-1]} \equiv \int d^2x\{[\gamma_a^+\psi_+^b + \gamma_a^-\psi_-^b]S_b^a(\phi) + \eta_i T^i(\phi)\} \tag{4.4}$$

The condition $S_L^0\Delta_{[-1]} = 0$ easily gives :

$$S_b^a(\phi) = 0 \; ; \quad \int d^2x g_{ij}(0)\partial_{+-}^2\phi^i T^j(\phi) = 0 \Leftrightarrow g_{ik}(0)T^k_{,j}(\phi) + g_{jk}(0)T^k_{,i}(\phi) = 0 \; .$$

This would mean that $T^i(\phi)$ is a Killing vector with respect to the flat approximation $g_{ij}(0)$ of the metric $g_{ij}[\phi]$. As a matter of facts, due to the simplicity of $\Delta_{[-1]}$, the cohomology of the complete S_L operator in the Faddeev-Popov charge sector -1 is easily obtained, and the vector $T^i[\phi]$ should satisfy :

$$\int d^2x \frac{\delta A^{inv.}}{\delta\phi^i(x)} T^i[\phi(x)] = 0 \; \Leftrightarrow \; T^i[\phi] \text{ is a Killing vector for the metric } \mathrm{g_{ij}}[\phi] \; .$$

Then, in the absence of Killing vectors, there are no Faddev-Popov negatively charged cocycles - nor coboundaries, see footnote 11.

4.1.2 The Faddeev-Popov 0 charge sector

The most general functional in the fields, sources, ghosts and their derivatives, of Faddeev-Popov charge 0, depends on 10 functions of ϕ and these 10 monomials can be ordered with

[10] The canonical dimensions of $C^\pm$, $\phi^i(x)$, $\psi_\pm^a(x)$, $\gamma_a^\pm(x)$ and $\eta_i(x)$ are respectively $-\frac{1}{2}$, 0, $+\frac{1}{2}$, $+\frac{3}{2}$ and +2.

[11] More negatively Faddeev-Popov charged functionals do not exist.

respect to the total spin of the fields, sources and ghosts composing them :

$$
\begin{aligned}
\Delta^0_{[0]} &\equiv \int d^2x \partial_+\phi^i\partial_-\phi^j T^1_{ij}(\phi) \\
\Delta^1_{[0]} &\equiv \int d^2x \{[\psi^a_+\partial_-\psi^b_+ + \psi^a_-\partial_+\psi^b_-]T^2_{ab}(\phi) + [\psi^a_+\psi^b_+\partial_-\phi^i + \psi^a_-\psi^b_-\partial_+\phi^i]T^3_{abi}(\phi) + \\
&+ [\gamma^+_a C^+\partial_+\phi^i + \gamma^-_a C^-\partial_-\phi^i]T^{5a}_i(\phi) + \eta_i[C^+\psi^a_+ + C^-\psi^a_-]T^{8i}_a(\phi)\} \\
\Delta^2_{[0]} &\equiv \int d^2x\{[\psi^a_+\psi^b_+\psi^c_-\psi^d_-]T^4_{abcd}(\phi) + \\
&+ [\gamma^+_a C^+\psi^b_+\psi^c_+ + \gamma^-_a C^-\psi^b_-\psi^c_-]T^{6a}_{bc}(\phi) + [\gamma^+_a C^-\psi^b_+\psi^c_- + \gamma^-_a C^+\psi^b_-\psi^c_+]T^{7a}_{bc}(\phi) + \\
&+ [\gamma^+_a\gamma^+_b(C^-)^2 + \gamma^-_a\gamma^-_b(C^+)^2]T^{9ab}(\phi) + [\gamma^+_a\gamma^-_b C^+C^-]T^{10ab}(\phi)\}
\end{aligned}
\tag{4.5}
$$

The condition $S^0_L\Delta_{[0]} = 0$ can be analysed in each spin sector separately :

i) $\Delta^0_{[0]}$ is not constrained, but as coboundaries exist :

$$
S^0_L\tilde{\Delta}^0_{[-1]} = S^0_L\left(\int d^2x\eta_i T^i(\phi)\right) ,
$$

this means that some freedom on $T^1_{ij}(\phi)$ corresponds to a trivial cohomology, *i.e.* to the expected effect on the metric of the field redefinition freedom :

$$
\phi^i \rightarrow \phi^i + T^i(\phi) ,
$$

ii) $\Delta^1_{[0]}$ is constrained, and the relations that one obtains among the 4 functions T^2, T^3, T^5 and T^8 give :

$$
\Delta^1_{[0]} \equiv S^0_L\left[-\int d^2x\{[\gamma^+_a\psi^b_+ + \gamma^-_a\psi^b_-]e^a_i(0)T^{8i}_b(\phi)\}\right] .
$$

This trivial cohomology corresponds to a $\psi^a_\pm(x)$ non-linear field redefinition :

$$
\psi^a_\pm \rightarrow \psi^a_\pm + e^a_i(0)T^{8i}_b(\phi)\psi^b_\pm ,
$$

iii) the cocycle condition $S^0_L\Delta^2_{[0]} = 0$ enforces the vanishing of T^4, T^6, T^7 T^9 and T^{10}: there are no cocycles (nor coboundaries) in that sub-sector.

To summarize, the cohomology of S^0_L in the Faddeev-Popov 0 charge sector corresponds to the arbitrariness of a "metric" $T^1_{ij}(\phi)$. This corresponds to the renormalizability in the space of metrics (*i.e. à la Friedan* [16]) as mentioned in the Introduction.

4.1.3 The Faddeev-Popov +1 charge sector

In that sector, the most general functional in the fields, sources, ghosts and their derivatives depends on 23 functions of ϕ and again these 23 monomials can be ordered with respect to the total spin of the fields, sources and ghosts composing them :

$$
\begin{aligned}
\Delta^0_{[+1]} &\equiv 0 \\
\Delta^1_{[+1]} &\equiv \int d^2x \left\{ (C^+\psi^a_+[\partial_+\phi^i\partial_-\phi^j U^1_{aij} + \partial_+\partial_-\phi^i U^2_{ai}] + C^+C^-\eta_i\partial_+\phi^j U^{3i}_j) + (+\leftrightarrow -) \right\} \\
\Delta^2_{[+1]} &\equiv \int d^2x \{ (C^+\psi^a_+ \left[\psi^b_+\psi^c_+\partial_-\phi^i V^1_{abc\,i} + \psi^b_-\psi^c_-\partial_+\phi^i V^2_{abc\,i} + \psi^b_+\partial_-\psi^c_+ V^3_{abc} + \psi^b_-\partial_+\psi^c_- V^4_{abc} \right] + \\
&+ (C^+)^2 \left[\{\gamma^+_a\psi^b_+ V^{5a}_{b\,i} + \gamma^-_a\psi^b_- V^{6a}_{b\,i}\}\partial_+\phi^i + \gamma^+_a\partial_+\psi^b_+ V^{7a}_b + \gamma^-_a\partial_+\psi^b_- V^{8a}_b \right] + \\
&+ C^+C^-\gamma^+_a \left[\psi^b_-\partial_+\phi^i V^{9a}_{b\,i} + \partial_+\psi^b_- V^{10a}_b \right] + \\
&+ \eta_i C^+\psi^a_+ \left[C^+\psi^b_+ V^{11\,i}_{ab} + C^-\psi^b_- V^{12\,i}_{ab} \right] + \eta_i(C^+)^2C^-\gamma^-_a V^{13a\,i}) + (+\leftrightarrow -)\} \\
\Delta^3_{[+1]} &\equiv \int d^2x \{ (C^+\psi^a_+\psi^b_+\psi^c_+\psi^d_-\psi^e_- W^1_{abcde} + \\
&+ C^+\psi^b_+\psi^c_+ \left[C^+(\gamma^+_a\psi^d_+ W^{2a}_{bcd} + \gamma^-_a\psi^d_- W^{3a}_{bcd}) + C^-\gamma^+_a\psi^d_- W^{4a}_{bcd} \right] + \\
&+ (C^+)^2\gamma^-_a \left[\gamma^-_b C^+\psi^c_+ W^{5ab}_c + \gamma^+_b C^-\psi^c_+ W^{6ab}_c + \gamma^-_b C^-\psi^c_- W^{7ab}_c \right]) + (+\leftrightarrow -)\} \qquad (4.6)
\end{aligned}
$$

The cocycle condition on $\Delta_{[+1]}$ is found to enforce relations among those 23 functions such as $\Delta_{[+1]}$ finally depends only on 9 functions and may be identified with the coboundary $S^0_L\Delta_{[0]}$, where $\Delta_{[0]}$ is equal to the $\Delta^1_{[0]} + \Delta^2_{[0]}$ of equations (4.5). As a consequence S^0_L has no anomaly.

It results from the whole subsection 4.1 that - at least in the absence of Killing vectors for the metric - the cohomology of S^0_L vanishes in the charged Faddeev-Popov sectors and is identified in the neutral sector by a generic symmetric metric tensor in the target space (up to an arbitrary change of coordinates).

4.2 The cohomology of S_L *(in the absence of Killing vectors for the classical metric)*

The hypothesis of our filtration theorem being satisfied, we get the desired results for the cohomology of the whole Slavnov operator S_L :

• the Slavnov identity is not anomalous [absence of Faddeev-Popov charge +1 cohomology], which means that, as expected, N=1 supersymmetry is renormalizable (in the 4 dimensional case this was proved in [19]). Notice that this property is not changed by a possible Faddeev-Popov charge -1 cohomology for S^0_L.

• moreover, the cohomology of S_L in the Faddeev-Popov neutral sector is identified by a generic, symmetric, metric tensor in the target space and may be obtained as :

$$\Delta_{[0]}[equ.(4.5)] \equiv \Gamma^{tot.}[equ.(2.17)]$$

with $g_{ij} \equiv t_{ij}$, e.t.c. In this case of d=2, N=1 non-linear σ models, the renormalization algorithm *a priori* does not change the number of parameters with respect to the one of the classical action[12]. Then we have the "stability" of the classical action in the space of metrics, *i.e.* the full renormalizability of the theory. Of course, as usual the trivial cohomology $S_L\Delta_{[-1]}(T^i[\phi], S^a_b[\phi])$ corresponds to the non-linear fields and sources reparametrisations, here according to :

$$\begin{array}{rclcl} \phi^i \to \phi^i + T^i(\phi) & , & \eta_i \to \eta_i - \eta_k T^k_{\ ,i} - S^a_{b\ ,i}(\phi)[\gamma^+_a \psi^b_+ + \gamma^-_a \psi^b_-] \\ \psi^a_\pm \to \psi^a_\pm - S^a_b(\phi)\psi^b_\pm & , & \gamma^\pm_a \to \gamma^\pm_a + S^b_a(\phi)\gamma^\pm_b \end{array} \tag{4.7}$$

5 Cohomology of supersymmetry in the presence of Killing vectors for the classical metric : *the example of coset-spaces*

As previously mentioned, the existence of Killing vectors is responsible for a non vanishing cohomology in the Faddeev-Popov charge -1 sector, which will restrict the one in the Faddeev-Popov 0 charge sector (the Action). As a matter of fact, the occurence of Killing vectors reveals the presence of extra symmetries whose renormalizability also has to be studied, from the very beginning, as these isometries are needed for a precise definition of the classical action and of the true physical parameters. As an example, in this section we extend to N=1 supersymmetric models the results of our previous analysis on the renormalizability of bosonic non-linear σ models built on compact homogeneous spaces G/H [8].

The non-linear isometries may be writen as :

$$\begin{aligned} \delta\phi^i &= \epsilon^j W^i_j[\phi] \\ \delta\psi^i_\pm &= \epsilon^j W^i_{j,k}[\phi]\psi^k_\pm \end{aligned} \tag{5.1}$$

where the W^i_j , j = 1,2,..n are n Killing vectors for the metric g_{ij} :

$$g_{il}\nabla_j W^l_k + g_{jl}\nabla_i W^l_k = 0 \ ,$$

and where ∇_i is the covariant derivative with (torsionless) symmetric connexion associated to the metric g_{ij} .

The homogeneity of the coset space G/H means that :

$$W^i_j[\phi] = \delta^i_j + ..., \tag{5.2}$$

With regard to the linear isometries, we know from the appendix A of [8] that they cause no difficulty in the renormalization program, at least when the corresponding Lie group H

[12] To be made more precise, this assertion supposes a true definition of the classical action, for example through extra isometries (see the next section).

is a compact one. In the following, we then restrict ourselves to H-invariant integrated local functionals in the fields, sources, ghosts and their derivatives.

Using tangent space fermions as defined in subsection 2.2, transformations (5.1) write :

$$\begin{aligned} \delta\phi^i &= \epsilon^j e^i_a W^a_j \\ \delta\psi^a_\pm &= \epsilon^j[\nabla_c W^a_j - \omega^a_{bc} W^b_j]\psi^c_\pm \,. \end{aligned} \tag{5.3}$$

Of course, the supersymmetry transformations (2.15) commute with the previous ones (5.3) and the commutation relations of the non-linear transformations (5.1) being those of a standard Lie algebra, equ.(2.2) still specifies to equ.(2.16). One then introduce constant anticommuting Faddeev-Popov ghosts C^j [13](of vanishing canonical dimension), and modify the total effective action of equ.(2.17):

$$\begin{aligned} \Gamma^{tot.} &= A^{inv.} + \int d^2x \{\gamma^+_a(x)\left(C^j[\nabla_c W^a_j - \omega^a_{bc} W^b_j]\psi^c_+ + iC^+ e^a_+ + \omega^a_{bc}\psi^c_+(C^+\psi^b_+ + C^-\psi^b_-)\right) \\ &+ \gamma^-_a(x)\left(C^j[\nabla_c W^a_j - \omega^a_{bc} W^b_j]\psi^c_- + iC^- e^a_- + \omega^a_{bc}\psi^c_-(C^+\psi^b_+ + C^-\psi^b_-)\right) \\ &+ \eta_i(x)(C^j e^i_a W^a_j + C^+ e^i_a \psi^a_+ + C^- e^i_a \psi^a_-)\} \\ &+ \frac{1}{4}\int d^2x \delta^{ab}\{\gamma^+_a(x)\gamma^+_b(x)(C^-)^2 + \gamma^-_a(x)\gamma^-_b(x)(C^+)^2 - 2\gamma^+_a(x)\gamma^-_b(x)(C^+C^-)\} \end{aligned} \tag{5.4}$$

The Slavnov identity of equ.(2.10) still holds and the linearized Slavnov operator (2.19) keeps the same structure :

$$\begin{aligned} S_L \equiv \int d^2x \; \{&\left(\frac{\delta\Gamma^{tot.}}{\delta\gamma^+_a}\right)\frac{\delta}{\delta\psi^a_+} + \left(\frac{\delta\Gamma^{tot.}}{\delta\gamma^-_a}\right)\frac{\delta}{\delta\psi^a_-} + \left(\frac{\delta\Gamma^{tot.}}{\delta\psi^a_+}\right)\frac{\delta}{\delta\gamma^+_a} + \left(\frac{\delta\Gamma^{tot.}}{\delta\psi^a_-}\right)\frac{\delta}{\delta\gamma^-_a} \\ + &\left(\frac{\delta\Gamma^{tot.}}{\delta\eta_i}\right)\frac{\delta}{\delta\phi^i} + \left(\frac{\delta\Gamma^{tot.}}{\delta\phi^i}\right)\frac{\delta}{\delta\eta_i}\} \end{aligned} \tag{5.5}$$

and is nilpotent : $(S_L)^2 = 0$, when acting in the space of H-invariant integrated local functionals. For further use, and thanks to the commutation of transformations (5.3) and (2.15), we split S_L in :

$$S_L = S^s_L + S^w_L \,,$$

where S^w_L is the nilpotent Slavnov operator associated to the homogenous transformations (5.3) and is linear in C^j and we have :

$$(S^s_L)^2 = (S^w_L)^2 = S^w_L S^s_L + S^s_L S^w_L = 0 \,.$$

The classical action and the nilpotent linearized Slavnov operator being so defined, we add to the grading operator (4.2) the operator counting the number of ghosts C^j . S^0_L is readily obtained as $S^{s|0}_L + S^{w|0}_L$ where $S^{s|0}_L$ was given in equ.(4.3) and :

$$S^{w|0}_L = C^i \int d^2x \frac{\delta}{\delta\phi^i(x)}$$

[13] Here again, as we did before when we did not introduce ghosts for the translations, we do not need to add ghosts associated to the linear isometries.

where the fundamental homogeneity property (5.2) has been used.

We are now in a position to analyse the cohomology of S_L^0, using the results of subsection 4.1 for $S_L^{s|0}$ and of [8] for $S_L^{w|0}$.

5.1 The cohomology of S_L^0

5.1.1 The Faddeev-Popov negatively charged sectors

The most general functional in the fields, sources, ghosts and their derivatives, of Faddeev-Popov charge -1 is still given by equ.(4.4). The condition $S_L^0 \Delta_{[-1]} = 0$ easily gives :

$$S_b^a(\phi) = 0 \; ; \quad T^i(\phi) = \text{constant} \; .$$

Moreover, this cohomology should be H-invariant. This occurs only when among the non-linear transformations W_j, there exists some, labelled by indices $\underline{j}$, $W_{\underline{j}}$, corresponding to a subgroup X of G that commutes with H. As explained in detail in [8], in such a case, some of the parameters that define the invariant action $\int d^2x g_{ij}[\phi]\partial_+\phi^i\partial_-\phi^j$ become unphysical ones. This corresponds to the fact that the non vanishing cohomology $T^{\underline{i}}$ in the Faddeev-Popov -1 charge sector restricts the cohomology of S_L in the Faddeev-Popov 0 charge sector (see the appendix and [14]).

5.1.2 The Faddeev-Popov 0 charge sector

The most general functional in the fields, sources, ghosts and their derivatives, of Faddeev-Popov charge 0 may be split into two parts : $\Delta^0_{[0]}$ and $\Delta^1_{[0]}$ according to their number of ghosts C^i. $\Delta^0_{[0]}$ is given by (4.5) and

$$\Delta^1_{[0]} = C^j \int d^2x \{[\gamma_a^+ \psi_+^b + \gamma_a^- \psi_-^b] S^a_{bj}(\phi) + \eta_i T^i_j(\phi)\} \equiv C^j \Delta^0_{[-1]\, j}$$

where $\Delta^0_{[-1]\, j}$ has the same expression as $\Delta_{[-1]}$ of (4.4) with tensors having one more index j. The condition $S_L^0 \Delta_{[0]} = 0$ is then analysed into 3 steps corresponding to the total C^j ghost number :

$$\bullet S_L^{s|0} \Delta^0_{[0]} = 0 \quad \overset{subsec.(4.1.2)}{\Leftrightarrow} \quad \Delta^0_{[0]} = \int d^2x \partial_+\phi^i\partial_-\phi^j T_{ij}[\phi] + S_L^{s|0} \Delta_{[-1]}[S^a_{1b},\, T^i_1]$$

where the symmetric tensor T_{ij} is, for the moment, unconstrained.

$$\bullet S_L^{w|0} \Delta^1_{[0]} = 0 \;\Leftrightarrow\; \Delta^1_{[0]} = S_L^{w|0} \Delta_{[-1]}[S^a_{2b},\, T^i_2]$$

$$\bullet S_L^{w|0} \Delta^0_{[0]} + S_L^{s|0} \Delta^1_{[0]} = 0 \Leftrightarrow\; S_L^{s|0} S_L^{w|0} \Delta_{[-1]}[S^a_{1b} - S^a_{2b},\, T^i_1 - T^i_2] = S_L^{w|0} \int d^2x \partial_+\phi^i\partial_-\phi^j T_{ij}[\phi]$$

$$\Rightarrow\; S^a_{1b} - S^a_{2b} = \text{constant} \;,\; T_{ij}[\phi] = -(g_{il}(0)[T^l_1 - T^l_2]_{,j} + g_{jl}(0)[T^l_1 - T^l_2]_{,i}) + \lambda_{ij}$$

where λ_{ij} is a constant H-invariant tensor.

Finally this gives :

$$\Delta_{[0]} = \lambda_{ij} \int d^2x \partial_+\phi^i\partial_-\phi^j + S_L^0 \Delta_{[-1]}[S_{1b}^a, T_2^i] \tag{5.6}$$

This means that, as in the purely bosonic case [8], the dimension of the cohomology space of S_L^0 in the Faddeev-Popov neutral sector is given by the number of H-invariant symmetric 2-tensors.

5.1.3 The Faddeev-Popov +1 charge sector

The most general functional in the fields, sources, ghosts and their derivatives, of Faddeev-Popov charge +1 may be split into three parts : $\Delta_{[+1]}^0$, $\Delta_{[+1]}^1$ and $\Delta_{[+1]}^2$ according to their number of ghosts C^i . $\Delta_{[+1]}^0$ is given by (4.6), $\Delta_{[+1]}^1 \equiv C^j\Delta_{[0]j}^0$ where $\Delta_{[0]j}^0$ has the same expression as $\Delta_{[0]}$ given by (4.5) with tensors having one more index j, and $\Delta_{[+1]}^2 \equiv C^jC^k\Delta_{[-1]jk}^0$ where $\Delta_{[-1]jk}^0$ has the same expression as $\Delta_{[-1]}$ given by (4.4) with tensors having two more indices j and k. The condition $S_L^0\Delta_{[+1]} = 0$ is then analysed into 4 steps corresponding to the total C^j ghost number :

$$\bullet S_L^{s|0}\Delta_{[+1]}^0 = 0 \overset{subsec.(4.1.3)}{\Leftrightarrow} \Delta_{[+1]}^0 = S_L^{s|0}\Delta_{[0]}^0$$

$$\bullet S_L^{w|0}\Delta_{[+1]}^2 = 0 \Leftrightarrow \Delta_{[+1]}^2 = S_L^{w|0}\Delta_{[0]}^1 = -C^kS_L^{w|0}\Delta_{[-1]k}^0[S_1,\ T_1]$$

$$\bullet S_L^{w|0}\Delta_{[+1]}^0 + S_L^{s|0}\Delta_{[+1]}^1 = 0 \Leftrightarrow -C^kS_L^{s|0}\left[\Delta_{[0]k}^0 - \int d^2x\frac{\delta}{\delta\phi^k(x)}\Delta_{[0]}^0\right] = 0$$

$$\overset{subsec.(4.1.2)}{\Leftrightarrow} \Delta_{[0]k}^0 = \int d^2xT_{ijk}^1\partial_+\phi^i\partial_-\phi^j - S_L^{s|0}\Delta_{[-1]k}^0[S_2,\ T_2] + \int d^2x\frac{\delta}{\delta\phi^k(x)}\Delta_{[0]}^0$$

$$\Rightarrow \Delta_{[+1]}^1 = C^k\int d^2xT_{ijk}^1\partial_+\phi^i\partial_-\phi^j + S_L^{s|0}\Delta_{[0]}^1[S_2,\ T_2] + S_L^{w|0}\Delta_{[0]}^0$$

$$\bullet S_L^{w|0}\Delta_{[+1]}^1 + S_L^{s|0}\Delta_{[+1]}^2 = 0 \Leftrightarrow C^kS_L^{w|0}\int d^2xT_{ijk}^1\partial_+\phi^i\partial_-\phi^j = C^kS_L^{s|0}S_L^{w|0}\Delta_{[-1]k}^0[S_1-S_2,\ T_1-T_2]$$

$$\Rightarrow [S_1-S_2]_{bk}^a = \partial_k[S_1-S_2]_b^a \ ; \ T_{ijk}^1 = \partial_kT_{ij}[\phi] - (g_{il}(0)[T_{1k}^l - T_{2k}^l]_{,j} + g_{jl}(0)[T_{1k}^l - T_{2k}^l]_{,i})$$

Finally this gives :

$$\Delta_{[+1]} = S_L^0\left[\Delta_{[0]}^0 + \Delta_{[0]}^1[S_{2b}^a,\ T_1^i] + \int d^2xT_{ij}[\phi]\partial_+\phi^i\partial_-\phi^j\right] = S_L^0\Delta_{[0]} \tag{5.7}$$

This means that the cohomology of S_L^0 in the Faddeev-Popov charge 1 sector vanishes.

5.2 The cohomology of S_L

It results from the whole subsection 5.1 that the cohomology of S_L^0 vanishes in the charged Faddeev-Popov sectors and is identified in the neutral sector by a generic H-invariant constant symmetric 2-tensor. Then, the hypothesis of our filtration theorem being satisfied, we get the desired results for the cohomology of the whole Slavnov operator S_L :

• the Slavnov identity is not anomalous [absence of Faddeev-Popov charge +1 cohomology],

• the cohomology of S_L in the Faddeev-Popov neutral sector is, as the classical action, identified by by a generic [14] H-invariant constant symmetric 2-tensor [stability of the theory].

This means that, as expected, N=1 supersymmetric non-linear σ models built on compact homogeneous spaces are renormalizable.

6 Concluding remarks

We have analysed in a regularization free manner the all-order renormalizability of N=1 supersymmetric non-linear σ models in component fields. Using a conveniently chosen grading, we proved the absence of supersymmetry anomaly and the renormalizability of the theory "in the space of metrics". For the special class of σ models built on an homogeneous manifold (usual non linear σ models on coset space), our work extends the renormalizability proof given for the bosonic case in ref.[8] to the N=1 supersymmetric case (up to infra-red analysis).

The quantization of the N=2 and 4 supersymmetric non-linear σ models of subsection 2.3 will be studied in a second paper of this series [15]. In particular, the rigorous proof of the renormalizability of N=1 supersymmetric non-linear σ models in component fields given here will allow us to analyse the extended supersymmetries in a N=1 superfield formalism (see also [3]).

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[14] up to some Faddeev-Popov charge -1 cohomology, which restricts the number of physical parameters of the classical action (subsection (5.1.1) and appendix 7.2).

7 Appendix : the filtration theorem in the presence of a non trivial cohomology for $S_L^{(0)}$ in the Faddeev-Popov charged sectors.

This appendix intends to give simple proofs of the results of the original papers ([13],[8] and [14]) in order to illustrate the fact that a non vanishing cohomology for $S_L^{(0)}$ in a given Faddeev-Popov charge ν sector *a priori* obstructs the construction of the cocycles of S_L in the Faddeev-Popov charge ν-1 and transforms into coboundaries of S_L some of the cohomology elements of S_L^0 in the Faddeev-Popov charge ν+1.

7.1 Presence of some cohomology in the Faddeev-Popov charge +1 sector.

As can be seen from the sketch of the proof given in subsection 3.2, this may prevent one from constructing Faddeev-Popov 0 charge cocycles of S_L starting from those of $S_L^{(0)}$. Indeed, suppose that there is some cohomology beginning at the filtration level ν :

$$S_L^{(0)}\Delta_{[1]} = 0 \;\Rightarrow\; \Delta_{[1]} = \Delta_{[1]}^{an.(\nu)} + S_L^{(0)}\Delta_{[0]}\;. \tag{7.1}$$

The construction described through equations (3.6) to (3.8) builds cocycles in the Faddeev-Popov 0 charge sector

$$S_L\Gamma|_{(\nu-1)} = 0 \;\Rightarrow\; \Gamma|_{(\nu-1)} = \Delta_{[0]}^{an.}|_{(\nu-1)} - \bar{\Delta}_{[0]}|_{(\nu-1)} + \left(S_L\Delta_{[-1]}\right)|_{(\nu-1)}.$$

At the next order one has :

$$S_L^{(0)}\Gamma^{(\nu)} + S_L^{(1)}\Gamma^{(\nu-1)} + .. + S_L^{(\nu)}\Gamma^{(0)} = 0$$

$$\Rightarrow\; S_L^{(0)}[\Gamma^{(\nu)} - S_L^{(1)}\Delta_{[-1]}^{(\nu-1)} + ..] + \{S_L^{(1)}\Delta_{[0]}^{an.(\nu-1)} - S_L^{(1)}\bar{\Delta}_{[0]}^{(\nu-1)} + ...\} = 0.$$

From $S_L^{(0)}\{S_L^{(1)}\Delta_{[0]}^{an.(\nu-1)} - S_L^{(1)}\bar{\Delta}_{[0]}^{(\nu-1)} + ...\} = 0$, the hypothesis (7.1) gives :

$$\{S_L^{(1)}\Delta_{[0]}^{an.(\nu-1)} - S_L^{(1)}\bar{\Delta}_{[0]}^{(\nu-1)} + ...\} = \Delta_{[1]}^{an.(\nu)} + S_L^{(0)}\bar{\Delta}_{[0]}^{(\nu)} \tag{7.2}$$

and finally :

$$S_L^{(0)}\left[\Gamma^{(\nu)} - S_L^{(1)}\Delta_{[-1]}^{(\nu-1)} + ... + \bar{\Delta}_{[0]}^{(\nu)}\right] + \Delta_{[1]}^{an.(\nu)} = 0\;,$$

which is self contradictory, <u>except if the coefficients</u> in the $\Delta_{[0]}^{an.(\nu-p)}$, $\bar{\Delta}_{[0]}^{(\nu-p)}$ involved in equ.(7.2) are related in such a way that $\Delta_{[1]}^{an.(\nu)}$ does not appear. In that case, one gets

$$\Gamma^{(\nu)} = S_L^{(1)}\Delta_{[-1]}^{(\nu-1)} + ... + \Delta_{[0]}^{an.(\nu)} - \bar{\Delta}_{[0]}^{(\nu)} + S_L^{(0)}\Delta_{[-1]}^{(\nu)}$$

and finally

$$\Gamma|_{(\nu)} = \Delta_{[0]}^{an.}|_{(\nu)} - \bar{\Delta}_{[0]}|_{(\nu)} + \left(S_L\Delta_{[-1]}\right)|_{(\nu)} \qquad Q.E.D.$$

Thus, in the case of a non vanishing intersection between the cohomology in the Faddeev-Popov charge 1 sector and the successive images through $S_L^{(1)}$, $S_L^{(2)}$... of the insertions $\Delta_{[0]}^{an.(\nu)}$, $\bar{\Delta}_{[0]}^{(\nu)}$ of Faddeev-Popov 0 charge, the aforementioned relations reduce the number of cocycles of S_L in the Faddeev-Popov 0 charge sector with respect to the ones of $S_L^{(0)}$.

This analysis will be useful for the study of N=2 and 4 supersymmetric non-linear σ models [15].

7.2 Presence of some cohomology in the Faddeev-Popov charge -1 sector.

As indicated in subsection 3.2, this will reduce the dimension of the cohomology space of S_L in the Faddeev-Popov 0 charge sector with respect to the one of $S_L^{(0)}$. A complete analysis is given in the appendix C of ref.[14] where it is shown that the cohomology of S_L in the Faddeev-Popov 0 charge sector is isomorphic to the repeated quotient of the cohomology of $S_L^{(0)}$ in the same Faddeev-Popov sector by the successive images through $S_L^{(1)}$, $S_L^{(2)}$... of the cohomology of $S_L^{(0)}$ in the Faddeev-Popov -1 charge sector. This reduction is now due to the fact that some cocycles for S_L built according to equations (3.6) to (3.8) may be coboundaries when there is some $S_L^{(0)}$ cohomology in the Faddeev-Popov -1 charge sector. For self-containedness, we give now some hints on this mechanism.

Suppose for example that there is some cohomology in the Faddeev-Popov -1 charge sector beginning at the filtration level ν :

$$S_L^{(0)}\Delta_{[-1]}^{an.(\nu)} = 0 \;\text{ with }\; \Delta_{[-1]}^{an.(\nu)} \neq S_L^{(0)}\Delta_{[-2]}^{(\nu)} \tag{7.3}$$

and consider $S_L^{(1)}\Delta_{[-1]}^{an.(\nu)}$. It is a $S_L^{(0)}$ cocycle in the Faddeev-Popov 0 charge sector and then may be writen as :

$$S_L^{(1)}\Delta_{[-1]}^{an.(\nu)} = S_L^{(0)}\tilde{\Delta}_{[-1]}^{(\nu+1)} + \Delta_{[0]}^{an.(\nu+1)} .$$

If this really occurs, *i.e.* if there is a non empty intersection between the cohomology of $S_L^{(0)}$ in the Faddeev-Popov 0 charge sector and the image through $S_L^{(1)}$ of the one in the Faddeev-Popov -1 charge, this particular cohomology trivializes itself. Indeed

$$\Delta_{[0]}^{an.(\nu+1)} = S_L^{(1)}\Delta_{[-1]}^{an.(\nu)} - S_L^{(0)}\tilde{\Delta}_{[-1]}^{(\nu+1)} \equiv (S_L^{(0)} + S_L^{(1)})\left(\Delta_{[-1]}^{an.(\nu)} - \tilde{\Delta}_{[-1]}^{(\nu+1)}\right) + \mathcal{O}(\bar{\bar{\Delta}}_{[0]}^{(\nu+2)}) ,$$

and so on on higher orders. This occurred in particular in subsections (5.1.1) and (5.1.2) where

$$\Delta_{[-1]}^{an.} = T^{\underline{i}} \int d^2x \eta_{\underline{i}} \equiv \Delta_{[-1]}^{an.(1)} \quad , \quad \Delta_{[0]}^{an.} = \lambda_{ij} \int d^2x \partial_+\phi^i \partial_-\phi^j \equiv \Delta_{[0]}^{an.(2)} .$$

Then, using the invariance under (5.1) of the classical action, one may check that :

$$S_L^{(1)}\Delta_{[-1]}^{an.(1)} = T^{\underline{i}} \int d^2x \frac{\delta\Gamma^{tot.}}{\delta\phi^{\underline{i}}}|_2 = T^{\underline{k}} g_{ij,\underline{k}}(0) \int d^2x \partial_+\phi^i \partial_-\phi^j + T^{\underline{k}} C^i W^j_{i,\underline{k}}(0) \int d^2x \eta_j$$

$$= S_L^0 \left[-T^{\underline{k}} W^j_{i,\underline{k}}(0) \int d^2x \eta_j \phi^i(x) \right] + T^{\underline{k}} \left[g_{il}(0)[W^l_{j,\underline{k}}(0) - W^l_{\underline{k},j}(0)] + (i \leftrightarrow j) \right] \int d^2x \partial_+ \phi^i \partial_- \phi^j$$

does intercept $\Delta_{[0]}^{an.(2)}$ if $T^{\underline{i}} \neq 0$.

This means that among the parameters λ_{ij}, the ones that are equal to $-T^{\underline{k}}[g_{il}(0) f^l_{j\underline{k}} + g_{jl}(0) f^l_{i\underline{k}}]$ - where $f^l_{i\underline{k}}$ are structure constants of the Lie algebra of G -, are unphysical parameters as they may be ruled out through a particular field redefinition (corresponding to a trivial cohomology). These λ_{ij} corresponds to X (and H)-invariant 2-tensors (see [8] for details).

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