Black Holes: The Inside Story

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Abstract.   Collapse to a black hole leaves a decaying tail of gravitational waves. The fallout from this tail, absorbed into the hole, is strongly blueshifted near the inner horizon, with dramatic effects on the internal geometry. Is the resulting singularity an all-embracing spacelike crunch, as envisioned in the strong cosmic censorship hypothesis? Or is there a mildly singular, lightlike precursor, characterized by inflation of the core mass? We review the evidence for each of these possibilities.

Descent into a black hole is fundamentally a progression in time (Inside a spherical hole, for instance, the radial co-ordinate r is timelike.) To unravel the hole's internal structure is therefore an *evolutionary* problem.

This has far-reaching implications. It means that, up to the stage when curvatures begin to approach Planck levels, the evolution can be followed even without a quantum theory of gravity. Causality does not permit our ignorance of the inner, high-curvature regions to infect the description of the overlying layers afforded by well-established (classical or semi-classical) theory. In this respect, a black hole is simpler than a star.

Moreover (in sharp contrast to the situation in cosmology), initial data for the evolution are known with precision, thanks to the no-hair property. Near the outer horizon, the geometry is that of a Kerr-Newman black hole, perturbed by a tail of gravitational waves whose flux decays as an inverse power v^{-p} of advanced time ($p = 4l+4$ for a multipole of order l [1]). Exploration of the hole's pre-Planckian layers — far from being metaphysical or hopelessly speculative — emerges as a standard (though intricate) applied-mathematical problem, not different in kind from following the motion of a fluid up to the onset of turbulence or a shock.

Twenty-five years ago, Penrose [2] adumbrated the key role of the inner or Cauchy horizon as a critical juncture in the evolution. Just as the event horizon marks the last outpost from which a doomed astronaut can still flash news to the outside, the Cauchy horizon (CH) — a lightlike 3-space characterized by $v = \infty$ — is his last opportunity to *receive* outside news. But then he gets *all* the news. In the few seconds remaining before he plunges through this barrier and into the core of the hole, the entire future of the outer universe passes in fast motion before his eyes!

This speedup of all processes entering the hole at late times — in particular, the fallout from the radiative tail of the collapse — is accompanied by a blueshift which has dramatic effects on the geometry near CH.

A spherical hole, bearing a charge Q and perturbed by a tail of spherical scalar waves, provides a simple prototype for the evolution. Charged and rotating holes have similar horizon structures, and there is evidence [3] (though certainly not universal agreement [4]) that spherical models capture the essential generic features. We here restrict attention to these models for simplicity.

A spherisymmetric 4-geometry is described by the metric

$$ds^2 = g_{ab}dx^a dx^b + r^2 d\Omega^2 \quad (a, b = 0, 1)$$

with $r(x^a)$ a function of a pair of arbitrary coordinates x^0, x^1 which label 2-spheres. Its gradient defines the mass function $m(x^a)$:

$$g^{ab}(\partial_a r)(\partial_b r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}.$$

Einstein's equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ can be recast as two-dimensionally covariant equations for the scalars r and m. The key result is the (1+1) - dimensional wave equation [5]

$$\Box m = -16\pi^2 r^3 T_{ab}T^{ab}$$

which shows explicitly how m is affected by the nonlinearity of the field equations.

Inside the hole, the source term $T_{ab}T^{ab}$ generally diverges as $v \to \infty$ because of the blueshift near CH. (We ignore the unrealistic case of pure inflow without backscatter, for which T_{ab} would be lightlike and the source term zero [6].) In the typical case of a power-law tail, where external observers register merely a slow rise of m toward a final static value m_0,

the experience of interior observers is quite different. Internally, the Weyl curvature and mass function are found [5] to diverge toward infinity like

$$m(v,r) \sim v^{-p} e^{\kappa_0 v} \quad (v \to \infty)$$

where κ_0 is the inner surface gravity associated with the asymptotic external state. (No trace of this drastic change is detectable externally, because news of it propagates with the speed of light, as a gravitational wave, and cannot emerge from the hole.)

This bizarre phenomenon has been dubbed "mass inflation". It draws upon the same source as cosmological mass inflation: unlimited convertibility of gravitational potential energy into material mass-energy. A closed universe and the interior of a black hole are both bottomless wells of gravitational energy.

To demystify this, a simple mechanical model may help. Consider a concentric pair of thin spherical shells which move radially at the speed of light, one inwards, the other outwards. Their mutual potential energy, of order $-m_{\text{in}} m_{\text{out}}/r$, imposes a "tax" — a binding energy — on the total mass-energy of the *outer* shell, whichever that happens to be. At the moment when they cross (at radius r_0, say) this tax is suddenly lifted from the infalling shell, and its gravitating mass boosted, say from m_{in} to m'_{in}. Exact calculation [7] shows that the new masses are

$$m'_{\text{in}} = m_{\text{in}} \left(1 - \frac{2m_{\text{out}}}{r_0} \right)^{-1}$$

$$m'_{\text{out}} = m_{\text{out}} \left(1 - \frac{2m'_{\text{in}}}{r_0} \right).$$

The total energy is, of course, conserved, as, indeed, follows directly from these equations:

$$m'_{\text{in}} + m'_{\text{out}} = m_{\text{in}} + m_{\text{out}}$$

In referring to this effect as "mass inflation", it is important to stress that we are not just using a catchphrase for something of merely formal significance. If the infalling light were absorbed by a lump of charcoal at the center, it would contribute the full value of its boosted mass m'_{in} to the lump.

For an encounter occurring just outside a horizon (i.e., if $r_0 = 2m_{\text{out}} + \epsilon$), the infalling mass can become arbitrarily large. This simple model already provides a fair schematic picture of what happens near the Cauchy horizon of a hole, with the two shells representing parts of the streams of infalling and backscattered radiation.

The divergence of mass and curvature at CH is spread over an area: the singularity is pancake-like and locally mild in the sense that, though tidal forces become infinite, they do not grow fast enough to demolish free-falling test objects before these actually reach CH [8, 9].

However, CH steadily shrinks in area as the transverse energy flux focuses its generators. Eventually, it tapers to a strong singularity at its future end. (The classical calculations

suggest that the strong singularity is spacelike and, generically, of chaotic (mixmaster) type. It would, however, be quite wrong to ignore quantum effects at this stage.)

The early studies [5, 8] led to a provisional working model of the black hole interior which is depicted in Figure 1. It shows a strong spacelike singularity, preceded by a milder precursor CH, characterized by mass inflation, which extends an infinite affine distance back into the past. In effect, CH forms a lightlike bridge which links the final, "hairy" crunch to the asymptotically stationary and hairless outer layers of the hole.

This picture is tentative, and a subject of debate. An opposing view, based on general stability arguments [4] and numerical integration of spherical models [10], is that the lightlike segment of the singularity does not survive generically, but is pre-empted by some kind of spacelike singularity.

The fate of this segment hinges (through Raychaudhuri's equation) on the strength of the transverse (i.e. "outgoing") flux which focuses its generators. (At early retarded times within the hole, this flux is dominated by back-scatter of the infalling tail.) A priori, it is not obvious that its falloff at early times will be rapid enough to allow the contraction of CH to begin from an asymptotically constant radius r_0. If not, the effect would be destruction of the Cauchy horizon. Early models [5, 8] elided this issue by setting initial conditions (with outflow turned on abruptly) at a retarded time after the event horizon.
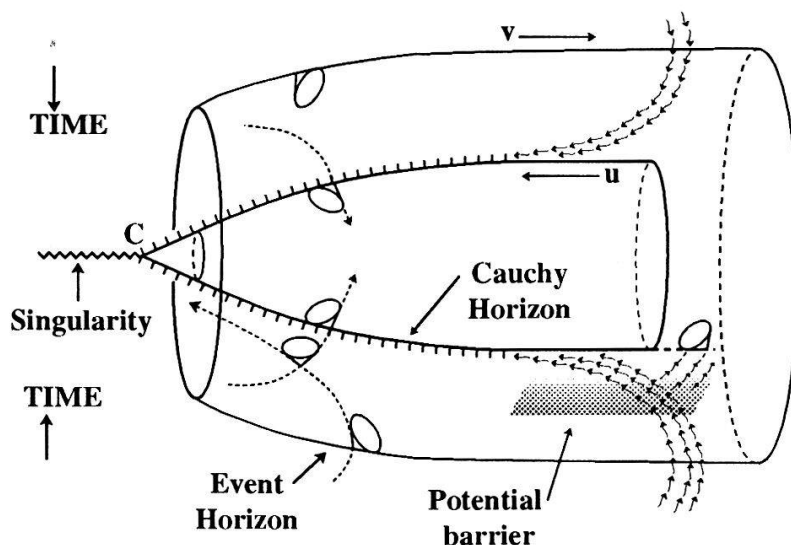

Fig. 1. Internal evolution of a spherical charged black hole (one angular variable suppressed), showing future light cones and a stream of infalling radiation, partially scattered off the potential barrier, with the remainder accumulating along the Cauchy horizon.

We have recently undertaken an examination of this question [11]. Fortunately, it separates into two essentially independent parts. The internal potential barrier which backscatters the infalling tail is located near $r \approx e^2/m_0$ between the horizons, well above the belt of large blueshift around CH. At this upper level, the geometry scarcely differs from the asymptotic

external configuration of mass m_0. The outflux may be simply estimated by treating the scattering on this static background. The influx $T_{vv} \sim v^{-p}$ generates an outflux $T_{uu} \sim \mid u \mid^{-p}$, where u is the static interior retarded time, increasing with depth from $u = -\infty$ at the event horizon. We have developed an analytic approximation to the solution of the spherical scalar-Einstein equations. This indicates that CH does, indeed, survive if T_{uu} decays according to a power-law as $u \to -\infty$.

A more accurate, global picture of the solution requires numerical integration. Sophisticated codes for handling the equations are available [1, 12], and are being adapted to the charged case by several groups [10, 13, 14].

The pioneering numerical study is due to Gnedin and Gnedin [15, 10]. They considered a scalar wave-pulse of finite v–duration imploding into a charged hole. This initial condition was set (a) on the event horizon in their first study [15], and (b) outside the horizon in the second [10]. Only in case (b) is an infalling radiative tail produced (by double scattering off the *external* barrier). Their results illustrate the dramatic effects of the tail. A Cauchy horizon is clearly evident in case (a) and much abbreviated or possibly absent in case (b). Gnedin and Gnedin [10] state that it is absent. But it seems to us that the present numerical accuracy (see their Figure 5) does not warrant any firm statement. With the efforts now being concentrated on this problem [13, 14], a definitive answer should not be long in coming.

In summary, we do not yet see any compelling reason to dismiss the naive picture suggested by the simplest spherical models and encapsulated in Figure 1.

Black hole speleology is pure theory. Only short-lived observers, already entombed, can hope to test its predictions. Yet human curiosity recognizes no boundaries, and has the right to call to its aid whatever rational means are at hand.

But that may not be the end of the story. Evidence is accumulating [16, 17] that our universe may be closed and fated to recollapse . In the last minutes of the crunch its black holes must merge, their mass-inflated cores no longer hidden beneath horizons. What happens next? Will this mass-inflated cosmos bounce? Here, certainly, we are in the realm of speculation.

ACKNOWLEDGMENTS

References

[1] Gundlach, C., Price, R.H. and Pullin, J. (1994). Phys. Rev. D**49**, 883.

[2] Penrose, R. (1968). In *Battelle Rencontres,* C. DeWitt and J.A. Wheeler, eds. (W.A. Benjamin, New York) p.222.

[3] Ori, A. (1992). Phys. Rev. Lett. **68**,2117.
Brady, P.R., Droz, S., Israel, W., and Morsink, S.M. Paper in preparation.
Israel, W. (1993). In *Current Topics in Astrofundamental Physics,* N. Sanchez and A. Zichichi, eds. (World Scientific, Singapore) p.430.

[4] Yurtsever, U. (1993). Class. Quant. Grav. **10**, L17

[5] Poisson, E. and Israel, W. (1990). Phys. Rev. **D41**, 1796.

[6] Hiscock, W.A. (1981). Physics Lett. **83**A, 110.

[7] Dray, T., and 't Hooft, G. (1985). Commun. Math. Phys. **38**,119.
Barrabès, C. Israel, W. and Poisson, E. (1990). Class Quant. Grav. **7**, L273.

[8] Ori, A. (1991). Phys. Rev. Lett. **67**,789.

[9] Balbinot, R., Brady, P.R., Israel, W. and Poisson, E. (1991). Physics Lett. **A161**, 223.

[10] Gnedin, M.L. and Gnedin, N.Y. (1993). Class Quant. Grav. **10**, 1083.

[11] Bonanno, A., Droz, S., Israel, W. and Morsink, S.M. (1994). Phys. Rev. Lett. (submitted), gr-qc/9403019.

[12] Choptuik, M.W. (1993). Phys. Rev. Lett. **70**,9.
Gómez, R., Winicour, J. and Isaacson, R.A. (1992). J. Comp. Phys. **98**,11.

[13] Gundlach, C. Personal communication.

[14] Choptuik, M.W. Personal communication.

[15] Gnedin, N.Y. and Gnedin, M.L. (1992). Sov. Astron. **36**,216.

[16] Tremaine, S. (1992). Physics Today (February 1992) p29.

[17] Fahlman, G., Kaiser, N., Squires G. and Woods D. (1994), Astrophys. J. (in press).