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New concepts in quantum gravity

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Abstract. Based on weak equivalence principle (WEP) and the basic principle of quantum mechanics, we introduce concepts of the quantum of gravitational charge, the quantum level for gravity, and the expression for the quantum of energy of gravitational fields. Combination of these concepts with the dual geometric-gauge field aspects of gravity shows the following: (1) the quantum of gravitational charge corresponds to the quantum of the minimal energy and is the coupling constant in gauge field aspect of gravity; (2) When geometric aspect converts to gauge field aspect, spacetime translation of geometric gravity converts to internal symmetries of gauge gravity, the spacetime of geometric gravity converts to internal spacetime, and the metric function $h^{\mu\nu}$ converts to the gauge potential $h^{a\mu}$. The gauge potential $h^{a\mu}$, thus, is a spin one field; (3) Gravity should be quantized at the minimal energy quantum level, rather than the elementary particle level; (4) the self-interaction related difficulties in quantazition of gravity, the puzzle of the dual role played by the $h^{\mu\nu}$, and the nonlocalization of $t^{\mu\nu}$, may be resolved.

1. Introduction

A long standing problem in gravitation physics is that there is no consistent quantum theory of gravity yet [1]. An essential difficulty in the quantization of geometric and gauge theories of gravity is caused by the dimensioness of the Newtonian constant G [2, 3] which is assumed to be the coupling constant. The geometric nature of GR and its failure to conform to the pattern of the other gauge theories make it difficult to understand gravity thoroughly and to quantize it perturbatively [4]. To resolve the G related difficulty and reveal similarities between gravity and other forces, the dualism of gravity has been proposed [5].

Another main difficulty in the quantization of geometric gravity is related with the highly non-linear terms $t^{\mu\nu}$ interpreted as the energy-momentum of gravity which represents the self-interaction. Actually, there even exist serious difficulties with $t^{\mu\nu}$ itself. The $t^{\mu\nu}$ does not transfer as tensor components, so it is called a pseudotensor. Moreover, the energy of gravity in geometric aspect is nonlocalizable, i.e., a coordinate transformation may make the energy density t^{00} to vanish. The problem of defining an energy-momentum tensor has been tackled by many authors [2, 6], and a variety of pseudotensors has been proposed as the expression of gravitational energy-momentum. It is, however, generally agreed that no completely satisfactory solution has been offered [7]. One way to show how $t^{\mu\nu}$ is interpreted as the energy-momentum of gravity is to follow the derivation of Gupta, Deser, Ward (GDW), etc. [2, 8]: start from a linear massless spin 2 field equation in the flat space,

$$\partial^{\mu} \partial_{\mu} A^{\nu\lambda} = 0,$$

then adjoin to the linear equation a source $t_0^{\mu\nu}$ attributed to the energy-momentum of the free spin 2 field, a further source $t_1^{\mu\nu}$ due to this one, etc. Repeating the same process one obtains an infinite number of terms in the Lagrangian, $L = \sum_{n=0}^{\infty} L_n$, and in the energy-momentum $t^{\mu\nu}$, $t^{\mu\nu} = \sum_{n=0}^{\infty} t_n^{\mu\nu}$. At the classical level, it is true that $t^{\mu\nu}$ have infinite terms, because there is no physical law to prohibit $t^{\mu\nu}$ from being infinitesimal. *Assume* the gauge field $A^{\mu\nu}$ is the metric function $h^{\mu\nu}$, then, the full Einstein equation emerges,

$$\partial_{\lambda} \partial^{\lambda} h^{\mu\nu} = -k(T^{\mu\nu} + t^{\mu\nu}).$$

But it is not clear why a gauge field $A^{\mu\nu}$ with the dimension of potential would be identical with a dimensionless metric function $h^{\mu\nu}$.

The GDW approach also shows the puzzle of the dual role played by $h^{\mu\nu}$ as both the quantity which describes the dynamics of gravity and the quantity which describes the spacetime structure. It has been realized that this puzzle may cause fundamental difficulties [9].

Before applying the dualism of gravity to resolve self-interaction related difficulties and quantizing gravitational gauge field, several fundamental questions need to be answered. First, what is the quantum of the gravitational mass charge. Second, at what level gravity should be quantized. Finally, if the gravitational quantum level (GQL) differs from the elementary particle level (EPL) as shown below, does one need to rethink of the concept of unification with gravity.

In this paper we review the dualism of gravity first, introduce the concept of a quantum of gravitational mass charge, find the quantum level for gravity, and then propose expressions of the quantum of gravitational field. We show that the introduction of those new concepts provides not only a possible resolution of the self-interaction related problem in the quantization of gravity, but also an insight into the concept of the unification with gravity.

2. The dualism of gravity

In this section we review the dualism of gravity [5]. In spite of geometric interpretation, Einstein equation

$$R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R = (8\pi G/c^4) T^{\mu\nu}, \tag{1}$$

has a well known non-abelian Yang–Mills form [10],

$$\frac{\partial G^{\mu\nu\lambda}}{\partial x^{\lambda}} = -\frac{4\pi G}{c^4} (T^{\mu\nu} + t^{\mu\nu}), \tag{2}$$

where the tensor gravitational field strength is

$$G^{\mu\nu\lambda} = (\tfrac{1}{4})(\bar{h}^{\mu\nu,\lambda} - \bar{h}^{\mu\lambda,\nu}),$$
$$\bar{h}^{\mu\nu} = h^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}h^{\alpha}_{\alpha}, \qquad h^{\mu\nu} = g^{\mu\nu} - \eta^{\mu\nu}. \tag{3}$$

The Lagrangian for equation (2) in quadratic terms of $G^{\mu\nu\lambda}$ is

$$L = -\frac{c^4}{4\pi G} G^{\mu\nu\lambda}G_{\mu\nu\lambda} + \bar{h}^{\mu\nu}T_{\mu\nu} + L^{(1)} \equiv L_0 + \bar{h}^{\mu\nu}T_{\mu\nu} + L^{(1)}. \tag{4}$$

The $L^{(1)}$ is determined by $t^{\mu\nu}$, the energy-momentum pseudotensor of gravity,

$$t^{\mu\nu} = \frac{\partial L^{(1)}}{\partial \bar{h}^{\mu\nu}} - \frac{\partial}{\partial x^{\lambda}} \frac{\partial L^{(1)}}{\partial \bar{h}^{\mu\nu,\lambda}}. \tag{5}$$

Now consider the G related difficulty. Einstein theory associates a physical entity $T^{\mu\nu}$ with geometry $R^{\mu\nu}$. Thus, there must be a dimensional constant to balance the units of both sides of equation, which is G. There is really an inevitable dilemma (called the G dilemma): without G, the geometric interpretation would be violated, with G as a coupling constant, gravity couldn't be quantized perturbatively. The G dilemma and the puzzle of the dual role of $h^{\mu\nu}$ indicate that there might be one or more major conceptual revolution before gravity is quantized [4, 11]. Note that, so far, the EPL has been assumed to be the quantum level for gravity. However, we emphasize that the quantum level for a specific gauge field depends on the quantum of the gauge field's charge, and, thus, is relative. The GQL differs from the EPL as shown in Section 5.

In order to resolve the G dilemma, we propose the dual geometric-gauge field aspects of gravity [5], which states: (1) When one quantizes gravity or studies the physical properties of gravity, gravity should be considered as a gauge field with an internal symmetry in the flat space; (2) When one deals with astronomical or classical phenomena, the geometric description of gravity with a spacetime symmetry is a familiar choice; (3) Two aspects including internal and spacetime symmetry groups should be reconciled, which ensures the correspondence principle. Einstein equation has to be treated as a physical law whenever one converts the system of unit from the MKS (or CGS) to other systems of unit, otherwise the geometric interpretation of Einstein theory would be destroyed. The dualism of gravity provides a possible resolution for the puzzle of the dual role of $h^{\mu\nu}$.

3. Conversion between internal and spacetime symmetries

To consider the gauge field aspect of gravity, we convert the gravitational mass charge m_g to $Q_g \equiv m_g\sqrt{4\pi G}$, unit $[Q_q] = m\sqrt{N}$ (m: meter, N: Newton), and $\bar{h}^{\mu\nu}/\sqrt{4\pi G} \to \bar{h}^{\mu\nu}$. This is the conversion from the MKS (or CGS) system of unit to the H-L system of unit. This conversion brings us several interesting results [5]. First let us study what happens to the symmetry property of gravity, and its consequences.

Under the conversion from geometric aspect to gauge field aspect, a spacetime symmetry group converts to an internal symmetry group, as long as the coupling constant is the gravitational mass charge Q_g.

At the classical level, the sources of gravity are just the energy and momentum distributions, associated through Noether's theorem with the spacetime translation group. Let us consider the spacetime translation group T^4 which produces the transformation

$$\exp\,[(i/\hbar)P^{\mu}\varepsilon_{\mu}], \qquad x'_{\mu} = x_{\mu} + \varepsilon_{\mu'} \qquad (x_0 = ct) \tag{6}$$

where ε_{μ} is spacetime displacement. In the MKS unit system,

$$\text{unit}\,[p^{\mu}\varepsilon_{\mu}/\hbar] = \text{unit}\,[m_i cx/\hbar] = \text{unit}\,[m_g cx/\hbar]. \qquad \text{(WEP)} \tag{7}$$

Let us reexpress Eq. (6) as

$$\exp\,[(i/\hbar)m_g \tau^{\mu}\theta_{g\mu}], \tag{8}$$

where $\tau^{\mu} \equiv (1, \beta)$, $\theta_{g\mu} \equiv c\varepsilon_{\mu}$, $\beta^j \equiv v^j/c$, $j = 1, 2, 3$, c is the speed of light. When considering the gauge field aspect, we convert to the H–L unit system. The nature of symmetry of gravity will still be the translation, but in an internal spacetime. In the H–L unit system, we have

$$\exp\,[(i/\hbar)Q_g \tau^{a}\theta_g^{a'}], \tag{9}$$

where $\theta_g^{a'} \equiv \theta_g^a/\sqrt{4\pi G}$, unit $[\theta_g^{a'}] = \sqrt{Ns}$, and unit $[m_g\theta_g/\hbar] \equiv$ unit $[Q_g\theta'_{ga}/\hbar]$. θ'_{ga} has the exact same unit as that of the internal space of QED. Here since τ^{μ} becomes the translation in the internal space in the gauge aspect of gravity, we have used the internal group index a to replace the spacetime group index μ in τ^{μ} and θ_g^{μ}. Comparing with other gauge groups, Q_g, τ^a, and θ_g^a correspond to coupling constant, generator of the internal group, and coordinate of internal space, respectively. The 4-dimension internal spacetime associated with gauge gravity will be converted to spacetime when converting to geometric aspect in the MKS unit system.

Although $T^{\mu\nu}$ is symmetric in μ and ν, these indices have different origins. One of them corresponds to the standard four-vector index of the current, whereas the other one labels the four current associated with the four independent translation in spacetime. Thus, one of indexes (μ and ν) of $T^{\mu\nu}$, say μ, is the group index. In the gauge field aspect, the group index becomes an internal group index, a. On the other hand, a gauge field carries a four-vector spacetime index and a group index. The arguments on indexes of both $T^{a\nu}$ and gauge field $h^{a\mu}$ are consistent. Therefore, gravitational gauge field $h^{a\mu}$ carries an internal translation group index and a spacetime index, and gauge bosons of gravity are spin one particles, which provides a testable feature of the dualism of gravity. In the gauge aspect of gravity with the *H-L* system of unit, we use $G^{a\nu\lambda}$, $h^{a\nu}$, $J^{a\nu}$, and $j^{a\nu}$, Einstein equation becomes

$$\frac{\partial G^{a\nu\lambda}}{\partial x^{\lambda}} = -(J^{a\nu} + j^{a\nu}), \tag{10}$$

$$L = -G^{a\nu\lambda}G_{a\nu\lambda} + \bar{h}^{a\nu}J_{a\nu} + L^{(1)}. \tag{11}$$

We have replaced $T^{a\nu}$ and $t^{a\nu}$ by $J^{a\nu}$ and $j^{a\nu}$, because, with an internal group index, $T^{a\nu}$ and $t^{a\nu}$ now are really currents of matter and gravitational field, respectively. We reserve the symbols $T^{\mu\nu}$ and $t^{\mu\nu}$ for energy-momentum.

That the metric $h^{\mu\nu}$ converts to the vector gauge potential $h^{a\mu}$ provides a new approach out of the difficulty of the nontensor character of $h^{\mu\nu}$. In the geometric aspect, the metric function $h^{\mu\nu}$, under a Lorentz transformation Λ^{μ}_{ρ}, will be subjected to an additional gauge transformation,

$$h'^{\mu\nu} = \Lambda^{\mu}_{\gamma}\Lambda^{\nu}_{\rho}h^{\gamma\rho} + \epsilon^{\mu,\nu} + \epsilon^{\nu,\mu},$$

where $\epsilon^{\mu} = x'^{\mu} - x^{\mu}$ is an infinitismal coordinate transformation. This implies the non-tensor character of metric $h^{\mu\nu}$ (see Ref. [2] and references within). Now in the gauge aspect, $h^{a\nu}$ is a gauge potential. Under a Lorentz transformation $\Lambda^{\mu\rho}$, $h^{a\mu}$ transforms like a vector,

$$h'^{\mu} = \Lambda^{\mu}_{\rho}h^{\rho}, \qquad h^{\rho} \equiv \tau_a h^{a\rho}. \tag{12}$$

The gauge transformation which $h^{a\mu}$ subjected to is

$$h^{\mu} \to h^{\mu} + \partial^{\mu}\phi,$$

where ϕ is an arbitrary scalar function.

Now the nonlinear current of gravitational gauge field $j^{a\nu}$ might still cause problem in the quantization of gravitational gauge field. We will show later that the introduction of the quantum level for gauge gravity may cure this problem.

4. Quantum of gravitational mass charge

The Einstein gauge field equation (10) is a classical equation. Before quantizing gravitational gauge field which interacts with a matter source, we have to find out what is the quantum level for gravity. It has been assumed that gravity would be quantized at the elementary particle level (EPL) (in this paper when we use the word "elementary particle", photons are excluded).

For a given gauge field, the quantum level is a level at which one has to consider *the discrete quanta of charges of the gauge field*, and then, quantizes the field interacting with discrete charges at that level. The quantum level of a gauge field is determined by the quantum of the charge of the gauge field. We argue, therefore, that a quantum level is relative, i.e., different gauge fields may have different quantum levels, because of that the quanta of different gauge fields are at different levels as shown later. The quantum level for a gauge field may be the classical level for another. We show below that the EPL is not the quantum level for gravity, but a classical level.

In QED, the quantum, $Q_{e,q}$, of electric charges is the charge carried by an electron in the *H-L* system of unit. Elementary particles carry either one or zero quantum of electric charge (in this paper we do not consider quarks, which does not affect our argument). So the EPL is the quantum level for QED. Similarly, the EPL is the quantum level for weak and strong interactions. When one considers a system with a source carrying $NQ_{e,q}$ and N is large, then it becomes a classical system.

Likewise, the quantum level for gravity is determined by the quantum, $Q_{g,q}$, of gravitational charges, i.e., the minimal gravitational charge. What is the quantum of gravitational charges? How many quanta of gravitational charge does an electron carry?

The quantum of gravitational charge may be determined by the WEP which says that the gravitational mass is equal to inertial mass, $m_g = m_i = E/c^2$, where E is the energy. Therefore the quantum of gravitational charge corresponds to the quantum of minimal energy. The quanta of energy are $\hbar\omega$. So far, to our knowledge, there is neither theoretical prediction nor experimental evidence of a minimal energy. However, there must be theoretically or practically a minimal energy and its quantum. For our discussion in this paper, it is sufficient to know the existence of the quantum of a minimal energy, $E_{\min}$. Thus, the quantum of gravitational charge, or the unit of gravitational charges, or the coupling constant of gravity, corresponds to the quantum of the minimal energy,

$$Q_{g,q} = \sqrt{4\pi G}\, m_g = \sqrt{4\pi G}\, E_{\min}/c^2, \tag{13}$$

which is the gravitational counterpart of the quantum of the electric charge. To show that the energy level of $Q_{g,q}$ is much lower than that of elementary particles, let us assign a value to $E_{\min}$, $E_{\min} \simeq \hbar(\omega = 1/s)$. Then

$$Q_{g,q} = 3.398 \times 10^{-56}\, m\sqrt{\mathrm{N}}. \qquad \text{(m: meter, N: Newton)}$$

The actual value of the minimal energy might be equal to or smaller than the assigned value, but it would not affect the argument here. An electron contains one quantum ($Q_{e,q}$) of electric charge and N quanta ($NQ_{g,q}$) of gravitational mass charges, where

$$N = \sqrt{4\pi G}\, m_e/Q_{g,q} \simeq 7.7633 \times 10^{20}, \tag{14}$$

i.e., an electron and, thus, elementary particles contains a huge number of quanta of gravitational mass charge.

There is no Fermion at the level of the minimal energy. The quanta of gravitational charges, therefore, are either scalar or spin one particles, bosons, with or without rest mass. The quantum of gravitational mass charge as a particle is described by a wave function. For a scalar quantum of gravitational mass charges with mass m (or massless $m = 0$), we have,

$$\partial^\alpha \partial_\alpha \psi + m^2 \psi = 0, \tag{15}$$

$$L_{\text{scalar}} = (\partial^\alpha \psi)^* \partial_\alpha \psi - m^2 \psi^* \psi. \tag{16}$$

In the gauge field aspect, gravity is treated as a normal gauge field. Therefore, interaction of the scalar quantum with gauge gravitational field $h^{a\mu}$ is introduced by the minimal substitution

$$\partial^\mu \to \partial^\mu + iQ_{g,q} h^\mu, \tag{17}$$

Then we have

$$[(\partial_\mu + iQ_{g,q}h_\mu)(\partial^\mu + iQ_{g,q}h^\mu) + m^2]\psi = 0, \tag{18}$$

$$\partial_\nu G^{a\mu\nu} = -(J^{a\mu} + j^{a\mu}), \tag{19}$$

$$J^{a\mu} = -iQ_{g,q}\tau^a[\psi^* \partial^\mu\psi - (\partial^\mu\psi^*)\psi] + 2Q^2_{g,q}h^{a\mu}\psi^*\psi, \tag{20}$$

$$L = -G^{a\mu\nu}G_{a\mu\nu} + L^{(1)} + [(\partial_\mu + iQ_{g,q}h_\mu)\psi]^*[(\partial^\mu + iQ_{g,q}h^\mu)\psi] - m^2\psi^*\psi, \tag{21}$$

where $L^{(1)}$ gives $j^{a\mu}$. We will discuss self-interaction $j^{a\mu}$ in Section 6 and 7.

A spin one quantum of gravitational mass charges with mass μ as a particle is described by a wave function ϕ^α which satisfies,

$$(\partial_\lambda\partial^\lambda + \mu^2)\phi^\alpha = 0, \tag{22}$$

$$L_{\text{vector}} = -\tfrac{1}{2}(\phi^{\alpha\lambda})^*\phi_{\alpha\lambda} + \mu^2\phi^{*\alpha}\phi_\alpha, \tag{23}$$

where

$$\phi^{\alpha\lambda} \equiv \partial^\alpha\phi^\lambda - \partial^\lambda\phi^\alpha. \tag{24}$$

In order to obtain the equations which describe the interaction of the spin one quantum of gravitational mass charge with gauge gravitational field, we apply the minimal principle again,

$$[(\partial^\lambda + iQ_{g,q}h^\lambda)(\partial_\lambda + iQ_{g,q}h_\lambda) + \mu^2]\phi^\alpha = 0, \tag{25}$$

$$\begin{aligned} &\partial_\lambda G^{a\alpha\lambda} = -(J^{a\alpha} + j^{a\alpha}),\\ &J^{a\alpha} = iQ_{g,q}\tau^a(\phi^*_\lambda\phi^{\alpha\lambda} - \phi^{*\alpha\lambda}\phi_\lambda) - 2Q^2_{g,q}h^{a\alpha}\phi^{*\lambda}\phi_\lambda\\ &\qquad + Q^2_{g,q}[\phi^{*\alpha}(h^{a\lambda}\phi\lambda) + (h^{a\lambda}\phi^*_\lambda)\phi^\alpha], \end{aligned} \tag{26}$$

$$\begin{aligned} L = &-G^{a\alpha\lambda}G_{a\alpha\lambda} + L^{(1)} - \tfrac{1}{2}[(\partial^\alpha + iQ_{g,q}h^\alpha)\phi^\lambda - (\partial^\lambda + iQ_{g,q}h^\lambda)\phi^\alpha]^*\\ &\times [(\partial_\alpha + iQ_{g,q}h_\alpha)\phi_\lambda - (\partial_\lambda + iQ_{g,q}h_\lambda)\phi_\alpha] + \mu^2\phi^{*\alpha}\phi_\alpha. \end{aligned} \tag{27}$$

The quantum of a non-relativistic gravitational mass charge may be described by Schrodinger equation

$$[-(\hbar^2/2m)\nabla^2 + V(x)]\phi = E\phi.$$

The introduction of the quantum of gravitational charges has several important impacts.

5. Quantum level for gravity

After introducing the concept of the quantum of gravitational charge, we may find the quantum level for gravity. First, an elementary particle contains a huge number of quanta of gravitational charges, thus, the EPL at which other three interactions are quantized and unified is not a quantum level for gravity, but a classical level. Gravity should be quantized at the gravitational quantum level

(GQL), i.e., the quantum level of gravitational charges, or the minimal energy quantum level, which is far below the EPL and is the bottom quantum level because there is no quantum carrying energy less than the minimal energy.

The choice of internal symmetry groups of gravity is constrained by the GQL. For example, since there is no Fermion at the GQL, SU(2) will not be the internal symmetry group for quantum gravity.

An interesting question is that: are there unknown short-range forces which glue quanta of gravitational charges (bosons) to form, at least part of, elementary particles (quarks)? This might provide part of a possible answer to the question: what is the origin of mass of elementary particles. A possible gravitational formation of elementary particles has been studied in a system of a large number of bosons bounded to each other by gravitational force and Yukawa potential [12]. Unknown force(s) (if any) would be very strong and have very short-range.

Table: Classical and quantum levels for all interactions

macro-world	——	classical level for gravitational and electromagnetic force; no weak and strong forces
	↑	four known forces glue elementary particles to form macro-world
elementary particle level	——	classical level for gravity; quantum level for electroweak and strong forces; no unknown forces
	↑	gravity and unknown superstrong force (if any) glue quanta of energy to form elementary particle(s)?
bottom quantum level	——	quantum level for gravity and unknown force(s) (if any); no electroweak and strong forces

Although the gravitational effects at the EPL are negligible, it has been pointed out that at the Planck energy level, gravitational effect on elementary particle physics should be taken into account. Now it is easy to see that at the Planck energy level one should consider the effects of classical gravity, by calculating that the Planck energy corresponds to N quanta of gravitational mass charge, where

$$N \simeq 1.855 \times 10^{43}.$$

The fact that gravity and other three interactions are quantized at different quantum levels makes us rethink of unification with gravity.

6. Resolution of $t^{\mu\nu}$ related problems

In this section we first make a remark on $t^{\mu\nu}$ from the point of view of both the dualism of gravity and the GQL. Then we propose a new approach to overcome the difficulties caused by the self-interaction in the quantization of gravity.

Einstein equation connects geometry with physical entity. One of ways to define energy-momentum of geometric gravity is to split the geometry terms into linear and nonlinear terms, then interpret the nonlinear terms as the energy-momentum pseudotensor $t^{\mu\nu}$ of gravity. This interpretation of $t^{\mu\nu}$ is an example of the mixture of concepts of geometry and physics entity: gravitational field, its energy-momentum, and spin of gauge boson of gravitational field are physical concepts, whereas $t^{\mu\nu}$ is made of geometric terms $h^{\mu\nu}$, $h^{\mu\nu,\lambda}$, etc. To consider $t^{\mu\nu}$ as the energy-momentum of gravitational field reflects the efforts of trying to interpret geometry by physical terms, and to study physical properties of gravity by geometric terms. Conceptually, to try to use geometric terms to represent energy-momentum of gravitational field is the same as to try to use geometric terms to represent energy-momentum of matter field, $T^{\mu\nu}$. If these efforts succeeded, Einstein equation would have a pure geometric form

geometry —— geometry,

and became a pure geometric theory. So far these efforts are not quite successful. This is part of reasons why we propose the dualism of gravity.

The dualism of gravity states that physical properties of gravity should be studied in the gauge aspect of gravity. Now we study the energy-momentum of gravitational field in the gauge aspect. First in the gauge aspect, $h^{a\mu}$ is no longer a geometric metric, but a gauge potential. Therefore it is impossible to vanish t^{00} by a coordinate transformation. Second, the dualism also requires that the correspondence principle should be satisfied. However the correspondence principle merely requires that a quantum theory must be consistent with the classical theory in the limit of large quantum numbers but does not specify uniquely the form of the quantum theory [13]. At the classical level the GDW derivation gives an infinite series of $t^{\mu\nu}$. At the GQL, however, this is no longer the case. There is the minimal energy, $E_{\rm min}$, corresponding to the GQL. This fact provides a physical constrain on the infinite series of $t^{\mu\nu}$, i.e., any $\int dv\, t_n^{00} < E_{\rm min}$ loses its physical meaning and, then, the infinite series of $t^{\mu\nu}$ should be cut off at this t_n^{00}. On the other hand, at the GQL, the quantum of gravitational mass charge as the source corresponds to the energy of order of $E_{\rm min}$. The energy of gravitational field of the source has, at most, the same order of energy of the source (If this is the case we will say that the source has a gravitational origin). Therefore, at the GQL, $t^{\mu\nu}$ contains finite term(s).

Actually, in the gauge field aspect, we deal with the spin 1 gravitational gauge field $h^{a\mu}$ and its current $j^{a\mu}$, instead of the geometric gravity $h^{\mu\nu}$ and its energy-momentum $t^{\mu\nu}$. Following the GDW procedure [8], we start with a field equation for a spin 1 massless particle in the absence of interaction

$$\partial_\nu G^{a\mu\nu} = 0, \tag{29}$$

where

$$G^{a\mu\nu} = \tfrac{1}{4}(\bar{h}^{a\mu,\nu} - \bar{h}^{a\nu,\mu}), \qquad a\text{: internal index.}$$

Equation (29) is given by a lagrangian

$$L_0 = -G^{a\mu\nu}G_{a\mu\nu}. \tag{30}$$

In the presence of gravitational self-interaction, equation (29) becomes

$$\partial_\nu G^{a\mu\nu} = -j^{a\mu}, \tag{31}$$

where $j^{a\mu}$ is the current of gravitational field and will be determined by successive approximations. The Lagrangian L_0 yields the current $j_0^{a\mu}$ of the free gravitational field. One, from WEP, has to consider $j_0^{a\mu}$ as a source, therefore, has to modify L_0 in such a way that in the resulting equation the $j_0^{a\mu}$ appears on the right-hand side of equation (29). For this, one has to take a Lagrangian of the form

$$L_0 + L_1.$$

However, then, the current of gravitational field becomes

$$j_0^{a\mu} + j_1^{a\mu}.$$

Again one has to modify $L_0 + L_1$ to

$$L_0 + L_1 + L_2$$

such that $j_0^{a\mu} + j_1^{a\mu}$ appears on the right-hand side of equation (29).

This series introduces an infinite number of terms in the Lagrangian

$$L = \sum_{n=0}^{\infty} L_n,$$

and the current

$$j^{a\mu} = \sum_{n=0}^{\infty} j_n^{a\mu},$$

and leads to the full nonlinear Einstein equation when converted to the geometric aspect of gravity. Ref. [14] shows how to convert from an internal current to a energy-momentum tensor. Note that one must have

$$j_n^{a\mu} > j_{n+1}^{a\mu}. \tag{32}$$

Similarly to the case of $t^{\mu\nu}$, at the GQL, $j^{a\mu}$ must be cut at $\int dv\, j_n^{a0} < Q_{g,q}\tau^a$.

7. Quantum of energy of gravitational field

Now we propose an alternative description of self-interaction $j^{a\mu}$ of a system consisting of a quantum of gravitational charge and gravitational field.

Before quantizing a gauge field, one must first find the expression for the quantum of charges at the quantum level, for example, the Dirac equation of an electron (positron) field in QED. Following the same idea, we have introduced the concept of the GQL and the expressions of the quantum of gravitational mass charge, equations (18–21, 25–27).

Now we consider the self-interaction aspect of gravitational field. There is energy stored in a gravitational field, say a gravitational wave. Although gravitational field up to this point has not been quantized yet, it is certain that the energy of gravitational field is quantized [2], because, from the principle of quantum mechanics, all kinds of energy are quantized. This fact makes the quantization of gravitational gauge field differ from that of QED in which photon is a consequence of the quantization of electromagnetic field. The quantum of energy of the gravitational field carries quantum of gravitational charge, and, thus, couples to gravitational field. This coupling represents the self-interaction of gravitational field, which is similar to a non-abilian Yang–Mills field.

To find the expression of the energy quantum of the gravitational field, let us start at the GQL. At this level both a quantum of gravitational charge and a quantum of the energy of the gravitational field are on the same footing to the source of gravitational field. Since the gravitational gauge field $h^{a\mu}$ carries one spacetime index μ and one internal index a, and, thus, is a vector field [5], it is reasonable to assume that the quantum of the energy of gravitational field is a spin one particle, called gravitophoton. As a spin one massless particle, a gravitophoton may be described by a wave function ζ^{α}. The coupling of a gravitophoton with gravitational field is described by the minimal principle. The Lagrangian of a system consisting of a spin one quantum of gravitational charge, gravitational field, and the quantum of gravitational field, is

$$\begin{aligned} L = & -G^{a\alpha\lambda}G_{a\alpha\lambda} - \tfrac{1}{2}[(\partial^{\alpha} + iQ_{g,q}h^{\alpha})\zeta^{\lambda} \\ & - (\partial^{\lambda} + iQ_{g,q}h^{\lambda})\zeta^{\alpha}]^{*}[(\partial_{\alpha} + iQ_{g,q}h_{\alpha})\zeta_{\lambda} \\ & - (\partial_{\lambda} + iQ_{g,q}h_{\lambda})\zeta_{\alpha}] - \tfrac{1}{2}[(\partial^{\alpha} + iQ_{g,q}h^{\alpha})\phi^{\lambda} \\ & - (\partial^{\lambda} + iQ_{g,q}h^{\lambda})\phi^{\alpha}]^{*}[(\partial_{\alpha} + iQ_{g,q}h_{\alpha})\phi_{\lambda} \\ & - (\partial_{\lambda} + iQ_{g,q}h_{\lambda})\phi_{\alpha}] + \mu^{2}\phi^{*a}\phi_{\alpha}. \end{aligned} \tag{33}$$

where $G^{a\alpha\lambda}$ and $h^{a\alpha}$ are the gravitational field strength and potential, respectively, ζ^{α} the wave function of the quantum of gravitational field, ϕ^{α} the wave function of the vector quantum of gravitational charge, and μ the mass of the ϕ^{α} field.

If the quantum of gravitational charge is a scalar field ψ with mass m, then the L is

$$\begin{aligned} L = & -G^{a\alpha\lambda}G_{a\alpha\lambda} - \tfrac{1}{2}[(\partial^{\alpha} + iQ_{g,q}h^{\alpha})\zeta^{\lambda} \\ & - (\partial^{\lambda} + iQ_{g,q}h^{\lambda})\zeta^{\alpha}]^{*}[(\partial_{\alpha} \\ & + iQ_{g,q}h_{\alpha})\zeta_{\lambda} - (\partial_{\lambda} + iQ_{g,q}h_{\lambda})\zeta_{\alpha}] \\ & + [(\partial^{\alpha} + iQ_{g,q}h^{\alpha})\psi]^{*}[(\partial_{\alpha} + iQ_{g,h}h_{\alpha})\psi] - m^{2}\psi^{*}\psi. \end{aligned} \tag{34}$$

The self-interaction of gravitational gauge field is described by $j^{a\alpha}$ which is obtained from equations (33 and 34),

$$\begin{aligned} j^{\alpha} = & iQ_{g,q}\{\zeta_{\lambda}^{*}[\partial^{\alpha}\zeta^{\lambda} - \partial^{\lambda}\zeta^{\alpha} + iQ_{g,q}(h^{\alpha}\zeta^{\lambda} - h^{\lambda}\zeta^{\alpha})] \\ & - [\partial^{\alpha}\zeta^{\lambda} - \partial^{\lambda}\zeta^{\alpha} + iQ_{g,q}(h^{\alpha}\zeta^{\lambda} - h^{\lambda}\zeta^{\alpha})]^{*}\zeta_{\lambda}\}, \end{aligned} \tag{35}$$

where ζ^{α} satisfies the following equation,

$$\partial_{\alpha}[\partial^{\alpha}\zeta^{\lambda} - \partial^{\lambda}\zeta^{\alpha} + iQ_{g,q}(h^{\alpha}\zeta^{\lambda} - h^{\lambda}\zeta^{\alpha})]$$
$$= -iQ_{g,q}h^{\alpha}[\partial_{\alpha}\zeta^{\lambda} - \partial^{\lambda}\zeta_{\alpha} + iQ_{g,q}(h_{\alpha}\zeta^{\lambda} - h^{\lambda}\zeta_{\alpha})]. \quad (36)$$

Note that we have used the complex wave functions ζ^{α}, ϕ^{α}, and ψ in equations (33–36) in order to keep the door opening to a possible existence of negative gravitational charge.

In the self-interaction j^{α} of equation (35), h^{α} and ζ^{α} are coupled. To decouple them, we may use Petian–Duffin–Kemmer theory [15]

$$L = -\zeta^{*}\beta_{\lambda}\,\partial^{\lambda}\zeta, \quad (37)$$

with

$$\beta_{\alpha}\beta_{\lambda}\beta_{v} + \beta_{v}\beta_{\lambda}\beta_{\alpha} = \delta_{\alpha\lambda}\beta_{v} + \delta_{v\lambda}\beta_{\alpha}, \quad (38)$$

and ζ has sixteen components and describes both spin 0 and spin 1 fields, here we only need the spin 1 field to describe the quantum of energy of gravitational field.

Now equations (33 and 34) become, respectively,

$$L = -G^{a\mu v}G_{a\mu v} - \zeta^{*}\beta_{\mu}(\partial^{\mu} + iQ_{g,q}h^{\mu})\zeta$$
$$-\tfrac{1}{2}[(\partial^{\mu} + iQ_{g,q}h^{\mu})\phi^{v} - (\partial^{v} + iQ_{g,q}h^{v})\phi^{\mu}]^{*}[(\partial_{\mu} + iQ_{g,q}h_{\mu})\phi_{v}$$
$$-(\partial_{v} + iQ_{g,q}h_{v})\phi_{\mu}] + m^{2}\phi^{*\lambda}\phi_{\lambda}, \quad (39)$$

and

$$L = -G^{a\mu v}G_{a\mu v} - \zeta^{*}\beta_{\mu}(\partial^{\mu} + iQ_{g,q}h^{\mu})\zeta$$
$$+[(\partial^{\alpha} + iQ_{g,q}h^{\alpha})\psi]^{*}[(\partial_{\alpha} + iQ_{g,q}h_{\alpha})\psi] - m^{2}\psi^{*}\psi. \quad (40)$$

The self-interaction of gravity is, from equations (39 and 40),

$$j^{\mu} = -iQ_{g,q}\zeta^{*}\beta^{\mu}\zeta. \quad (41)$$

8. Summary and discussion

We show that some existing difficulties, such as the G dilemma, differences between geometric gravity and Yang–Mills theory, nonlocalization of $t^{\mu v}$, non-tensor character of $h^{\mu v}$, and $t^{\mu v}$ related difficulties, are due to the mixture of geometric and physical aspects of gravity. In order to resolve the dimensional Newtonian constant G problem and the highly non-linear self-interaction problem in the quantization of gravity, we have (1) proposed the dual geometric-gauge field aspects of gravity; (2) introduced the new concepts of the quantum of gravitational charge, the graitational quantum level (GQL), and the quantum of energy of gravitational field; and (3) proposed two approaches to describe the self-interaction of gravitational field. These concepts and approaches may be applied to other theories of gravity.

Gravity exhibits both geometric and gauge field aspects. One of two aspects alone proves inadequate for the full description. However, the mixture of two aspects causes serious difficulties. Once two aspects are separated, difficulties may either disappear or be resolved. The Yang–Mills gauge field aspect is indeed the intrinsic nature of gravity. The way we treat gravity as a gauge field is quite different from that of other gauge theories of gravity:

	dualism	other gauge theories of gravity
coupling constant	$Q_{g,q}$	G
gauge potential	$h^{a\nu}$	tetrad and affine connection
conversion between internal and external symmetries	yes	no
quantum level	GQL	EPL
gauge boson	spin one	spin two
self-interaction	$j^{a\mu}$ [Eq. (41)]	$t^{\mu\nu}$

To quantize a gauge field, we have to determine what is the quantum level for the gauge field and what is the quantum of the charge of the gauge field first. Based on WEP and the basic principle in quantum mechanics, we introduce the quantum of gravitational charge which is carried by the quantum of a minimal energy. The quantum of gravitational charge is so small that an elementary particle carries a huge number of them. Therefore the EPL is not the quantum level for gravity. The minimal energy quantum level is the GQL, which is the bottom quantum level.

Keeping the GQL in mind, then, we study the self-interaction of gravity, e.g., the $t^{\mu\nu}$ problem in the quantization of gravity. We find that at the GQL the $t^{\mu\nu}$ no longer contains infinite nonlinear terms, but is cut at $\int dv\, t_n^{00} < E_{\min}$ term.

Actually in the gauge aspect of gravity, $t^{\mu\nu}$ no longer appears in the field equation, rather the current $j^{a\mu}$ of gravitational gauge field. We propose two different approaches to describe the self-interaction of gravity. Although the gravitational effects on elementary particles are negligibly small, we do have a detectible effect that the gauge boson of gravitational gauge field is the spin one particle.

All of results obtained above indicate that the dualism of gravity, the concepts of the quantum of gravitational charge, the GQL and the quantum of energy of gravitational gauge field contain the ingredients for a satisfactory description of gravity, and are fruitful.

There is an interesting possibility that there might be a very strong and very short-range force(s) at the GQL which is responsible for the formation of, at least part of, elementary particles. When go up to the EPL, the unknown force disappears, and electroweak and strong forces appear to play. Then go up to the macro-world, weak and strong forces disappear, only two long-range forces left.

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Appendix: Comparison between Einstein and Maxwell Equations

		Maxwell	Einstein
Geometric equation	MKS unit		$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi G T^{\mu\nu}$ $L = -(16\pi G)^{-1}\sqrt{g}R + L_{\text{matter}}$
Gauge field equation	MKS unit	$\frac{\partial F^{\nu\lambda}}{\partial x^{\lambda}} = 4\pi K J^{\nu}$	$\frac{\partial G^{\mu\nu\lambda}}{\partial x^{\lambda}} = -4\pi G(T^{\mu\nu} + t^{\mu\nu})$
		$F^{\nu\lambda} = A^{\nu,\lambda} - A^{\lambda,\nu}$	$G^{\mu\nu\lambda} = \frac{1}{4}(\bar{h}^{\mu\nu,\lambda} - \bar{h}^{\mu\lambda,\nu})$
		$L = -\frac{F^{\nu\lambda}F_{\nu\lambda}}{16\pi K} - J^{\mu}A_{\mu}$	$L = -\frac{G^{\mu\nu\lambda}G_{\mu\nu\lambda}}{4\pi G} + \bar{h}^{\mu\nu}T_{\mu\nu} + L^{(1)}$
		$F = Kqq/(r^2)$	$F = Gm_g m_g/(r^2)$
		unit $[q]$ = c	unit $[m_g]$ = kg
		unit $[A^{\mu}]$ = mN/c	unit $[h^{\mu\nu}] = 1$
		$U(1)$: $\exp[(i/\hbar)q\theta_e]$	T^4: $\exp[(i/\hbar)p^{\mu}\epsilon_{\mu}]$
		unit $[\theta_e]$ = mNs/c	unit $[x]$ = m
	Heaviside-Lorentz unit	$F^{\nu\lambda,\lambda} = J^{\nu}$	$G^{a\nu\lambda,\lambda} = -(J^{a\nu} + j^{a\nu})$
		$L = -\frac{1}{4}F^{\nu\lambda}F_{\nu\lambda}$ $-\psi^*\gamma_{\mu}(\partial^{\mu} + iQ_{e,q}A^{\mu})\psi$	$L = -G^{a\nu\lambda}G_{a\nu\lambda} + L_{\text{matter}}$ $-\zeta^*\beta_{\mu}(\partial^{\mu} + iQ_{g,q}h^{\mu})\zeta$
		$F = Q_e Q_e/(4\pi r^2)$	$F = Q_g Q_g/(4\pi r^2)$
		unit $[Q_e] = \text{m}\sqrt{\text{N}}$	unit $[Q_g] = \text{m}\sqrt{\text{N}}$
		unit $[A^{\mu}] = \sqrt{\text{N}}$	unit $[h^{a\nu}] = \sqrt{\text{N}}$
		$U(1)$: $\exp[(i/\hbar)Q_e\theta_e]$	T^4_{internal}: $\exp[(i/\hbar)Q_g\tau^a\theta_{ga}]$
		unit $[\theta_e] = \text{s}\sqrt{\text{N}}$	unit $[\theta_g^a] = \text{s}\sqrt{\text{N}}$

*c: coulomb, m: meter, N: Newton, s: second, kg: kilogram