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Chiral Current Algebras in three–dimensional BF–Theory with boundary

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ABSTRACT

We consider the three–dimensional BF–model with planar boundary in the axial gauge. We find two–dimensional conserved chiral currents living on the boundary and satisfying Kac–Moody algebras.

1 Introduction

In 1981 K.Symanzik faced the problem of studying renormalizable Quantum Field Theories (QFTs) in presence of a boundary [1]. He introduced boundary conditions for the quantum fields by adding to the action surface interaction terms, compatible with power counting and locality.

The introduction of a boundary is particularly interesting in topological QFTs for at least two important reasons. First, it is known that topological field theories have no local observables but in the case in which the base manifold has a boundary [2, 3]. Secondly, all rational conformal field theories can be classified in terms of Chern-Simons theories built on three-dimensional manifolds with spatial boundary [2, 4]. This connection is made explicit by noting that chiral currents satisfying a Kac–Moody algebra live on the two-dimensional boundary of the three-dimensional base manifold [2, 5].

In particular, the chiral current algebra living on the boundary of a three-dimensional Chern-Simons theory has been derived in [6] and [7] with an approach closely related to Symanzik's ideas and the existence of chiral currents on the boundary and their anomalous Kac–Moody algebra has been derived in the framework of BRS formalism. The authors of [6] add to the action local boundary terms compatible with power counting, using a covariant gauge fixing. The approach followed in [7] is different in that the equations of motion, rather than the action, are modified by appropriate boundary terms, and non-covariant axial gauge is preferred.

Recently, many efforts have been made to clarify in general the relation between gauge theories in N dimensions and current algebra in $N-1$ dimensions [3, 8]. The Chern–Simons theory is not well-suited for this investigation, because of the difficulties of defining it in an arbitrary number of dimensions, in particular in the non-abelian case.

This difficulty does not appear in the other important topological QFTs of the Schwartz-type, the BF models [9, 10], which can be defined in any number of dimensions. The three-dimensional case is of particular physical relevance, because it coincides with the three-dimensional Einstein-Hilbert gravity [9, 11], and because in three dimensions the model can be provided with a true coupling constant in form of a cosmological constant. The finiteness of this model has been proved to all orders in perturbation theory in the Landau gauge [12].

In this paper we study three-dimensional BF theory with a planar boundary in the axial gauge, following the approach of [7]. Such a gauge choice is quite natural when studying a QFT with boundary, because the Poincaré invariance is lost *a priori* due to the presence itself of the boundary and the main reason for a covariant choice of the gauge fixing is thus failing. It is on the other hand well known that the axial gauge fixing is not a complete one [13]. It remains indeed a residual gauge invariance on the planar boundary which can be expressed by a local Ward identity. The existence of such a "residual" Ward identity is the main advantage of this non-covariant choice, because just from it one derives the algebra for the conserved chiral currents on the boundary. Nevertheless, the axial gauge is in general affected by a number of problems only partially avoided by the ultraviolet finiteness of the theory we consider. For example we must postulate that the quantum fields vanish at infinity. The problems deriving from the adoption of an axial gauge in the three-dimensional Chern-Simons theory with boundary are faced and solved in [7]. The same arguments can be applied to our case. The aim of this work is rather to investigate the existence of conserved chiral currents on the boundary and their algebraic structure.

The plan of the paper is the following: in section 2 we illustrate the model in unbounded space–time. In section 3 and 4 we find the free propagators of the theory with boundary in the ghost and in the gauge sector respectively. In section 5 we compute the chiral current algebra on the boundary, and finally in section 6 we draw some concluding remarks.

2 The model in unbounded flat space–time

The classical action for the three–dimensional BF–system with cosmological constant $\lambda \geq 0$ is

$$S_{BF} = \frac{1}{2}\int_M d^3x\ \varepsilon^{\mu\nu\rho}\left\{F^a_{\mu\nu}B^a_\rho + \frac{\lambda}{3}f^{abc}B^a_\mu B^b_\nu B^c_\rho\right\}\ , \tag{2.1}$$

where M is the flat space–time, B^a_μ is a one–form and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu\ .$$

As usual, f^{abc} are the structure constants of a compact simple gauge group G.

We make use of the light–cone coordinates

$$\begin{aligned} u &= x^1 \\ z &= \frac{x^0 + x^2}{\sqrt{2}} \\ \bar{z} &= \frac{x^0 - x^2}{\sqrt{2}}\ . \end{aligned} \tag{2.2}$$

Correspondingly the components of the gauge fields A^a_μ and B^a_μ read

$$\begin{aligned} A^a_u &= A^a_1 \\ A^a &= \frac{A^a_0 + A^a_2}{\sqrt{2}} \\ \bar{A}^a &= \frac{A^a_0 - A^a_2}{\sqrt{2}} \\ B^a_u &= B^a_1 \\ B^a &= \frac{B^a_0 + B^a_2}{\sqrt{2}} \\ \bar{B}^a &= \frac{B^a_0 - B^a_2}{\sqrt{2}} \end{aligned} \tag{2.3}$$

The action for the model becomes

$$\begin{aligned} S_{BF} = &\int dud^2z\{B^a(\bar{\partial}A^a_u - \partial_u\bar{A}^a + f^{abc}\bar{A}^bA^c_u) + \bar{B}^a(\partial_uA^a - \partial A^a_u + f^{abc}A^b_uA^c) \\ &+B^a_u(\partial\bar{A}^a - \bar{\partial}A^a + f^{abc}A^b\bar{A}^c) + \lambda f^{abc}B^a\bar{B}^bB^c_u\}\ . \end{aligned} \tag{2.4}$$

For the aim of this work, a convenient choice of the gauge fixing term is

$$\begin{aligned} S_{gf} = \int dud^2z\{&b^aA^a_u + \bar{c}^a(\partial_uc^a + f^{abc}A^b_uc^c + \lambda f^{abc}B^b_u\phi^c) \\ &+d^aB^a_u + \bar{\phi}^a(\partial_u\phi^a + f^{abc}A^b_u\phi^c + f^{abc}B^b_uc^c)\}\ , \end{aligned} \tag{2.5}$$

which corresponds to the "axial" gauge

$$A_u^a = B_u^a = 0\ . \tag{2.6}$$

In (2.5) c^a, $\bar{c}^a$, b^a and ϕ^a, $\bar{\phi}^a$, d^a are respectively ghost, antighost and Lagrange multipliers fields for the gauge fields A_μ^a and B_μ^a. In Table 1 are displaied for both fields and coordinates the canonical dimensions as well as the ghost number and helicity assignments.

	z	$\bar{z}$	u	A^a	$\bar{A}^a$	A_u^a	c^a	$\bar{c}^a$	b^a	B^a	$\bar{B}^a$	B_u^a	ϕ^a	$\bar{\phi}^a$	d^a
dim	-1	-1	-1	1	1	1	1	1	2	1	1	1	1	1	2
helicity	-1	1	0	1	-1	0	0	0	0	1	-1	0	0	0	0
$\Phi\Pi$	0	0	0	0	0	0	1	-1	0	0	0	0	1	-1	0

Table 1. Dimensions, $\Phi\Pi$ charges and helicities.

With the addition to the action of a gauge–fixing term, the general covariance of the theory, if any, is lost. Here, moreover, the use of a non–covariant gauge fixing breaks the three–dimensional Poincaré invariance to the two–dimensional one in the plane $\{z, \bar{z}\}$. The gauge is not completely fixed by (2.5) and, as in [7], the Ward identity expressing the residual gauge invariance of the theory will play a key role in determining the chiral current algebra on the plane boundary $u = 0$.

The classical action $S = S_{BF} + S_{gf}$ is invariant under the nilpotent BRS transformations [12]

$$\begin{aligned}
sA_\mu^a &= -(D_\mu c)^a - \lambda f^{abc} B_\mu^b \phi^c \\
sc^a &= \frac{1}{2} f^{abc} \left(c^b c^c + \lambda \phi^b \phi^c \right) \\
s\bar{c}^a &= b^a \\
sb^a &= 0 \\
sB_\mu^a &= -(D_\mu \phi)^a - f^{abc} B_\mu^b c^c \\
s\phi^a &= f^{abc} \phi^b c^c \\
s\bar{\phi}^a &= d^a \\
sd^a &= 0\ ,
\end{aligned} \tag{2.7}$$

where the covariant derivative is defined by $(D_\mu X)^a = \partial_\mu X^a + f^{abc} A_\mu^b X^c$. Besides the BRS symmetry (2.7), the theory is invariant by the "parity" coordinate transformations $z \leftrightarrow \bar{z}$, $u \to -u$, to which corresponds the discrete field symmetry

$$\begin{array}{rl}
A^a \leftrightarrow \bar{A}^a & B^a \leftrightarrow \bar{B}^a \\
A_u^a \to -A_u^a & B_u^a \to -B_u^a \\
c^a \to \bar{\phi}^a & \phi^a \to \bar{c}^a \\
\bar{c}^a \to -\phi^a & \bar{\phi}^a \to -c^a \\
b^a \to -b^a & d^a \to -d^a
\end{array} \tag{2.8}$$

At tree level, the generating functional $Z_c(J_\psi)$ of the connected Green functions is obtained from the classical action $S(\psi)$ by a Legendre transformation:

$$Z_c(J_\psi) = S(\psi) + \int dud^2z \sum_\psi J_\psi^a \psi^a, \tag{2.9}$$

where J^a_ψ are the sources for the fields, denoted collectively by ψ^a. The following equations of motion for the gauge fields and multipliers are derived:

$$
\begin{aligned}
&\partial B^a_u - \partial_u \bar{B}^a + f^{abc}\bar{A}^b B^c_u - f^{abc}A^b_u \bar{B}^c + J^a_{\bar{A}} = 0 \\
&\partial_u B^a - \bar{\partial} B^a_u + f^{abc}A^b_u B^c - f^{abc}A^b B^c_u + J^a_{\bar{A}} = 0 \\
&\partial \bar{B}^a - \bar{\partial} B^a + f^{abc}A^b \bar{B}^c - f^{abc}\bar{A}^b B^c + b^a - f^{abc}\bar{c}^b c^c - f^{abc}\bar{\phi}^b \phi^c + J^a_{A_u} = 0 \\
&A^a_u + J^a_b = 0 \\
&\bar{\partial} A^a_u - \partial_u \bar{A}^a + f^{abc}\bar{A}^b A^c_u + \lambda f^{abc}\bar{B}^b B^c_u + J^a_{\bar{B}} = 0 \\
&\partial_u A^a - \partial A^a_u + f^{abc}A^b_u A^c - \lambda f^{abc}B^b B^c_u + J^a_{\bar{B}} = 0 \\
&\partial \bar{A}^a - \bar{\partial} A^a + f^{abc}A^b \bar{A}^c + \lambda f^{abc}B^b \bar{B}^c + d^a - f^{abc}\bar{\phi}^b c^c - \lambda f^{abc}\bar{c}^b \phi^c + J^a_{B_u} = 0 \\
&B^a_u + J^a_d = 0 \ ,
\end{aligned}
\tag{2.10}
$$

while for the ghost fields one has:

$$
\begin{aligned}
&\partial_u \bar{c}^a + f^{abc}A^b_u \bar{c}^c + f^{abc}B^b_u \bar{\phi}^c - J^a_c = 0 \\
&\partial_u c^a + f^{abc}A^b_u c^c + \lambda f^{abc}B^b_u \phi^c - J^a_{\bar{c}} = 0 \\
&\partial_u \bar{\phi}^a + f^{abc}A^b_u \bar{\phi}^c + \lambda f^{abc}B^b_u \bar{c}^c - J^a_\phi = 0 \\
&\partial_u \phi^a + f^{abc}A^b_u \phi^c + f^{abc}B^b_u c^c - J^a_{\bar{\phi}} = 0 \ .
\end{aligned}
\tag{2.11}
$$

In order to write a Slavnov identity, we couple external sources to the non linear variations of quantum fields in (2.7)

$$S_s = \int dud^2z \left\{ \Omega^{a\mu} sA^a_\mu + L^a sc^a + \rho^{a\mu} sB^a_\mu + D^a s\phi^a \right\} . \tag{2.12}$$

The dimensions and the Faddeev–Popov charges of the four sources are listed in Table 2.

	$\Omega^{a\mu}$	L^a	$\rho^{a\mu}$	D^a
dim	2	3	2	3
$\Phi\Pi$	-1	-2	-1	-2

Table 2. Dimensions and $\Phi\Pi$-charges of the external fields.

The complete action $\Sigma = S_{BF} + S_{gf} + S_s$ satisfies the Slavnov identity

$$\mathcal{S}(\Sigma) = \int dud^2z \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + \frac{\delta\Sigma}{\delta\rho^{a\mu}}\frac{\delta\Sigma}{\delta B^a_\mu} + \frac{\delta\Sigma}{\delta D^a}\frac{\delta\Sigma}{\delta\phi^a} + b^a\frac{\delta\Sigma}{\delta\bar{c}^a} + d^a\frac{\delta\Sigma}{\delta\bar{\phi}^a} \right) = 0 \ . \tag{2.13}$$

The three–dimensional BF–system exhibits two ghost equations of motion [12], which in the axial gauge are local symmetries of the action, broken at the classical level by terms linear in the quantum fields:

$$\mathcal{F}^a(x)\Sigma = \Delta^a_{(f)}(x) \tag{2.14}$$

$$\mathcal{G}^a(x)\Sigma = \Delta^a_{(g)}(x) \ , \tag{2.15}$$

where

$$\mathcal{F}^a(x) = \frac{\delta}{\delta c^a} + f^{abc}\bar{c}^b\frac{\delta}{\delta b^c} + f^{abc}\bar{\phi}^b\frac{\delta}{\delta d^c} \tag{2.16}$$

$$
\begin{aligned}
\mathcal{G}^a(x) &= \frac{\delta}{\delta\phi^a} + f^{abc}\bar{\phi}^b\frac{\delta}{\delta b^c} + \lambda f^{abc}\bar{c}^b\frac{\delta}{\delta d^c} && (2.17)\\
\Delta^a_{(f)}(x) &= -\partial_\mu\Omega^{a\mu} + \partial_u\bar{c}^a + f^{abc}(\Omega^{b\mu}A^c_\mu - L^b c^c + \rho^{b\mu}B^c_\mu - D^b\phi^c) && (2.18)\\
\Delta^a_{(g)}(x) &= -\partial_\mu\rho^{a\mu} + \partial_u\bar{\phi}^a + f^{abc}(\rho^{b\mu}A^c_\mu - D^b c^c + \lambda\Omega^{b\mu}B^c_\mu - \lambda L^b\phi^c)\ . && (2.19)
\end{aligned}
$$

Anticommuting (2.14) and (2.15) with the Slavnov operator (2.13) one gets respectively

$$
\begin{aligned}
&\mathcal{F}^a(x)\mathcal{S}(\gamma) + B_\gamma(\mathcal{F}^a(x)\gamma - \Delta^a_{(f)}(x)) = H^a(x)\gamma && (2.20)\\
&\mathcal{G}^a(x)\mathcal{S}(\gamma) + B_\gamma(\mathcal{G}^a(x)\gamma - \Delta^a_{(g)}(x)) = N^a(x)\gamma\ , && (2.21)
\end{aligned}
$$

where γ is a generic functional of the fields, B_γ is the anticommuting linearized Slavnov operator

$$
\begin{aligned}
B_\gamma = \int dud^2z \biggl(&\frac{\delta\gamma}{\delta\Omega^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\gamma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\gamma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\gamma}{\delta c^a}\frac{\delta}{\delta L^a}\\
&+ \frac{\delta\gamma}{\delta\rho^{a\mu}}\frac{\delta}{\delta B^a_\mu} + \frac{\delta\gamma}{\delta B^a_\mu}\frac{\delta}{\delta\rho^{a\mu}} + \frac{\delta\gamma}{\delta D^a}\frac{\delta}{\delta\phi^a} + \frac{\delta\gamma}{\delta\phi^a}\frac{\delta}{\delta D^a}\\
&+ b^a\frac{\delta}{\delta\bar{c}^a} + d^a\frac{\delta}{\delta\bar{\phi}^a}\biggr)\ , && (2.22)
\end{aligned}
$$

and

$$
\begin{aligned}
H^a(x) &= \partial_\mu\frac{\delta}{\delta A^a_\mu} - \partial_u b^a + \sum_\psi f^{abc}\psi^b\frac{\delta}{\delta\psi^c} && (2.23)\\
N^a(x) &= \partial_\mu\frac{\delta}{\delta B^a_\mu} - \partial_u d^a + f^{abc}\biggl[A^b_\mu\frac{\delta}{\delta B^c_\mu} + c^b\frac{\delta}{\delta\phi^c} + d^b\frac{\delta}{\delta b^c} + \bar{\phi}^b\frac{\delta}{\delta\bar{c}^c} + \rho^{b\mu}\frac{\delta}{\delta\Omega^{c\mu}} + D^b\frac{\delta}{\delta L^c}\\
&\quad + \lambda\left(B^b_\mu\frac{\delta}{\delta A^c_\mu} + \phi^b\frac{\delta}{\delta c^c} + \bar{c}^b\frac{\delta}{\delta\bar{\phi}^c} + b^b\frac{\delta}{\delta d^c} + \Omega^{b\mu}\frac{\delta}{\delta\rho^{c\mu}} + L^b\frac{\delta}{\delta D^c}\right)\biggr] && (2.24)
\end{aligned}
$$

If, in particular, γ stands for the action Σ, the local operators $H^a(x)$ and $N^a(x)$ represent exact classical symmetries of the theory which, written in terms of $Z_c(J_\psi)$, read

$$
\begin{aligned}
&H^a(x)Z_c(J_\psi) = -\partial_\mu J^{a\mu}_A - \partial_u\frac{\delta Z_c}{\delta J^a_b} + \sum_\psi f^{abc}J^b_\psi\frac{\delta Z_c}{\delta J^c_\psi} = 0 && (2.25)\\
&N^a(x)Z_c(J_\psi) = -\partial_\mu J^{a\mu}_B - \partial_u\frac{\delta Z_c}{\delta J^a_d}\\
&\quad + f^{abc}\biggl[J^{b\mu}_B\frac{\delta}{\delta J^{c\mu}_A} + J^b_\phi\frac{\delta}{\delta J^c_c} + J^b_b\frac{\delta}{\delta J^c_d} + J^b_{\bar{c}}\frac{\delta}{\delta J^c_{\bar{\phi}}} + \rho^{b\mu}\frac{\delta}{\delta\Omega^{c\mu}} + D^b\frac{\delta}{\delta L^c}\\
&\quad + \lambda\left(J^{b\mu}_A\frac{\delta}{\delta J^{c\mu}_B} + J^{b\mu}_c\frac{\delta}{\delta J^c_\phi} + J^b_{\bar{\phi}}\frac{\delta}{\delta J^c_{\bar{c}}} + J^b_d\frac{\delta}{\delta J^c_b} + \Omega^{b\mu}\frac{\delta}{\delta\rho^{c\mu}} + L^b\frac{\delta}{\delta D^c}\right)\biggr] Z_c = 0 && (2.26)
\end{aligned}
$$

The operators $H^a(x)$ and $N^a(x)$ are the local versions of those found in [12] in the Landau gauge. We have here recovered a general property of the axial gauge, *i.e.* that the Slavnov identities of the theory take the local form (2.25,26)[13].

3 Free propagators of the theory with boundary: the ghost sector

We consider now the model built on the flat space–time $\mathbf{R}^3$ divided into two parts $\mathbf{R}^3_+$ and $\mathbf{R}^3_-$ by the plane $u = 0$ and we propose to compute the propagators of the theory

$$\frac{\delta^2 Z_c(J_\psi)}{\delta J^a_{\psi_1}(x_1)\delta J^b_{\psi_2}(x_2)}\Big|_{J^a_\psi=0} = \Delta^{ab}_{\psi_1\psi_2}(x_1,x_2) = \langle\psi^a_1(x_1)\psi^b_2(x_2)\rangle \quad , \tag{3.1}$$

taking into account the effect of the boundary.

We shall seize upon the procedure illustrated in [7], taking as outstanding points of the theory the two requirements of decoupling and locality.

First of all, we demand that the boundary decouples the regions $\mathbf{R}^3_+$ and $\mathbf{R}^3_-$ of $\mathbf{R}^3$. In terms of propagators this means

$$\Delta^{ab}_{\psi_1\psi_2}(x_1,x_2) = 0 \quad \text{if} \quad u_1u_2 < 0 \ . \tag{3.2}$$

Such a condition is fulfilled by a two–point function of the form

$$\Delta^{ab}_{\psi_1\psi_2}(x_1,x_2) = \delta^{ab}[\theta_+\Delta^+_{\psi_1\psi_2}(x_1,x_2) + \theta_-\Delta^-_{\psi_1\psi_2}(x_1,x_2)] \ , \tag{3.3}$$

where $\theta_\pm = \theta(\pm u_1)\theta(\pm u_2)$ and $\theta(u)$ is the step function defined as

$$\theta(u) = \begin{cases} 1 & \text{if } u \geq 0 \\ 0 & \text{if } u < 0 \end{cases} \tag{3.4}$$

We then consider as fundamental the equations of motion of the quantum fields, modified by boundary terms, in such a way to recover the standard ones (2.10) and (2.11) away from the boundary (locality condition).

Let us begin analyzing in detail the ghost sector. We then will apply the same technique to the gauge sector, whose results will be given in the next section. From the free ghost equations of motion

$$\begin{array}{ll} \partial_u \dfrac{\delta Z_c}{\delta J^a_{\bar c}(x)} - J^a_c(x) = 0 & \partial_u \dfrac{\delta Z_c}{\delta J^a_c(x)} - J^a_{\bar c}(x) = 0 \\ \partial_u \dfrac{\delta Z_c}{\delta J^a_{\bar\phi}(x)} - J^a_\phi(x) = 0 & \partial_u \dfrac{\delta Z_c}{\delta J^a_\phi(x)} - J^a_{\bar\phi}(x) = 0 \end{array} \tag{3.5}$$

one gets the equations for the propagators

$$\begin{array}{ll} \partial_{u'}\Delta^{ab}_{c\bar c}(x,x') = \delta^{ab}\delta^3(x-x') & \partial_{u'}\Delta^{ab}_{cc}(x,x') = 0 \\ \partial_{u'}\Delta^{ab}_{c\bar\phi}(x,x') = 0 & \partial_{u'}\Delta^{ab}_{c\phi}(x,x') = 0 \\ \partial_{u'}\Delta^{ab}_{\bar c c}(x,x') = \delta^{ab}\delta^3(x-x') & \partial_{u'}\Delta^{ab}_{\bar c\phi}(x,x') = 0 \\ \partial_{u'}\Delta^{ab}_{\phi\bar c}(x,x') = 0 & \partial_{u'}\Delta^{ab}_{\phi\bar\phi}(x,x') = \delta^{ab}\delta^3(x-x') \\ \partial_{u'}\Delta^{ab}_{\bar\phi c}(x,x') = 0 & \partial_{u'}\Delta^{ab}_{\bar\phi\phi}(x,x') = \delta^{ab}\delta^3(x-x') \end{array} \tag{3.6}$$

The most general solution of equations (3.6) fulfilling the decoupling condition and compatible with helicity conservation, scale invariance and regularity, is

$$\Delta^{ab}(x,x') = \delta^{ab}[\theta_+\Delta^+(x,x') + \theta_-\Delta^-(x,x')] \ , \tag{3.7}$$

where

$$\Delta^{+}(x,x') = \begin{pmatrix} 0 & -T_\rho(x,x') & 0 & \alpha\delta^2(z-z') \\ T_\rho(x',x) & 0 & \beta\delta^2(z-z') & 0 \\ 0 & -\beta\delta^2(z-z') & 0 & -T_\sigma(x,x') \\ -\alpha\delta^2(z-z') & 0 & T_\sigma(x',x) & 0 \end{pmatrix} \tag{3.8}$$

and $\Delta^{-}(x,x')$ is obtained from $\Delta^{+}(x,x')$ by parity. In writing (3.8) we have adopted a matrix notation according to the order c^a, $\bar{c}^a$, ϕ^a, $\bar{\phi}^a$. The ghost propagator $\Delta^{ab}(x,x')$ depends on four constant parameters α, β, ρ, σ and on the tempered distribution

$$T_\xi(x,x') \equiv [\theta(u-u') + \xi]\delta^2(z-z') \; . \tag{3.9}$$

The next step is constituted by the most general modification of the free ghost equations of motion by a boundary term which respects, besides the locality condition, also the helicity and $\phi\pi$–charge conservation:

$$\begin{aligned} \partial_u \bar{c}^a - J^a_c &= \delta(u)[\mu_+ \bar{c}^a_+ + \mu_- \bar{c}^a_- + k_+ \bar{\phi}^a_+ + k_- \bar{\phi}^a_-] \\ \partial_u \bar{\phi}^a - J^a_\phi &= \delta(u)[\alpha_+ \bar{c}^a_+ + \alpha_- \bar{c}^a_- + \beta_+ \bar{\phi}^a_+ + \beta_- \bar{\phi}^a_-] \\ \partial_u \phi^a - J^a_{\bar{\phi}} &= -\delta(u)[\mu_+ \phi^a_- + \mu_- \phi^a_+ + k_+ c^a_- + k_- c^a_+] \\ \partial_u c^a - J^a_{\bar{c}} &= -\delta(u)[\alpha_+ \phi^a_- + \alpha_- \phi^a_+ + \beta_+ c^a_- + \beta_- c^a_+] \end{aligned} \tag{3.10}$$

where $\mu_\pm$, $k_\pm$, $\alpha_\pm$, $\beta_\pm$ are constant parameters.

In (3.10), the latter two equations are derived from the former two by parity. The r.h.s. of the above equations depend on the two-dimensional fields which live on the opposite sides of the boundary:

$$\psi^a_\pm(z,\bar{z}) \equiv \lim_{u\to 0^\pm} \psi^a(u,z,\bar{z}) \; . \tag{3.11}$$

Some of the parameters appearing in (3.10) can be eliminated by imposing that eqs. (3.10) are compatible with each other. Indeed, from the anticommutation relations between the ghost equations of motion, one finds

$$\begin{aligned} k_+ &= k_- \equiv k \\ \beta_- &= \mu_+ \\ \beta_+ &= \mu_- \\ \alpha_+ &= \alpha_- \equiv \nu \end{aligned} \tag{3.12}$$

From the equations (3.10) follow sixteen equations for the free propagators in presence of the plane boundary $u = 0$, which reduce to eight independent non–linear equations for the eight parameters α, β, ρ, σ, μ_+, μ_-, k, ν:

$$\begin{aligned} &(1+\rho)(1-\mu_+) + \alpha k = 0 \\ &\sigma(1+\mu_-) - \alpha k = 0 \\ &\beta(\mu_+ - 1) + k(1+\sigma) = 0 \\ &\beta(1+\mu_-) + k\rho = 0 \\ &\rho(1+\mu_+) + \beta\nu = 0 \\ &(1+\sigma)(1-\mu_-) - \beta\nu = 0 \\ &\alpha(1+\mu_+) - \nu\sigma = 0 \\ &\alpha(1-\mu_-) + \nu(1+\rho) = 0 \end{aligned} \tag{3.13}$$

Of course one could solve directly the non-linear set of equations (3.13), but the task of finding the solutions of our problem is made much simpler by the observation that the action S possesses a further discrete simmetry involving the ghost fields only:

$$\begin{aligned} \bar{c}^a &\leftrightarrow \phi^a \\ c^a &\leftrightarrow \bar{\phi}^a \,. \end{aligned} \tag{3.14}$$

This new symmetry, imposed on the free ghost equations of motion (3.10) and on the propagators (3.3), respectively gives the following two groups of constraints on the parameters:

$$\begin{aligned} &\mu_+ = -\mu_- \equiv \mu \\ &k = \nu = 0 \end{aligned} \tag{3.15}$$

and

$$\begin{aligned} &\sigma = -(1+\rho) \\ &\alpha = \beta = 0 \end{aligned} \tag{3.16}$$

Eqs. (3.13) considerably simplifies to

$$\begin{aligned} &(1+\rho)(1-\mu) = 0 \\ &\rho(1+\mu) = 0 \end{aligned} \tag{3.17}$$

which has two distinct solutions:

	ρ	μ
I	0	1
II	-1	-1

Let us take the solution I to discover the corresponding boundary conditions on the ghost fields. The propagator matrix $\Delta^+(x,x')$ relative to the half-space $\mathbf{R}^3_+$ is

$$\Delta^+(x,x') = \begin{pmatrix} 0 & -\theta(u-u')\delta^2 & 0 & 0 \\ \theta(u'-u)\delta^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta(u'-u)\delta^2 \\ 0 & 0 & -\theta(u-u')\delta^2 & 0 \end{pmatrix} \tag{3.18}$$

From (3.18) we have, for any ghost field $\xi^a(x)$

$$\begin{aligned} &\lim_{u\to 0^+} \langle c^a(x)\xi^b(x')\rangle = 0 \\ &\lim_{u\to 0^+} \langle \bar{\phi}^a(x)\xi^b(x')\rangle = 0 \,. \end{aligned} \tag{3.19}$$

We then realize that the ghost field boundary conditions corresponding to the solution I are of Dirichlet type

$$c^a_+(z) = \bar{\phi}^a_+(z) = 0 \tag{3.20}$$

and, by parity, one gets from $\Delta^-(x,x')$

$$c^a_-(z) = \bar{\phi}^a_-(z) = 0 \ . \tag{3.21}$$

To conclude the analysis of the first solution, we write the ghost equations of motion modified by the presence of the plane boundary $u = 0$

$$\begin{aligned}
&\partial_u \bar{c}^a + f^{abc} A^b_u \bar{c}^c + f^{abc} B^b_u \bar{\phi}^c - J^a_c = \delta(u)(\bar{c}^a_+(z) - \bar{c}^a_-(z)) \\
&\partial_u c^a + f^{abc} A^b_u c^c + \lambda f^{abc} B^b_u \phi^c - J^a_{\bar{c}} = 0 \\
&\partial_u \bar{\phi}^a + f^{abc} A^b_u \bar{\phi}^c + \lambda f^{abc} B^b_u \bar{c}^c - J^a_\phi = 0 \\
&\partial_u \phi^a + f^{abc} A^b_u \phi^c + f^{abc} B^b_u c^c - J^a_{\bar{\phi}} = \delta(u)(\phi^a_+(z) - \phi^a_-(z)) \ .
\end{aligned} \tag{3.22}$$

One easily sees that the solution II corresponds to the Dirichlet boundary conditions

$$\begin{aligned}
&\phi^a_+ = \phi^a_- = 0 \\
&\bar{c}^a_+ = \bar{c}^a_- = 0 \ ,
\end{aligned} \tag{3.23}$$

while the ghost equations of motion read

$$\begin{aligned}
&\partial_u \bar{c}^a + f^{abc} A^b_u \bar{c}^c + f^{abc} B^b_u \bar{\phi}^c - J^a_c = 0 \\
&\partial_u c^a + f^{abc} A^b_u c^c + \lambda f^{abc} B^b_u \phi^c - J^a_{\bar{c}} = \delta(u)(c^a_+(z) - c^a_-(z)) \\
&\partial_u \bar{\phi}^a + f^{abc} A^b_u \bar{\phi}^c + \lambda f^{abc} B^b_u \bar{c}^c - J^a_\phi = \delta(u)(\bar{\phi}^a_+(z) - \bar{\phi}^a_-(z)) \\
&\partial_u \phi^a + f^{abc} A^b_u \phi^c + f^{abc} B^b_u c^c - J^a_{\bar{\phi}} = 0
\end{aligned} \tag{3.24}$$

4 Free propagator of the theory with boundary: the gauge sector

According to the lines followed in the previous section, from the free equations of motion of the gauge fields and multipliers follow 8×8 equations for the propagators, many of which are equivalent to each other. The most general solution compatible with helicity conservation, scale invariance and regularity is, according to the decoupling condition,

$$\Delta^{ab}(x,x') = \delta^{ab}[\theta_+ \Delta^+(x,x') + \theta_- \Delta^-(x,x')] \ , \tag{4.1}$$

where we adopted a matrix notation following the order A, $\bar{A}$, A_u, b, B, $\bar{B}$, B_u, d, and $\Delta^+(x,x')$ is displaied in table 3, while $\Delta^-(x,x')$ is obtained from $\Delta^+(x,x')$ by a parity transformation. The matrix of propagators (4.1) depends on ten constant parameters a_i, $i = 1, \ldots, 10$. The inclusion of a boundary term in the free equations of motion of the gauge fields and multipliers leads to

$$\begin{aligned}
&\bar{\partial} B^a_u - \partial_u \bar{B}^a + J^a_A = \delta(u)[\alpha_1(\bar{A}^a_+ + \bar{A}^a_-) + \alpha_2 \bar{B}^a_+ + \alpha_3 \bar{B}^a_-] \\
&\partial_u B^a - \partial B^a_u + J^a_{\bar{A}} = \delta(u)[\alpha_1(A^a_+ + A^a_-) + \alpha_3 B^a_+ + \alpha_2 B^a_-] \\
&\partial \bar{B}^a - \bar{\partial} B^a + b^a + J^a_{A_u} = 0 \\
&A^a_u + J^a_b = 0 \\
&\bar{\partial} A^a_u - \partial_u \bar{A}^a + J^a_B = \delta(u)[\alpha_3 \bar{A}^a_+ + \alpha_2 \bar{A}^a_- + \alpha_4(\bar{B}^a_+ + \bar{B}^a_-)] \\
&\partial_u A^a - \partial A^a_u + J^a_{\bar{B}} = \delta(u)[\alpha_2 A^a_+ + \alpha_3 A^a_- + \alpha_4(B^a_+ + B^a_-)] \\
&\partial \bar{A}^a - \bar{\partial} A^a + d^a + J^a_{B_u} = 0 \\
&B^a_u + J^a_d = 0 \ ,
\end{aligned} \tag{4.2}$$

The above equations (4.2), which depend on four constant parameters α_j, $j = 1, \ldots, 4$, respect the parity transformations (2.8) and are compatible with each other.

From the consistency between eqs. (4.2) and the form of the propagator $\Delta^{ab}(x, x')$ follows a set of non–linear equations for the parameters a_i and α_j

$$
\begin{aligned}
&(a_3 - a_4)(\alpha_2 + 1) + a_2\alpha_1 = 0 \\
&(1 + a_7)(1 - \alpha_3) - a_2\alpha_1 = 0 \\
&(a_6 - a_7)(1 + \alpha_2) + a_5\alpha_1 = 0 \\
&a_4(1 - \alpha_3) - a_1\alpha_1 = 0 \\
&a_9(1 + \alpha_2) + a_7\alpha_1 = 0 \\
&a_9(1 - \alpha_3) - \alpha_1(1 + a_3 - a_4) = 0 \\
&a_{10}(1 + \alpha_2) + \alpha_1(a_6 - a_7) = 0 \\
&a_8(1 - \alpha_3) - a_4\alpha_1 = 0 \\
&a_1(1 - \alpha_2) - a_4\alpha_4 = 0 \\
&a_5(1 + \alpha_3) + \alpha_4(a_6 - a_7) = 0 \\
&a_2(1 - \alpha_2) - \alpha_4(1 + a_7) = 0 \\
&a_2(1 + \alpha_3) + \alpha_4(a_3 - a_4) = 0 \\
&a_4(1 - \alpha_2) - \alpha_4 a_8 = 0 \\
&(a_6 - a_7)(1 + \alpha_3) + \alpha_4 a_{10} = 0 \\
&(1 + a_3 - a_4)(1 - \alpha_2) - \alpha_4 a_9 = 0 \\
&a_7(1 + \alpha_3) + \alpha_4 a_9 = 0
\end{aligned}
\tag{4.3}
$$

The equations (4.3) form a set of sixteen independent non–linear equations for fourteen parameters. The introduction of a boundary contribution in the equations of motion (4.2) causes a boundary breaking of the Ward identities (2.25) and (2.26), which we force to be present only at classical level. We can reach that goal by imposing that the breaking is linear in the quantum fields[1]:

$$
\begin{aligned}
\alpha_3 &= \alpha_2 \\
\alpha_4 &= \lambda\alpha_1 .
\end{aligned}
\tag{4.4}
$$

The resulting Ward identities are

$$
\begin{aligned}
\int_{-\infty}^{+\infty} du H^a(x) Z_c(J_\psi) &= -\alpha_1(\partial\bar{A}^a_+ + \partial\bar{A}^a_- + \bar{\partial}A^a_+ + \bar{\partial}A^a_-) \\
&\quad -\alpha_2(\partial\bar{B}^a_+ + \partial\bar{B}^a_- + \bar{\partial}B^a_+ + \bar{\partial}B^a_-) && (4.5) \\
\int_{-\infty}^{+\infty} du N^a(x) Z_c(J_\psi) &= -\lambda\alpha_1(\partial\bar{B}^a_+ + \partial\bar{B}^a_- + \bar{\partial}B^a_+ + \bar{\partial}B^a_-) \\
&\quad -\alpha_2(\partial\bar{A}^a_+ + \partial\bar{A}^a_- + \bar{\partial}A^a_+ + \bar{\partial}A^a_-) && (4.6)
\end{aligned}
$$

The identities (4.5) and (4.6) concern the plane $u = 0$ and they are consequences of the fact that the axial gauge we adopted does not completely fix the gauge. With all the remarks and prescriptions made in [7], we shall take (4.5) and (4.6) as the Ward identities expressing the residual gauge invariance of the theory. Equivalently we postulate that

[1] We are indebted to O.Piguet for suggesting us this point

$\int_{-\infty}^{+\infty} du\partial_u b^a(x) = \int_{-\infty}^{+\infty} du\partial_u d^a(x) = 0$. The choice of an axial gauge in fact does not guarantee that $b^a(z,\bar{z},\pm\infty) = d^a(z,\bar{z},\pm\infty) = 0$.

By imposing the two–dimensional identities (4.5) and (4.6) on the two-point functions, one gets a further set of constraints for the fourteen parameters a_i and α_j:

$$\begin{aligned}
&\alpha_1 a_2 + \alpha_2(a_3 - a_4) - \alpha_1 a_1 - \alpha_2 a_4 = 1\\
&\alpha_1 a_2 + (1 + a_7)\alpha_2 - a_5\alpha_1 - (a_6 - a_7)\alpha_2 = 1\\
&\alpha_1 a_7 + \alpha_2 a_9 - \alpha_1 a_4 - a_8\alpha_2 = 0\\
&\alpha_1(1 + a_3 - a_4) + \alpha_2 a_9 - (a_6 - a_7)\alpha_1 - a_{10}\alpha_2 = 0\\
&\alpha_2 a_2 + \lambda\alpha_1(a_3 - a_4) - a_1\alpha_2 - \lambda\alpha_1 a_4 = 0\\
&\alpha_2 a_2 + \lambda\alpha_1(1 + a_7) - a_5\alpha_2 - \lambda(a_6 - a_7)\alpha_1 = 0\\
&\alpha_2 a_7 + \lambda\alpha_1 a_9 - \alpha_2 a_4 - \lambda\alpha_1 a_8 = 1\\
&\alpha_2(1 + a_3 - a_4) + \lambda\alpha_1 a_9 - (a_6 - a_7)\alpha_2 - \lambda\alpha_1 a_{10} = 1
\end{aligned} \tag{4.7}$$

There are four independent solutions of the two sets of equations (4.3) and (4.7), which are reported in Table 4. Notice that the solutions III and IV in the table depend on the cosmological constant λ, and exist only for $\lambda \neq 0$.

5 Chiral algebra on the plane boundary

The four solutions in Table 4 can be worked out directly by solving the equations (4.3) and (4.7). From the matrix of propagators $\Delta^{ab}(x, x')$ we can read the boundary conditions relative to each of them:

$$I \;:\; A^a_+ = \bar{A}^a_- = B^a_+ = \bar{B}^a_- = 0 \tag{5.1}$$

$$II \;:\; \bar{A}^a_+ = A^a_- = \bar{B}^a_+ = B^a_- = 0 \tag{5.2}$$

$$III \;:\; A^a_+ - \sqrt{\lambda}B^a_+ = \bar{A}^a_+ + \sqrt{\lambda}\bar{B}^a_+ = \bar{A}^a_- - \sqrt{\lambda}\bar{B}^a_- = A^a_- + \sqrt{\lambda}B^a_- = 0 \tag{5.3}$$

$$IV \;:\; A^a_+ + \sqrt{\lambda}B^a_+ = \bar{A}^a_+ - \sqrt{\lambda}\bar{B}^a_+ = \bar{A}^a_- + \sqrt{\lambda}\bar{B}^a_- = A^a_- - \sqrt{\lambda}B^a_- = 0 \tag{5.4}$$

An alternative way to proceed of course could have been the converse one, *i. e.* to figure out all possible behaviours of the gauge fields on the boundary and then to read from the matrix $\Delta^{ab}(x, x')$ which conditions on the parameters are derived.

One finds that the conditions written above are the only ones compatible with the locality and decoupling requirements and, eqs. (5.1,2,3,4) holding, corresponding to each of them the eqs. (4.3) and (4.7) are easily solved to give the four solutions of Table 4.

The solution I corresponds to the modified equations of motion

$$\begin{aligned}
&\bar{\partial}B^a_u - \partial_u\bar{B}^a + f^{abc}\bar{A}^b B^c_u - f^{abc}A^b_u\bar{B}^c + J^a_{\bar{A}} = -\delta(u)\bar{B}^a_+\\
&\partial_u B^a - \partial B^a_u + f^{abc}A^b_u B^c - f^{abc}A^b B^c_u + J^a_{\bar{A}} = -\delta(u)B^a_-\\
&\partial\bar{B}^a - \bar{\partial}B^a + f^{abc}A^b\bar{B}^c - f^{abc}\bar{A}^b B^c + b^a - f^{abc}\bar{c}^b c^c - f^{abc}\bar{\phi}^b\phi^c + J^a_{A_u} = 0\\
&A^a_u + J^a_b = 0\\
&\bar{\partial}A^a_u - \partial_u\bar{A}^a + f^{abc}\bar{A}^b A^c_u + \lambda f^{abc}\bar{B}^b B^c_u + J^a_{\bar{B}} = -\delta(u)\bar{A}^a_+\\
&\partial_u A^a - \partial A^a_u + f^{abc}A^b_u A^c - \lambda f^{abc}B^b B^c_u + J^a_{\bar{B}} = -\delta(u)A^a_-\\
&\partial\bar{A}^a - \bar{\partial}A^a + f^{abc}A^b\bar{A}^c + \lambda f^{abc}B^b\bar{B}^c + d^a - f^{abc}\bar{\phi}^b c^c - \lambda f^{abc}\bar{c}^b\phi^c + J^a_{B_u} = 0\\
&B^a_u + J^a_d = 0\ ,
\end{aligned} \tag{5.5}$$

and the two-dimensional Ward identities (4.5) and (4.6) become

$$\int_{-\infty}^{+\infty} du H^a(x) Z_c(J_\psi) = \partial \bar{B}_+^a + \bar{\partial} B_-^a \tag{5.6}$$

$$\int_{-\infty}^{+\infty} du N^a(x) Z_c(J_\psi) = \partial \bar{A}_+^a + \bar{\partial} A_-^a \tag{5.7}$$

From (5.6) and (5.7) one gets the following relations between the Green functions:

$$\begin{aligned}
&\bar{\partial}\langle B^a(z)\mathcal{A}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\mathcal{B}^{a_{N+1}\ldots a_M}_{z_{N+1},\ldots,z_M}\rangle_- = \\
&= \ -\delta_{1N}\delta_{NM}\delta^{aa_1}\partial\delta^2(z-z_1) \\
&\quad +\sum_{k=1}^{N} f^{aa_kb}\delta^2(z-z_k)\langle A^b(z)A^{a_1}(z_1)\ldots\widehat{A^{a_k}(z_k)}\ldots A^{a_N}(z_N)\mathcal{B}^{a_{N+1}\ldots a_M}_{z_{N+1},\ldots,z_M}\rangle_- \\
&\quad +\sum_{k=N+1}^{M} f^{aa_kb}\delta^2(z-z_k)\langle B^b(z)\mathcal{A}^{a_1\ldots a_N}_{z_1,\ldots,z_N}B^{a_{N+1}}(z_{N+1})\ldots\widehat{B^{a_k}(z_k)}\ldots B^{a_M}(z_M)\rangle_- \ ,
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
&\bar{\partial}\langle A^a(z)\mathcal{A}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\mathcal{B}^{a_{N+1}\ldots a_M}_{z_{N+1},\ldots,z_M}\rangle_- = \\
&= \ -\delta_{0N}\delta_{1M}\delta^{aa_1}\partial\delta^2(z-z_1) \\
&\quad +\lambda\sum_{k=1}^{N} f^{aa_kb}\delta^2(z-z_k)\langle B^b(z)A^{a_1}(z_1)\ldots\widehat{A^{a_k}(z_k)}\ldots A^{a_N}(z_N)\mathcal{B}^{a_{N+1}\ldots a_M}_{z_{N+1},\ldots,z_M}\rangle_- \\
&\quad +\sum_{k=N+1}^{M} f^{aa_kb}\delta^2(z-z_k)\langle A^b(z)\mathcal{A}^{a_1\ldots a_N}_{z_1,\ldots,z_N}B^{a_{N+1}}(z_{N+1})\ldots\widehat{B^{a_k}(z_k)}\ldots B^{a_M}(z_M)\rangle_- \ ,
\end{aligned} \tag{5.9}$$

where (z) stands for $(z, \bar{z})$, the hat means omission of its argument and

$$\mathcal{A}^{a_1\ldots a_N}_{z_1,\ldots,z_N} \equiv A^{a_1}(z_1)\ldots A^{a_N}(z_N) \tag{5.10}$$

$$\mathcal{B}^{a_{N+1}\ldots a_M}_{z_{N+1},\ldots,z_M} \equiv B^{a_{N+1}}(z_{N+1})\ldots B^{a_M}(z_M) \ . \tag{5.11}$$

Let us consider first the case $\lambda = 0$. For the operators

$$A^a(z,\bar{z}) = \lim_{u\to 0^-} A^a(x) \qquad B^a(z,\bar{z}) = \lim_{u\to 0^-} B^a(x) \tag{5.12}$$

we derive the conservation laws

$$\bar{\partial} A^a(z,\bar{z}) = \bar{\partial} B^a(z,\bar{z}) = 0 \tag{5.13}$$

and the commutation relations

$$\begin{aligned}
\left[A^a(z), A^b(z')\right] &= 0 \\
\left[B^a(z), B^b(z')\right] &= f^{abc}\delta(z-z')B^c(z) \\
\left[A^a(z), B^b(z')\right] &= f^{abc}\delta(z-z')A^c(z) - \delta^{ab}\delta'(z-z')
\end{aligned} \tag{5.14}$$

This algebra can be interpreted as a semidirect sum of the Kac–Moody algebra satisfied by B and its adjoint representation with a central extension in the mixed commutators.

If $\lambda > 0$, the operators

$$
\begin{aligned}
K^a(z,\bar{z}) &= \frac{1}{2}\lim_{u\to 0^-}\left(\frac{1}{\sqrt{\lambda}}A^a(x)+B^a(x)\right)\\
T^a(z,\bar{z}) &= \frac{1}{2}\lim_{u\to 0^-}\left(-\frac{1}{\sqrt{\lambda}}A^a(x)+B^a(x)\right)
\end{aligned}
\tag{5.15}
$$

are conserved currents:

$$
\bar{\partial}K^a(z,\bar{z}) = \bar{\partial}T^a(z,\bar{z}) = 0 \tag{5.16}
$$

and satisfy the commutation relations

$$
\begin{aligned}
&\left[K^a(z),K^b(z')\right] = f^{abc}\delta(z-z')K^c(z) - \frac{1}{2\sqrt{\lambda}}\delta^{ab}\delta'(z-z')\\
&\left[T^a(z),T^b(z')\right] = f^{abc}\delta(z-z')T^c(z) + \frac{1}{2\sqrt{\lambda}}\delta^{ab}\delta'(z-z')\\
&\left[K^a(z),T^b(z')\right] = 0
\end{aligned}
\tag{5.17}
$$

which is the direct sum of two Kac–Moody algebras with central extension $\pm\frac{1}{2\sqrt{\lambda}}$.

The case $\lambda \neq 0$ is not a common feature of the generic d–dimensional BF–system. In four dimensions, for instance, the model cannot be provided of a cosmological constant. We could therefore guess that only the algebra corresponding to $\lambda = 0$ survives in such cases.

By parity we can find the conservation laws and the algebraic structure which live on the opposite side of the boundary.

The solution II is easily recognized to give the same algebraic structure of the first solution, therefore we skip to the solution III, for which the equations of motion read

$$
\begin{aligned}
&\bar{\partial}B^a_u - \partial_u\bar{B}^a + f^{abc}\bar{A}^bB^c_u - f^{abc}A^b_u\bar{B}^c + J^a_{\bar{A}} = \delta(u)\frac{1}{\sqrt{\lambda}}(\bar{A}^a_+ + \bar{A}^a_-)\\
&\partial_uB^a - \partial B^a_u + f^{abc}A^b_uB^c - f^{abc}A^bB^c_u + J^a_{\bar{A}} = \delta(u)\frac{1}{\sqrt{\lambda}}(A^a_+ + A^a_-)\\
&\partial\bar{B}^a - \bar{\partial}B^a + f^{abc}A^b\bar{B}^c - f^{abc}\bar{A}^bB^c + b^a - f^{abc}\bar{c}^bc^c - f^{abc}\bar{\phi}^b\phi^c + J^a_{A_u} = 0\\
&A^a_u + J^a_b = 0\\
&\bar{\partial}A^a_u - \partial_u\bar{A}^a + f^{abc}\bar{A}^bA^c_u + \lambda f^{abc}\bar{B}^bB^c_u + J^a_B = \delta(u)\sqrt{\lambda}(\bar{B}^a_+ + \bar{B}^a_-)\\
&\partial_uA^a - \partial A^a_u + f^{abc}A^b_uA^c - \lambda f^{abc}B^bB^c_u + J^a_{\bar{B}} = \delta(u)\sqrt{\lambda}(B^a_+ + B^a_-)\\
&\partial\bar{A}^a - \bar{\partial}A^a + f^{abc}A^b\bar{A}^c + \lambda f^{abc}B^b\bar{B}^c + d^a - f^{abc}\bar{\phi}^bc^c - \lambda f^{abc}\bar{c}^b\phi^c + J^a_{B_u} = 0\\
&B^a_u + J^a_d = 0\ ,
\end{aligned}
\tag{5.18}
$$

The Ward identities expressing the residual gauge invariance on the boundary are

$$
\int_{-\infty}^{+\infty} du H^a(x)Z_c(J_\psi) = -\frac{1}{\sqrt{\lambda}}(\partial\bar{A}^a_+ + \partial\bar{A}^a_- + \bar{\partial}A^a_+ + \bar{\partial}A^a_-) \tag{5.19}
$$

$$
\int_{-\infty}^{+\infty} du N^a(x)Z_c(J_\psi) = -(\partial\bar{B}^a_+ + \partial\bar{B}^a_- + \bar{\partial}B^a_+ + \bar{\partial}B^a_-) \tag{5.20}
$$

The relations between Green functions are better understood in terms of the fields

$$
P^a(x) = \frac{A^a(x)+\sqrt{\lambda}B^a(x)}{\sqrt{1+\lambda}} \tag{5.21}
$$

$$Q^a(x) \quad = \quad \frac{A^a(x) - \sqrt{\lambda}B^a(x)}{\sqrt{1+\lambda}} \,, \tag{5.22}$$

for which the Dirichlet boundary conditions hold:

$$Q^a_+(z,\bar z) = \bar P^a_+(z,\bar z) = \bar Q^a_-(z,\bar z) = P^a_-(z,\bar z) = 0 \,. \tag{5.23}$$

For the new variables the identities (5.19) and (5.20) imply the conservation laws

$$\bar\partial P^a_+(z,\bar z) = 0 \qquad \partial \bar Q^a_+(z,\bar z) = 0 \tag{5.24}$$

and the chiral current algebra

$$\begin{aligned}
&\bar\partial\langle P^a(z)\mathcal{P}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\bar{\mathcal{Q}}^{a_{N+1}\ldots a_M}_{\bar z_{N+1},\ldots,\bar z_M}\rangle_+ = \\
&= c(\lambda)_+\delta_{1N}\delta_{NM}\delta^{aa_1}\partial\delta^2(z-z_1) + c(\lambda)_-\delta_{0N}\delta_{1M}\delta^{aa_1}\bar\partial\delta^2(z-z_1) \\
&\quad -c(\lambda)_+\sum_{i=1}^{N} f^{aa_ib}\delta^2(z-z_i)\langle P^b(z)P^{a_1}(z_1)\ldots\widehat{P^{a_i}(z_i)}\ldots P^{a_N}(z_N)\bar{\mathcal{Q}}^{a_{N+1}\ldots a_M}_{\bar z_{N+1},\ldots,\bar z_M}\rangle_+ \\
&\quad -c(\lambda)_-\sum_{i=N+1}^{M} f^{aa_ib}\delta^2(z-z_i)\langle\bar Q^b(z)\mathcal{P}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\bar Q^{a_{N+1}}(z_{N+1})\ldots\widehat{\bar Q^{a_i}(z_i)}\ldots\bar Q^{a_M}(z_M)\rangle_+ \,,
\end{aligned} \tag{5.25}$$

and

$$\begin{aligned}
&\partial\langle \bar Q^a(z)\mathcal{P}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\bar{\mathcal{Q}}^{a_{N+1}\ldots a_M}_{\bar z_{N+1},\ldots,\bar z_M}\rangle_+ = \\
&= c(\lambda)_-\delta_{1N}\delta_{NM}\delta^{aa_1}\partial\delta^2(z-z_1) + c(\lambda)_+\delta_{0N}\delta_{1M}\delta^{aa_1}\bar\partial\delta^2(z-z_1) \\
&\quad -c(\lambda)_-\sum_{i=1}^{N} f^{aa_ib}\delta^2(z-z_i)\langle P^b(z)P^{a_1}(z_1)\ldots\widehat{P^{a_i}(z_i)}\ldots P^{a_N}(z_N)\bar{\mathcal{Q}}^{a_{N+1}\ldots a_M}_{\bar z_{N+1},\ldots,\bar z_M}\rangle_+ \\
&\quad -c(\lambda)_+\sum_{i=N+1}^{M} f^{aa_ib}\delta^2(z-z_i)\langle\bar Q^b(z)\mathcal{P}^{a_1\ldots a_N}_{z_1,\ldots,z_N}\bar Q^{a_{N+1}}(z_{N+1})\ldots\widehat{\bar Q^{a_i}(z_i)}\ldots\bar Q^{a_M}(z_M)\rangle_+ \,,
\end{aligned} \tag{5.26}$$

where

$$c(\lambda)_\pm = \frac{\sqrt{\lambda}(1\pm\sqrt{\lambda})}{1+\lambda} \tag{5.27}$$

and

$$\mathcal{P}^{a_1\ldots a_N}_{z_1,\ldots,z_N} \;\equiv\; P^{a_1}(z_1)\ldots P^{a_N}(z_N) \tag{5.28}$$

$$\bar{\mathcal{Q}}^{a_{N+1}\ldots a_M}_{\bar z_{N+1},\ldots,\bar z_M} \;\equiv\; \bar Q^{a_{N+1}}(\bar z_{N+1})\ldots\bar Q^{a_M}(\bar z_M) \,. \tag{5.29}$$

It is easily seen that eqs (5.25) and (5.26) are compatible only if $\lambda = 1$. We finally derive for the operators

$$K^a(z,\bar z) = -\sqrt{2}\lim_{u\to o^+} P^a(x) \qquad \bar T^a(z,\bar z) = -\sqrt{2}\lim_{u\to o^+}\bar Q^a(x) \tag{5.30}$$

the conservation laws

$$\bar\partial K^a(z,\bar z) = \partial\bar Q^a(z,\bar z) = 0 \tag{5.31}$$

and the chiral current algebra

$$\begin{aligned}
&\Big[K^a(z),K^b(z')\Big] = f^{abc}\delta(z-z')K^c(z) + \frac{1}{2}\delta^{ab}\delta'(z-z') \\
&\Big[\bar T^a(\bar z),\bar T^b(\bar z')\Big] = f^{abc}\delta(z-z')\bar T^c(\bar z) - \frac{1}{2}\delta^{ab}\delta'(\bar z-\bar z') \\
&\Big[K^a(z),\bar T^b(\bar z')\Big] = 0 \,.
\end{aligned} \tag{5.32}$$

The solution IV corresponds to the exchange $K^a(x) \leftrightarrow T^a(x)$.

6 Conclusions

We studied the three–dimensional BF model with a planar boundary in the axial gauge. We found the most general equations of motion for the quantum fields of the theory with boundary compatible with the fundamental requirements of locality and decoupling.

The choice of an axial gauge allowed us to write the Slavnov identities expressing the residual gauge invariance on the planar boundary. From these identities we found three possible distinct anomalous chiral current algebras, with two conserved chiral currents for each of them.

In the case $\lambda = 0$ we obtain a semidirect sum of a Kac–Moody algebra satisfied by a chiral conserved current and its adjoint representation acting on a current of the same chirality (5.14). For a generic $\lambda \neq 0$ we find a direct sum of two independent Kac–Moody algebras satisfied by conserved currents of the same chirality (5.17). In the particular case $\lambda = 1$, we find a third possible algebraic structure: a direct sum of Kac–Moody algebras satisfied by conserved currents of opposite chirality (5.32).

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$$\Delta^+(x,x') = \begin{pmatrix} \frac{a_1}{2\pi i(z-z')^2} & a_2\delta^2 & 0 & \partial T_{a_3}(x',x) & \frac{a_4}{2\pi i(z-z')^2} & \partial T_{a_3-a_4}(x',x) & 0 & (a_1+a_2)\partial\delta^2 \\ a_2\delta^2 & \frac{a_5}{2\pi i(\bar z-\bar z')^2} & 0 & -\bar\partial T_{a_6}(x,x') & T_{a_7}(x,x') & \frac{a_6-a_7}{2\pi i(\bar z-\bar z')^2} & 0 & -(a_2+a_5)\bar\partial\delta^2 \\ 0 & 0 & 0 & -\delta^3 & 0 & 0 & 0 & 0 \\ -\partial T_{a_3}(x,x') & \bar\partial T_{a_6}(x',x) & -\delta^3 & (2a_9+a_8+a_{10})\partial\bar\partial\delta^2 & -(a_8+a_9)\partial\delta^2 & (a_9+a_{10})\bar\partial\delta^2 & 0 & (1+a_3+a_6)\partial\bar\partial\delta^2 \\ \frac{a_4}{2\pi i(z-z')^2} & T_{a_7}(x',x) & 0 & (a_8+a_9)\partial\delta^2 & \frac{a_8}{2\pi i(z-z')^2} & a_9\delta^2 & 0 & \partial T_{a_4+a_7}(x',x) \\ T_{a_3-a_4}(x,x') & \frac{a_6-a_7}{2\pi i(\bar z-\bar z')^2} & 0 & -(a_9+a_{10})\bar\partial\delta^2 & a_9\delta^2 & \frac{a_{10}}{2\pi i(\bar z-\bar z')^2} & 0 & -\bar\partial T_{a_3-a_4+a_6-a_7}(x,x') \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta^3 \\ -(a_1+a_2)\partial\delta^2 & (a_2+a_5)\bar\partial\delta^2 & 0 & (1+a_3+a_6)\partial\bar\partial\delta^2 & -\partial T_{a_4+a_7}(x,x') & \bar\partial T_{a_3-a_4-a_6-a_7}(x',x) & -\delta^3 & (2a_2+a_1+a_5)\partial\bar\partial\delta^2 \end{pmatrix}$$

Table 3. $\Delta^+(x,x')$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	α_1	α_2	α_3	α_4
I	0	0	-1	-1	0	0	0	0	0	0	0	1	1	0
II	0	0	-1	0	0	0	-1	0	0	0	0	-1	-1	0
III	$-\frac{1}{2}\sqrt{\lambda}$	$\frac{1}{2}\sqrt{\lambda}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{\lambda}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{\lambda}}$	$\frac{1}{2\sqrt{\lambda}}$	$-\frac{1}{2\sqrt{\lambda}}$	$\frac{1}{\sqrt{\lambda}}$	0	0	$\sqrt{\lambda}$
IV	$\frac{1}{2}\sqrt{\lambda}$	$-\frac{1}{2}\sqrt{\lambda}$	-1	$-\frac{1}{2}$	$\frac{1}{2}\sqrt{\lambda}$	0	$-\frac{1}{2}$	$\frac{1}{2\sqrt{\lambda}}$	$-\frac{1}{2\sqrt{\lambda}}$	$\frac{1}{2\sqrt{\lambda}}$	$-\frac{1}{\sqrt{\lambda}}$	0	0	$-\sqrt{\lambda}$

Table 4. Solutions of the sets of equations (4.3) and (4.7).