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Autor: Hilke, M. / Flores, J.C.
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Diffusion in the kicked quantum rotator by random corrections to a linear and sine field

M. Hilke[1a] and J.C. Flores[1b]

[a] *Département de Physique Théorique, Université de Genève, 24 quai Ernest Ansermet, CH - 1211 Genève 4, Switzerland*

[b] *Purdue University, Physics building West Lafayette, IN 47907, USA*

We discuss the diffusion in momentum space, of the kicked quantum rotator, by introducing random corrections to a linear and sine external field. For the linear field we obtain a linear diffusion behavior identical to the case with zero average in the external field. But for the sine field, accelerator modes with quadratic diffusion are found for particular values of the kicking period.

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It has been shown that the wave function of the periodically kicked rotator, is localized in momentum space and does not exhibit diffusion [1, 2]. On the other hand, in the classical model a chaotic i.e. a diffusive regime was found in momentum space. Until today the origin of the difference between the quantum model and the classical model is not clearly understood. It therefore seems interesting to study how quantum localisation effects could be destoyed. One possibility seems to be 'randomization'.
In ref.[3] for instance, it is argued that an unbounded regime in energy occurs when the time between two kicks is random. Ref.[4] pointed out that a diffusive behavior is found, if the amplitude of every kick is random with zero average. For a non-zero average of the external force, the behavior is still unknown. We present here some results for a linear and sine average force.
Explicitly we consider the hamiltonian of the kicked rotator:

$$H = -\frac{\partial^2}{\partial\theta^2} + \sum_{j=0}^{J} (V(\theta) + \varepsilon_j(\theta))\,\delta(t - \tau j) \tag{1}$$

where $-\pi < \theta < \pi$, $\varepsilon_j(\theta)$ is a stochastic independent process for any j. $V(\theta)$ is the deterministic amplitude and τ the period.
The case $V(\theta) = 0$ was solved in ref.[4] and diffusion in momentum space was found. The case where $V(\theta) \neq 0$ and $\varepsilon_j(\theta)$ is small was solved in ref.[5] by a perturbative method. Here again diffusion was found through 'randomization'.
We will show in this paper more general results.

The general state of the system $\Psi(\theta, t)$, between two kicks, can be written as

$$\Psi(\theta, t) = \sum_{n=-\infty}^{\infty} \Psi_n(t) e^{in\theta} \ , \ n \in Z \tag{2}$$

where $\Psi(t)$ are time-dependent Fourier coefficients. From the Schrödinger equation related to the hamiltonian (1) and the matching condition at $t_j = \tau \cdot j$: $\Psi(\theta, t_j^+) = e^{-i(V+\varepsilon_j)}\Psi(\theta, t_j^-)$, the following recursive relation between t_{j+1}^+ and t_j^+ is therefore obtained:

$$\Psi_m(t_{j+1}^+) = \sum_n J_{m-n}^j e^{-i\tau n^2} \Psi_n(t_j^+) \tag{3}$$

where

$$J_l^j = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta e^{il\theta} e^{-i(V(\theta)+\epsilon_j(\theta))} \tag{4}$$

is a random quantity because of ϵ_j and $^{-+}$ denote before and after the kick. Lets define the elements of the density matrix (at time t_j^+) in the usual form, namely

$$\rho_{mn}^j = \Psi_m(t_j^+)\Psi_n^*(t_j^+). \tag{5}$$

Then from (3) we get:

$$\rho_{mn}^{j+1} = \sum_{l,s} J_{m-l}^j J_{n-s}^{*j} e^{-i\tau(l^2-s^2)}\rho_{ls}^j. \tag{6}$$

Physical quantities like energy or momentum are related to the sample averaging density matrix $\langle\rho_{mn}^j\rangle$, where $\langle\cdot\rangle$ denotes the average on the random quantities ϵ_j.
We consider that the correlation C, between the random phases $e^{i\epsilon_j(\theta)}$ and $e^{-i\epsilon_j(\phi)}$ is only depending on the distance, explicitly

$$C(\theta-\phi) = \langle e^{i(\epsilon_j(\theta)-\epsilon_j(\phi))}\rangle \tag{7}$$

this assumes translational invariance. We suppose moreover that the correlation C is an even function i.e. $C(\theta) = C(-\theta)$.
The power spectrum C_n of the correlation (7) is given by

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta C(\theta) e^{-in\theta} \tag{8}$$

Using (8) and (4) we have

$$\langle J_m^j J_n^{*j}\rangle = \sum_l C_l J_{m+l} J_{n+l}^* \tag{9}$$

where

$$J_l = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{il\theta} e^{-iV(\theta)} \tag{10}$$

is related to the deterministic contribution $V(\theta)$. It is interesting to note that $C_l = \delta_{l,0}$ corresponds to the model without "disorder" [1, 2], which could be

interpreted as a system with high correlation length i.e. $C(\theta) = const.$ Using the fact that the sum over l of $e^{il\theta}$ gives a $\delta(\theta)$, we find

$$\sum_l l(J_{n+l}J_l^*) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta V'(\theta)e^{in\theta} \tag{11}$$

$$\sum_l l^2(J_{n+l}J_l^*) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta \left(V'^2(\theta) - iV''(\theta)\right)e^{in\theta} \tag{12}$$

thus the first and the second moment of the kinetic momentum ($\hbar = 1$).

$$[l]_j \equiv \sum_l l\langle\rho_{ll}^j\rangle \tag{13}$$

$$[l^2]_j \equiv \sum_l l^2\langle\rho_{ll}^j\rangle \tag{14}$$

are related at t_{j+1} and t_j, by

$$[l]_{j+1} = [l]_j + \frac{1}{2\pi}\sum_{l,s} e^{-i\tau(l^2-s^2)}\langle\rho_{ls}^j\rangle\int_{-\pi}^{\pi} d\theta V'(\theta)e^{-i(l-s)\theta} \tag{15}$$

and

$$\begin{aligned}[l^2]_{j+1} = [l^2]_j + D &+ \frac{1}{2\pi}\sum_{l,s} e^{-i\tau(l^2-s^2)}\langle\rho_{ls}^j\rangle\int_{-\pi}^{\pi} d\theta(V'^2(\theta) + (l+s)V'(\theta))e^{-i(l-s)\theta} \\ &+ \frac{2}{2\pi}\sum_{l,s} s\, e^{-i\tau(l^2-s^2)}\langle\rho_{ls}^j\rangle\int_{-\pi}^{\pi} d\theta V'(\theta)e^{-i(l-s)\theta}\end{aligned} \tag{16}$$

where the constant D is given by

$$D = \sum_l l^2 C_l \tag{17}$$

as

$$C(\theta) = \sum_l C_l e^{il\theta} \tag{18}$$

we have $C'(0) = i\sum_l lC_l = 0$ ($C(\theta)$ is even) and $-C"(0) = \sum_l l^2 C_l = D \geq 0$ because $C(\theta)$ has its maximum value at $\theta = 0$. The fact that $\sum_l C_l = 1$ and $\sum_l lC_l = 0$ was used for deriving (15) and (16), as well as $\sum_l\langle\rho_{ll}^j\rangle = 1$.
It is interesting to note that (15) and (16) are valid for any potential $V(\theta)$, but resolving them is very complex because they involve also the non-diagonal terms $\langle\rho_{ls}^j\rangle$.

- The model which is easily solved, is the linear potential i.e. $V(\theta) = K\theta$ with $-\pi < \theta < \pi$ and K beeing a constant. Using (15) and (16) we get

$$\begin{aligned} [l]_{j+1} &= [l]_j + K \\ [l^2]_{j+1} &= [l^2]_j + D + K^2 + 2K[l]_j \end{aligned} \tag{19}$$

As the dispersion in kinetic momentum space is

$$\sigma_j^2 \equiv [l^2]_j - [l]_j^2 \tag{20}$$

it follows from (19) and (20) that

$$\sigma_j^2 = D \cdot j \tag{21}$$

Thus a diffusive behaviour in momentum space is found.
It is interesting to note that from (18) the diffusion coefficient can be related to the correlation C by

$$D = -\frac{d^2}{d\theta^2} C(\theta)|_{\theta=0} \tag{22}$$

which is independent of the deterministic amplitude K and then valid in the zero average model (K=0).

- Lets consider now the case $V(\theta) = K sin(\theta)$.
We obtain after numerous calculations

$$\begin{aligned} [l^2]_{j+1} &= [l^2]_j + D + K^2/2 + f(\tau) \\ [l]_{j+1} &= [l]_j \end{aligned} \tag{23}$$

where

$$\begin{aligned} f(\tau) = \; & \frac{K}{2} \sum_s (2s+1) \left(e^{-i\tau(2s+1)} \langle \rho^j_{s+1,s} \rangle + e^{i\tau(2s+1)} \langle \rho^j_{s,s+1} \rangle \right) \\ & + \frac{K^2}{4} \sum_s \left(e^{-i4\tau s} \langle \rho^j_{s+1,s-1} \rangle + e^{i4\tau s} \langle \rho^j_{s-1,s+1} \rangle \right) \end{aligned} \tag{24}$$

this function is bounded by the following value:

$$|f(\tau)| \leq K^2 \cdot j \tag{25}$$

The extreme value $f = K^2 \cdot j$ corresponds to the case $\tau = 2\pi n$. For this case we have the same situation as in the classical rotator where "accelerator modes" exist [2], with an energy dispersion proportional to j^2, i.e. $\sigma_j^2 = K^2 \cdot j^2 + (D + K^2/2) \cdot j$.
For irrational values of τ the dispersion would probally just increase proportionally to j.
If $\tau = \pi(2n+1)$ we actually find $f = 0$. It is interesting to note that for $D = 0$ the same diffusion coefficient is found as in the classical model of the kicked rotator without disorder.
Taking $V(\theta) = Kcos(\theta)$ doesn't change the conclusions. This result can be generalised for any potential $V(\theta) = \sum C_n e^{in\theta}$, where a quadratic diffusion is found for the resonant case.

Conclusions: We have found a diffusive behaviour in both, the linear and sine kicked rotator, by introducing randomness. The diffusion coefficient for the sine field depends on the correlation function between the random phases $e^{i\epsilon_j(\theta)}$, the field amplitude and the period. Whereas the diffusion coefficient of the linear field, only depends on the correlation function between the random phases and not on the amplitude of the field. This behaviour is in agreement with the result of E. Ott et al. [5] who considered a small perturbative noise. The results of the randomized sine quantum kicked rotator are very similar to the classical sine kicked rotator. We can therefore question ourselves on the physical origin of 'randomization' for quantum systems. As was partially pointed out in ref. [6, 7], it seems conceavable to consider that the Hamiltonian of a microscopic system, for instance the quantum rotator, connot be known exactly, but that a statistical description ('randomization' maybe) seems more adequate.

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