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ON THE DESCRIPTION OF SINGLE MASSLESS QUANTUM OBJECTS

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Abstract: We advance an understanding of a single free quantum object as an extended time-stable finite (3+1)-dimensional object, intrinsically connected with a particular periodic process with period T, such that its integral energy E is related to T by the Planck's relation $ET=h$. It is explicitly shown that the nonlinear local conservation laws for the energy-momentum tensor of the electromagnetic field admit a large class of such "pulsating" soliton-like massless solutions. Explicit mathematical expressions for the Planck's constant h and for a local intrinsic angular momentum of these solutions are proposed.

1. Outlining the idea

At the very beginning of modern quantum physics Einstein [1] and de Broglie [2] made attempts to consider photons and electrons as localized objects, i.e. finite extended lumps, or lumps-like formations, moving as a whole along some fixed direction, but after Born's probabilistic interpretation [3] of quantum phenomena became wide-spread and almost generally accepted by the majority of leading physicists, the interest to describe single quantum objects or events seemed to disappear. It must be stressed however that neither Einstein nor de Broglie accepted ever entirely this probabilistic point of view on the physical world.

A new interest to this problem appeared during the last 25 years, based mainly on some success in dealing with nonlinear equations, the so called "soliton science" [4,5], but a deep enough development of this point of view is still missing. A constructive and encouraging interest in this field is shown nowadays in the papers of Barut [6,7], and there one can find a simple and clear notion of the concept "quantum particle" or "single quantum object". For the sake of clearness I shall begin with an outline of this notion in such a way as I accept it.

Under free quantum object I understand any finite extended time-stable physical object whose existence is demonstrated only through a particular intrinsic periodic process (T denotes the period) and whose integral energy E satisfies Planck's relation $ET = h$,where h is the Planck's constant. Of course the simplest case is when this periodicity is described well enough by the elementary periodic functions "$\sin x$" and "$\cos x$". "Extendedness" means here that at any moment t , even at $t\to\infty$, our object occupies finite 3-region $\Omega_t \subset \mathbb{R}^3$ with unspecified topology, except connectedness, and "finiteness" means nonsingular field functions, being zero at points out of Ω_t for any t. Any free quantum object moves as a whole with constant velocity $v \leq c$. If $v < c$ it is expectable that a frame in which the field functions take the simple form $f(x,y,z)\cos(\nu t+\varphi)$ exists and $\varphi(x,y,z)$ and $f(x,y,z)$ are different from zero only inside Ω_t. This means that any comoving observer finds at any point of Ω_t the same periodicity. Functions $f(x,y,z)$ and $\varphi(x,y,z)$ describe the structure of the object. If $v = c$, the case we are interested in this paper, there is no such frame and choosing axis z as a direction of motion, the field functions are expected to take the form $f(x,y,z\pm ct)\cos(\nu t+\varphi)$. We note that Maxwell's pure field equations can not describe such objects because their solutions in the whole space are just superposition of plane running waves.Moreover, since any component of the field satisfies the wave equation, the solution of the Cauchy problem with strictly localized initial condition shows that this initial excitation "blows up" radially and goes to infinity with the velocity of light after which the points of the 3-space "forget" about what happened. Note that any plane wave $f(z\pm vt)$ is in fact a "dead structure", it has no intrinsic "life", because the co-moving observer does not see any changes occurring to the wave. On the contrary, quantum objects have intrinsic "life" and according to the above made assumption it demonstrates itself entirely through some periodic process. The relation $ET = h$ connects the integral and intrinsic characteristics of quantum objects. As an illustration recall the sine-Gordon equation

$$\phi_{zz} - \phi_{tt} = A\sin\phi$$

and its 1-soliton and "breather" solutions: the 1-soliton solution is a "dead" structure, and the "breather" solution "breathes" in the co-moving frame. Since the "breather" solution does not satisfy Planck's relation it cannot be a model of any quantum object.

Let's turn our attention now to the local energy-momentum properties of a single quantum object. The classical field theoretic approach, assumed in this paper, requires existence of energy-momentum tensor T^{ν}_{μ} with $T^{4}_{4}>0$ being the energy density. Let P_{μ} be the corresponding integral energy-momentum; then if $P_{\mu}P^{\mu}=0$ the physical object is called massless, and if $P_{\mu}P^{\mu}>0$ the proper mass "m" of the object is defined by

$$m = c^{-2}\left[P_4{}^2 - c^2\vec{P}^2\right]^{\frac{1}{2}}.$$

We shall localize this condition, namely, we shall call the corresponding objects *isotropic* or *nonisotropic* according to $T_{\mu\nu}T^{\mu\nu}=0$ or $T_{\mu\nu}T^{\mu\nu}\neq 0$ respectively. This generalization seems reasonable because our quantum objects move as a whole,i.e. every point of them moves with the same velocity.

On the other hand T^{ν}_{μ} can be found if the lagrangian is known and it satisfies, by definition, the equations $\nabla_{\nu}T^{\nu}_{\mu}=0$, which together with space-time symmetries (i.e.isometries) guarantee conservation of the integral energy-momentum. The integral energy E is given by

$$E = \int_{\Omega} T^{4}_{4} \mathrm{dxdydz} \ ,$$

where T^{4}_{4} is expressed through the field functions, which are determined as solutions of the field equations. This procedure, applied to the plane-wave solution of Maxwell's equations, for instance, meets a very serious difficulty - the above integral goes to infinity since the plane-wave solution occupies infinite region and $T^{4}_{4}>0$. So, Maxwell's equations are not good for our purpose. On the other hand the field functions $F_{\mu\nu}$ satisfy the nonlinear equations $\nabla_{\nu}T^{\nu}_{\mu}=0$, and combining them with the local isotropy condition $T_{\mu\nu}T^{\mu\nu} = \left[\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right]^{2} + \left[\frac{1}{2}F_{\mu\nu}(*F)^{\mu\nu}\right]^{2}=0$, where $*$ is the Hodge operator, we get 6 equations. After solving these equations we can impose some additional conditions (or equations) on the solution found, and these additional conditions are meant to express some important features of the intrinsic periodic process, considered as characteristic for the isotropic quantum object.

The outlined idea is applicable in principle for any appropriate classical field (with lagrangian or with known energy tensor) and the purpose of this paper is to consider what happens in case of Maxwell field $F_{\mu\nu}$, assuming that in general $F_{\mu\nu}$ do not satisfy the pure field equations $dF=0$, $d*F=0$.

2. Formulation of the model and results

We start with mentioning our basic assumptions. First of all our free quantum object will live in a Minkowski's space. From pure mathematical point of view this means that some field with appropriate features is assumed to exist on $(\mathbb{R}^4,\eta)$, where η is a chosen pseudo-euclidean metric. In natural coordinates $(x^1,x^2,x^3,x^4)=(x,y,z,\xi=ct)$ we have

$$ds^2=-dx^2-dy^2-dz^2+d\xi^2 \quad , \text{ i.e. } \quad \eta_{\mu\nu}=\begin{Vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix} .$$

Having η we have the volume form $\sqrt{|\eta|}\,dx\wedge dy\wedge dz\wedge d\xi$, $|\eta|=|\det(\eta_{\mu\nu})|$, the Hodge $*$-operator and the divergence (coderivative) operator $\delta=(-1)^p *^{-1}d*$, we recall that $\delta\circ\delta=0$. Our next assumption concerns the character of the field: we assume the field to be represented by a differential 2-form $F\in\Lambda^2_b(\mathbb{R}^4,\eta)$, "b" means that the components $F_{\mu\nu}$ are bounded functions in canonical coordinates and for any ξ are different from zero only inside some Ω_ξ, where Ω_ξ is a finite connected region in $\mathbb{R}^3$ for any ξ. From F we get $*F$, δF and $\delta*F$. Since $\delta(\delta F)=0$ and $F_{\mu\nu}$ are bounded and $\mathbb{R}^3$-finite functions the two integrals

$$q_1(F)=\int(\delta F)_4 dxdydz \quad , \qquad q_2(F)=\int(\delta*F)_4 dxdydz$$

are finite and do not depend on time. Therefore the Lorentz invariant quantity

$$Q(F)=\frac{q_1^2+q_2^2}{c}$$

also does not depend on time. Choosing $\dim F_{\mu\nu}$=dim [energy density]$^{1/2}$ we get dimQ=dim [action]. Clearly, the set of all 2-forms on our Minkowski's space consists of non-intersecting subsets, giving the same Q. In particular, it is easy to see that the subset of 2-forms, giving Q=0, is a vector space, containing all solutions of Maxwell equations.

Remark 1.Since the components $F_{\mu\nu}$ of the field do not satisfy Maxwell's equations the physical interpretation of $F_{\mu\nu}$ becomes perhaps not quite clear. What is kept from Maxwell theory is the physical dimension of $F_{\mu\nu}$, namely, [energy density]$^{1/2}$. Although $\delta F\neq 0$ and $\delta * F\neq 0$ these two 4-vectors have nothing to do with any electric or magnetic currents. Of course, they carry some information about our single free quantum object, and that's all, no characteristics of any other objects are introduced by them.

After these two preliminary assumptions of general character we impose the following two conditions on $F\in\Lambda^2_Q(\mathbb{R}^4,\eta)$, with $Q\neq 0$. Let $T^{\nu}_{\mu} = \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\delta^{\nu}_{\mu} - F_{\mu\sigma}F^{\nu\sigma}$ be the standard energy-tensor of the electromagnetic field,obtained through variation with respect to the metric coefficients of the classical action integral. We assume T^{ν}_{μ} to be the energy-momentum of our field.

We require

$$1^\circ.\ T_{\mu\nu}T^{\mu\nu} = \left[\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} \right]^2 + \left[\tfrac{1}{2}F_{\mu\nu}(*F)^{\mu\nu}\right]^2 = I^2_1 + I^2_2 = 0$$

$$2^\circ.\ \nabla_{\mu}T^{\mu}_{\nu} = 0.$$

From 1° it follows [8] that at any point all eigen values of T^{ν}_{μ}, $F_{\mu\nu}$ and $(*F)_{\mu\nu}$ are equal to zero. Also, T^{ν}_{μ} and $F_{\mu\nu}$ admit just one common isotropic eigen direction and all other eigen directions are space-like. So, we have the isotropic vector fields $\pm V^{\mu}$, $V_{\mu}V^{\mu}=0$. Then there exist two 1-forms A and B, such that $F=A\wedge V$ and $*F=B\wedge V$. Clearly, A and B are defined up to additive factors ϑ_1 and ϑ_2, such that $V\wedge\vartheta_1= V\wedge\vartheta_2=0$. Now we obtain

$$0=I_1=-(A_{\mu}V^{\mu})^2,\ 0=I_2=-(B_{\mu}V^{\mu})^2,\ \text{i.e.}$$

A and B are orthogonal to V. Also, $T^{\nu}_{\mu}=-(A)^2V_{\mu}V^{\nu}=-(B)^2V_{\mu}V^{\nu}$, i.e. $(A)^2=(B)^2$. Besides, $T^{\nu}_{\mu}A^{\mu}=T^{\nu}_{\mu}B^{\mu}=0$, i.e. A and B are eigen vectors of T^{ν}_{μ}, consequently, $(A)^2<0$ and $(B)^2<0$ – the two 1-forms A and B are space-like. Now from 2° it follows

$$\nabla_{\nu}T^{\nu}_{\mu}= \nabla_{\nu}(-A^2V_{\mu}V^{\nu})=-A^2V^{\nu}\nabla_{\nu}V_{\mu}- V_{\mu}\nabla_{\nu}(A^2V^{\nu})=0,\text{i.e.}$$

$$V^{\nu}\nabla_{\nu}V^{\mu} =-\left[(A^2)^{-1}\nabla_{\nu}(A^2V^{\nu})\right]V^{\mu}.$$

This last relation means [9] that the integral curves of V^{μ} are isotropic geodesics. Now by means of an appropriate Lorentz transformation of coordinates we can achieve the trajectories of V^{μ}, which are straight lines, to lie in the (z,ξ)-planes. In these new coordinates we have $V_{\mu}=(0,0,\varepsilon\lambda,\lambda)$, where $\varepsilon=\pm1$. By obvious reasons we assume $\lambda=1$.

Now from $F_{\mu\nu}V^{\nu}=0$, $(*F)_{\mu\nu}V^{\nu}=0$, $F_{\mu\nu}=A_{\mu}V_{\nu}-A_{\nu}V_{\mu}$, $*F_{\mu\nu}=B_{\mu}V_{\nu}-B_{\nu}V_{\mu}$ it follows $F_{12}=F_{34}=0$, $F_{13}=\varepsilon F_{14}=\varepsilon A_1$, $F_{23}=\varepsilon F_{24}=\varepsilon A_2$,

$$A_3=\varepsilon A_4,\ B_3=\varepsilon B_4,\ B_1=-\varepsilon A_2,\ B_2=\varepsilon A_1,\ \varepsilon=\pm1.$$

Moreover, since A_4 and B_4 cannot be determined by $F_{\mu\nu}$ we shall assume $A_4=B_4=0$. Denoting $F_{14}=u$, $F_{24}=p$ we obtain

$$F=\varepsilon u dx\wedge dz + u dx\wedge d\xi + \varepsilon p dy\wedge dz + p dy\wedge d\xi$$

$$*F=-p dx\wedge dz - \varepsilon p dx\wedge d\xi + u dy\wedge dz + \varepsilon u dy\wedge d\xi$$

$$\nu \longrightarrow$$

$$T_{\mu}{}^{\nu} = \begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -(u^2+p^2) & \varepsilon(u^2+p^2) \\ 0 & 0 & -\varepsilon(u^2+p^2) & (u^2+p^2) \end{Vmatrix}$$

and the conservation equations $\nabla_{\nu}T^{\nu}_{\mu}=0$ reduce to

$$(u^2+p^2)_{\xi} - \varepsilon(u^2+p^2)_{z} = 0.$$

The general solution of this equation is $u^2+p^2=\phi(x,y,\xi+\varepsilon z)$, which suggests introduction of new functions ϕ and φ according to

$$\phi = \sqrt{u^2+p^2} \quad , \ \varphi = \frac{u}{\sqrt{u^2+p^2}} \ , \ \phi\geq 0, \ \ |\varphi|\leq 1.$$

So ϕ, the square of which represents the energy density, is a running wave, and since u and p are finite functions with respect to (x,y,z) ϕ is finite too. The apparent dependence of ϕ on (x,y,z) is to be determined by some initial condition, which means that the spatial structure of the solution can not be determined by the conservation law. Entirely unknown however is the (bounded) function φ. Clearly φ describes some intrinsic features of the object and the energy density ϕ^2 is indifferent to these features. The case φ=const is out of interest because it leads to running wave character of $F_{\mu\nu}$

<u>Remark 2</u>. Recall that since $\nabla_{\nu}T^{\nu}_{\mu}=0$ the integral energy E and momentum $\vec{P}=(0,0,\frac{\pm E}{c})$ do not depend on time. Obviously $E^2-c^2\vec{P}^2=0$ and the quantity Q naturally defines some time-interval $T=\frac{Q}{E}$ and a natural length cT.

The following is readily obtained:

$$\delta F = (u_\xi - \varepsilon u_z)dx + (p_\xi - \varepsilon p_z)dy + \varepsilon(u_x + p_y)dz + (u_x + p_y)d\xi$$

$$\delta * F = -(\varepsilon p_\xi - p_z)dx + (\varepsilon u_\xi - u_z)dy - (p_x - u_y)dz - \varepsilon(p_x - u_y)d\xi$$

$$\delta F_\mu \delta F^\mu = -(u_\xi - \varepsilon u_z)^2 - (p_\xi - p_z)^2 = -\phi^2(\varphi_\xi - \varepsilon\varphi_z)^2 = (\delta * F)_\mu (\delta * F)^\mu$$

$$\delta F_\mu (\delta * F)^\mu = 0$$

$$F_{\mu\nu}\delta F^\nu = 0, \quad (*F)_{\mu\nu}(\delta * F)^\nu = 0.$$

The last two relations mean that δF^μ is an eigen vector of $F_{\mu\nu}$ and $\delta * F^\mu$ is an eigen vector of $*F_{\mu\nu}$ and also these last relations may be used as natural generalization of Maxwell's equations $\delta F=0$, $\delta * F=0$. Worth it also to note the relation $\nabla_\nu T^\nu_\mu = F_{\mu\nu}\delta F^\nu + (*F)_{\mu\nu}(\delta * F)^\nu$. On the other hand it is easily seen that the obtained F satisfies: $F_{\mu\nu} = \hat{F}_{\sigma\tau} L^\sigma_\mu L^\tau_\nu$, where

$$\hat{F}_{\mu\nu} = \begin{array}{c} \nu \longrightarrow \\ \left\| \begin{matrix} 0 & 0 & \varepsilon\phi & \phi \\ 0 & 0 & 0 & 0 \\ -\varepsilon\phi & 0 & 0 & 0 \\ -\phi & 0 & 0 & 0 \end{matrix} \right\| \end{array}, \quad L^\nu_\mu = \begin{array}{c} \mu \longrightarrow \\ \left\| \begin{matrix} \varphi & -\sqrt{1-\varphi^2} & 0 & 0 \\ \sqrt{1-\varphi^2} & \varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \right\| \end{array}.$$

This shows that $F_{\mu\nu}$ is obtained from its canonical form $\hat{F}_{\mu\nu}$ through some transformation L^ν_μ which, clearly, is connected with the group SO(2)(rotation around z-axis). So we have got a suggestion to determine this transformation L assuming it is just the differential of the flow of the vector field

$Z = \bar{\varepsilon}\left[-y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}\right] + cT\frac{\partial}{\partial \xi}$, $\bar{\varepsilon}=\pm 1$ determines the rotation direction and has nothing to do with the above ε. The flow of Z is determined by the equations

$$\frac{dx}{d\tau} = -\bar{\varepsilon}y, \quad \frac{dy}{d\tau} = \bar{\varepsilon}x, \quad \frac{dz}{d\tau} = 0, \quad \frac{d\xi}{d\tau} = cT$$

and initial conditions $x(0)=x_o$, $y(0)=y_o$, $z(0)=z_o$, $\xi(0)=\xi_o$. The flow looks as follows

$$x=x_o\cos(\bar{\varepsilon}\tau)-y_o\sin(\bar{\varepsilon}\tau) \qquad z=z_o$$

$$y=x_o\sin(\bar{\varepsilon}\tau)+y_o\cos(\bar{\varepsilon}\tau) \qquad \xi=\xi_o+cT\tau.$$

Now according to the above made suggestion we have to write down the following equation

$$\frac{\partial x^\sigma}{\partial x_o^\mu}=L^\sigma_\mu.$$

Taking in view that $\tau=\dfrac{\xi-\xi_o}{cT}$ and denoting $b=-\dfrac{\bar{\varepsilon}\xi_o}{cT}$ we get

$$\varphi = \cos(\bar{\varepsilon}\frac{\xi}{cT}+b).$$

The intrinsic "life" of our object is a very simple one and the intrinsic frequency "ν" of the solution F is expressed through the integral characteristics E and Q of the solution, i.e. $E=Q\nu$, which coincides with Planck's relation $E=h\nu$ if Q is identified with h. After simple computation for Q is obtained the following

$$Q = \frac{1}{c}\left\{\left[\int\phi_x dxdydz\right]^2+\left[\int\phi_y dxdydz\right]^2\right\},$$

which shows that in this case Q does not depend on φ, but depends on the transversal derivatives of ϕ.

3. Conclusion

So, the picture we get is the following: a connected finite and "pulsating" 3-lump with unknown 3-topology moves as a whole through space along a fixed 3-space direction $\vec{n}$ with the velocity of light and its particular nature consists (besides motion with the velocity of light) in the availability of a couple of two space-like 1-forms A and

B . These two 1-forms are micro-descendants of the 4-dimensional versions of the electric and magnetic vectors in Maxwell theory and meet the relations $A^2=B^2$, $A_\mu B^\mu=0$, also they are orthogonal to $\vec{n}$. The couple (A,B) rotates at every point of the "lump" with the same frequency $\nu=\frac{E}{Q}$. A natural invariant integral characteristic of this rotation is Q, since, obviously, Q is the real action of the "lump" for 1 period $T=\frac{1}{\nu}$. Thus, the "lump" does not rotate as a whole, but it can exist only through a, so to say, "progressive local rotation". The 2-form $S=\frac{1}{c}A\wedge B$ characterizes locally this rotation since

$$|S|=\frac{1}{c}\,|A\wedge B|=\frac{1}{c}\,|A_\mu A^\mu|=\frac{1}{c}\,|B_\mu B^\mu|=\frac{\Phi^2}{c}$$

and integrating $|S|$ over the 4-region $\Omega\mathrm{x}cT$ we obtain

$$\int_{\Omega\mathrm{x}cT}|S|\,\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\xi = E.T = Q.$$

Geometrically, Q is the integral area (in action units) covered by the rotating orthogonal 2-frame (A,B) during one period T. This motivates the 2-form S to be considered as local intrinsic angular (spin) momentum tensor of the object. Another approach is to consider the tensor field $V^\mu S_{\alpha\beta}=-V^\mu S_{\beta\alpha}$, having zero divergence: $\nabla_\mu V^\mu S_{\alpha\beta}=0$, and the conserved quantity $\int V_4 S_{12}\mathrm{d}x\mathrm{d}y\mathrm{d}z=\pm E$, which could be interpreted as integral spin momentum for one second, and giving $\pm Q$ for one period T. The two signs $\pm$ come from the two opposite directions of motion. As for the integral wave vector k_μ it can be introduced by the relation $k_\mu=\frac{1}{Q}\,p_\mu$, where $p_\mu=(0,0,\frac{\pm E}{c},\frac{E}{c})$ is the integral 4-momentum of the solution. So, instead of $\phi(x,y,\xi+\varepsilon z)$ we can write $\phi(x,y,k_\mu x^\mu)$, but the same cannot be done with the phase function $\varphi(\xi)$, which is not a running wave.

Having such single solutions at hand various further attempts can be made, such as description of flow of non-interacting solutions with possible interference, making corresponding statistical considerations, introduction of new equations, e.g. $F_{\mu\nu}\delta F^{\nu}=0$, appropriate definition of the classical electromagnetic wave, reflection phenomena and so on. Some of these problems are now under consideration. However, the important problem of interaction with mass-particles has not been considered so far because a good enough 3-dimensional soliton-like model of mass-particles is still missing and, I think, more efforts have to be concentrated on this problem.

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