Zeitschrift: Helvetica Physica Acta

Band: 65 (1992)

Heft: 4

Artikel: Antinucleon-nucleon scattering formalism and possible tests of CPT invariance

Autor: LaFrance, P. / Lehar, F. / Loiseau, B.

DOI: https://doi.org/10.5169/seals-116506

Helv Phys Acta
Vol. 65 (1992)

0018-0238/92/040611-30$1.50+0.20/0

Antinucleon-nucleon scattering formalism and possible tests of CPT invariance

By P. LaFrance[1,4], F. Lehar[2], B. Loiseau[3] and P. Winternitz[4]

[1]Université du Québec à Trois-Rivières, Département de Physique, C.P. 500, Trois-Rivières, Québec G9A 5H7, Canada
[2]DAPNIA-SPP, CEN-Saclay, 91191 Gif sur Yvette CEDEX, France
[3]Division de Physique Théorique*, IPN, 91406 Orsay CEDEX, France and LPTPE, Université Pierre et Marie Curie, 4, place Jussieu, 75252 Paris CEDEX 05, France
[4]Centre de Recherches Mathématiques, Université de Montréal, C.P. 6128, Montréal H3C 3J7, Québec, Canada

(17. IX. 1991, revised 23. XII. 1991)

Abstract. A detailed formalism of antinucleon-nucleon scattering in a convenient form for experimentalists is presented. Special attention is devoted to the discrete C, P and T symmetries as well as to a discussion of possible direct tests of the fundamental CPT invariance. A phase shift analysis of antinucleon-nucleon data is discussed, including electromagnetic and one-pion exchange contributions.

1. Introduction

This article is to be viewed as the fourth in a series devoted to the detailed formalism of the two-body scattering of spin $\frac{1}{2}$ particles, i.e. reactions of the type

$$1+2 \Rightarrow 3+4, \qquad s_i = \tfrac{1}{2}, \quad i = 1, \dots 4. \tag{1.1}$$

The first article [1] was devoted to the elastic scattering of two identical particles, e.g. proton-proton scattering, or neutron-proton scattering under the assumption of exact isospin invariance. The second article [2] treated the case of the elastic scattering of nonidentical particles, e.g. $np \Rightarrow np$ with isospin invariance not imposed. The third article [3] in the series presented the case of a reaction that is not self-conjugate under time reversal. This can be viewed as the formalism of time reversal tests in elastic reactions such as $pp \Rightarrow pp$, or $np \Rightarrow np$, or the formalism of inelastic reactions referred to e.g. $\Sigma^- p \Rightarrow \Lambda n$, independently of the validity of time reversal invariance (TRI).

The above articles presented a scattering formalism with 5, 6 and 8 amplitudes, respectively. Lorentz (or Galilei) invariance was always imposed, as was parity conservation and the Pauli principle for identical particles. TRI was imposed in the first two. These articles will be referred to as I, II and III, respectively.

---

*) Unité de Recherche des Universités Paris 11 et Paris 6 associée au CNRS.

The purpose of the present article is to complete the series by adapting the formalism to the case of antinucleon-nucleon scattering. Our task is simplified by the great similarity between the $\bar{p}p \Rightarrow \bar{p}p$ and $\bar{n}p \Rightarrow \bar{n}p$ formalism on the one hand and the $pp \Rightarrow pp$ and $np \Rightarrow np$ one on the other hand. Throughout the article we use the similarities but stress the differences. Special attention is devoted to the discrete C, P and T symmetries and in particular to a discussion of possible direct tests of the fundamental CPT invariance. The role of G-parity is discussed.

Our aim is to provide a detailed formalism of antinucleon-nucleon scattering in a convenient form for experimentalists. One of our motivation is related with improvements of the LEAR facility at CERN [4, 5] and the KAON factory project at TRIUMF. The beam intensity increases and antiprotons are regularly delivered at LEAR making a new generation of precise experiments possible. The fact that polarization experiments have already been performed at LEAR raises similar questions of amplitude reconstruction, phase shift analysis, etc. as in nucleon-nucleon scattering. In the $\bar{p}p$ elastic scattering case for instance, several collaborations like the PS173 experiment [6], PS172 [7], PS198 [8] and PS199 (elastic scattering and charge-exchange) are using a polarized target for their measurements while a project for the future intends to produce an accelerated polarized antiproton beam [9]. This would allow in principle a complete study of the spin dependence of the $\bar{N}N$ elastic scattering matrix.

Experiments done at LEAR have been stimulating theoretical investigations and provided new insights into many physical problems, such as the existence or nonexistence of baryonium states (antidiquark-diquark states) [10, 11], other mesonic resonances (quasinuclear bound states of a nucleon and an antinucleon) [12, 13] such as the AX(1565) [14] or atomic bound states from bag models [15]. The measured polarization observables help to discriminate between the various optical and phenomenological potential models [16–20]. Detailed information on the antinucleon-nucleon scattering amplitudes would also contribute, via analyticity and crossing symmetry, to our knowledge of the nucleon-nucleon interaction.

In Section 2 we discuss the very general form of the scattering matrix (16 amplitudes) and crossing relations expressed in the invariant basis. All experimental quantities with at most 2-spin indices are expressed in the center-of-mass system (CM) in terms of the amplitudes in Section 3. Section 4 is devoted to all possible direct tests of CPT invariance in a single reaction, namely $\bar{p}p$ elastic, both in the CM and in the laboratory system (ls). The electromagnetic corrections present in the $\bar{N}N$ system are calculated in Section 5 and the one-pion exchange (OPE) amplitudes in this context are given in Section 6. The formalism and guidelines for a $\bar{N}N$ phase shift analysis using the results of the two preceding sections are discussed in Section 7. Conclusions are drawn in Section 8.

2. The scattering matrix and crossing relations

Let us first consider a general binary reaction (1.1) involving four massive particles of spin 1/2. We assume that the scattering matrix M is invariant under

Lorentz or Galilei transformations (and hence under rotations in the CM). If no discrete symmetries are assumed (parity, time reversal invariance (TRI) or charge conjugation invariance (CC)), the scattering matrix will involve 16 independent amplitudes that are functions of energy and scattering angle.

We shall first write this matrix in the CM system, introducing 16 invariant amplitudes. We have

$$\begin{aligned} M(\vec{k}', \vec{k}) = \tfrac{1}{2}\{&(a+b) + (a-b)\sigma_{1n}\sigma_{2n} + (c+d)\sigma_{1m}\sigma_{2m} + (c-d)\sigma_{1l}\sigma_{2l} \\ &+ e(\sigma_{1n}+\sigma_{2n}) + h(\sigma_{1l}\sigma_{2m} - \sigma_{1m}\sigma_{2l}) + q(\sigma_{1l}+\sigma_{2l}) + r(\sigma_{1m}-\sigma_{2m}) \\ &+ s(\sigma_{1l}\sigma_{2n} + \sigma_{1n}\sigma_{2l}) + t(\sigma_{1m}\sigma_{2n} - \sigma_{1n}\sigma_{2m}) \\ &+ F_1(\sigma_{1n}-\sigma_{2n}) + F_2(\sigma_{1m}+\sigma_{2m}) + F_3(\sigma_{1l}-\sigma_{2l}) \\ &+ G_1(\sigma_{1l}\sigma_{2m} + \sigma_{1m}\sigma_{2l}) + G_2(\sigma_{1m}\sigma_{2n} + \sigma_{1n}\sigma_{2m}) \\ &+ G_3(\sigma_{1n}\sigma_{2l} - \sigma_{1l}\sigma_{2n})\} \end{aligned} \tag{2.1}$$

where

$$\vec{n} = (\vec{k} \times \vec{k}')/|\vec{k} \times \vec{k}'|, \qquad \vec{l} = (\vec{k}' + \vec{k})/|\vec{k}' + \vec{k}|, \qquad \vec{m} = (\vec{k}' - \vec{k})/|\vec{k}' - \vec{k}|. \tag{2.2}$$

$\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli 2×2 matrices and $\vec{k}$ and $\vec{k}'$ are unit vectors in the direction of the incident and scattered particles in the CM, respectively.

Let us now restrict to the special case of antinucleon-nucleon scattering, i.e. to the reactions

$$\begin{array}{lll} \text{a)}\ \bar{p}p \Rightarrow \bar{p}p, & \text{b)}\ \bar{n}n \Rightarrow \bar{n}n, & \text{c)}\ \bar{n}p \Rightarrow \bar{n}p, \\ \text{d)}\ \bar{p}n \Rightarrow \bar{p}n, & \text{e)}\ \bar{n}n \Rightarrow \bar{p}p, & \text{f)}\ \bar{p}p \Rightarrow \bar{n}n. \end{array} \tag{2.3}$$

Assuming that isospin is conserved in the $\bar{N}N$ interaction we can express the scattering matrices for all reactions in (2.3) as

$$\begin{aligned} \langle \bar{p}p|M|\bar{p}p\rangle &= \langle \bar{n}n|M|\bar{n}n\rangle = \tfrac{1}{2}(M_0 + M_1), \\ \langle \bar{p}p|M|\bar{n}n\rangle &= \langle \bar{n}n|M|\bar{p}p\rangle = \tfrac{1}{2}(M_1 - M_0) \\ \langle \bar{p}n|M|\bar{p}n\rangle &= \langle \bar{n}p|M|\bar{n}p\rangle = M_1 \end{aligned} \tag{2.4}$$

where M_0 and M_1 are the isospin $I = 0$ and $I = 1$ sets of amplitudes, respectively.

Reactions (2.3a, b) are self-conjugate under CPT and also under charge conjugation, (2.3a, b, c, d) are self conjugate under time reversal (being elastic). All these reactions are self conjugate under G-parity [21].

In Table 1 we list the properties of the amplitudes in (2.1) with respect to parity P, particle-antiparticle conjugation (or C-parity) and time reversal T.

The scattering formalism with five amplitudes a, b, c, d, e was developed in I, the amplitude $f \equiv F_1$ was included in II (for $np \Rightarrow np$ scattering). Eight amplitudes were allowed in III, namely a, b, c, d, e, $f \equiv F_1$, $g \equiv G_1$ and h. In proton-proton scattering f and h violate the Pauli principle, g and h violate TRI. In neutron-proton scattering f and h violate the generalized Pauli principle, i.e. isospin invariance.

Table 1
Invariance properties of the invariant amplitudes in equation (2.1) with respect to parity P, charge conjugation C, time reversal T and their product CPT. ("$+$" = invariant, "$-$" = changes sign under the transformation).

Amplitude	P	C	T	CPT
a	+	+	+	+
b	+	+	+	+
c	+	+	+	+
d	+	+	+	+
e	+	+	+	+
h	+	−	−	+
q	−	−	+	+
r	−	+	−	+
s	−	−	+	+
t	−	+	−	+
F_1	+	−	+	−
F_2	−	−	−	−
F_3	−	+	+	−
G_1	+	+	−	−
G_2	−	−	−	−
G_3	−	+	+	−

For antinucleon-nucleon scattering C-conjugation plays the role of the Pauli principle in nucleon-nucleon scattering. Thus, if P, T and C are conserved separately, only the five amplitudes $a, \ldots, e$ contribute to $\bar{p}p \Rightarrow \bar{p}p$ and $\bar{n}n \Rightarrow \bar{n}n$ scattering. The reactions (2.3c) and (2.3d) are not self-conjugate under CC, hence the amplitude F_1 is also allowed to be present (as it is in $np \Rightarrow np$ scattering), but it violates G-parity and is consequently expected to be small. Reactions (2.3e) and (2.3f) on the other hand, are not elastic, hence T-reversal in itself poses no restrictions and the amplitude G_1 is allowed. Again, it violates G-parity and is hence expected to be small (as compared to $a, \ldots, e$).

The spin structure in the elastic reaction $\bar{p}p \Rightarrow \bar{p}p$ provides a unique possibility to test CPT invariance directly [22–26]. A glance at Table 1 shows that while 11 amplitudes violate one or more of the symmetries C, P and T, only 6 of them, namely F_i and G_i ($i = 1, 2, 3$) violate CPT invariance.

Finally let us add a few words on analyticity and crossing symmetry. The elements of the scattering matrix are assumed to be analytic functions of the kinematic variables. Furthermore the same matrix considered in different kinematic regions describes the different channels $NN \Rightarrow NN$ and $\bar{N}N \Rightarrow \bar{N}N$. The crossing matrices for helicity amplitudes can be found e.g. in Refs [21, 27–29]. Here we present it for the invariant amplitudes $A = (a, b, c, d, e)$ assuming that the five amplitude formalism applies. We have

$$\bar{A} = M_C A \tag{2.5}$$

where

$$M_C = \begin{bmatrix} \dfrac{\cos 2(\chi+\theta)}{\Delta} & 0 & 0 & 0 & \dfrac{i\sin 2(\chi+\theta)}{\Delta} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \dfrac{i\sin 2(\chi+\theta)}{\Delta} & 0 & 0 & 0 & \dfrac{\cos 2(\chi+\theta)}{\Delta} \end{bmatrix} \tag{2.6}$$

and

$$\cos\chi = [st/(s-4m^2)(t-4m^2)]^{1/2}, \qquad \sin\chi = 2m[u/(s-4m^2)(t-4m^2)]^{1/2}$$

$$\cos\theta = (t-u)/(s-4m^2), \qquad \sin\theta = 2\sqrt{ut}/(s-4m^2) \tag{2.7}$$

$$\Delta = \cos 4(\chi+\theta) \quad \text{and} \quad i=\sqrt{-1}.$$

The Mandelstam variables s, t and u (not to be confused with the amplitudes s and t of eq. (2.1)), in terms of the particle four-momenta p_i are

$$s=(p_1+p_2)^2, \qquad t=(p_1-p_3)^2 \quad \text{and} \quad u=(p_1-p_4)^2.$$

This crossing relation is valid for the strong and the electromagnetic interactions. However, in the latter case, it must be remembered that charge conjugation changes the sign of the charges involved.

3. Experimental quantities

If all the fundamental discrete symmetries are assumed to be valid, then the scattering formalism for $\bar{N}N \Rightarrow \bar{N}N$ scattering coincides with that of $NN \Rightarrow NN$ scattering. Thus the formulas for all CM and ls experimental quantities in terms of the scattering amplitudes can be taken from I for reactions (2.3a) and (2.3b); from II for (2.3c) and (2.3d) with $F_1 \equiv f$, and from III for (2.3e) and (2.3f) with $G_1 \equiv g$. All amplitudes in equation (2.1) except a, b, c, d, e and respectively f for reactions (2.3c), (2.3d) or g for (2.3e), (2.3f), should be set equal to zero.

In view of the fundamental importance of the CPT theorem it is worthwhile to discuss possible tests in detail. To do this we develop the 16 amplitude formalism of equation (2.1) at least for polarization tensors involving two or less spin labels. The resulting formulas are used in tests of CPT invariance in $\bar{p}p \Rightarrow \bar{p}p$ scattering. The formalism of course applies to arbitrary reactions of the type (1.1).

We first introduce an abbreviated notation for the scattering matrix M, putting

$$M = \tfrac{1}{2}\{M_0 + M_{1a}\sigma_{1a} + M_{2a}\sigma_{2a} + M_{ab}\sigma_{1a}\sigma_{2b}\} \tag{3.1}$$

where summation from 1 to 3 over repeated labels is understood. Comparing equations (3.1) and (2.1) we have

$$
\begin{aligned}
&M_0 = a + b, \quad M_{1l} = q + F_3, \quad M_{1m} = r + F_2, \quad M_{1n} = e + F_1,\\
&M_{2l} = q - F_3, \quad M_{2m} = -r + F_2, \quad M_{2n} = e - F_1,\\
&M_{ll} = c - d, \quad M_{ml} = -h + G_1, \quad M_{nl} = s + G_3,\\
&M_{lm} = h + G_1, \quad M_{mm} = c + d, \quad M_{nm} = -t + G_2,\\
&M_{ln} = s - G_3, \quad M_{mn} = t + G_2, \quad M_{nn} = a - b.
\end{aligned}
\tag{3.2}
$$

Using the same notations as in earlier articles I, II, III, we write a general observable as

$$\sigma X_{pqij} = \tfrac{1}{4} \operatorname{Tr} \sigma_{1p}\sigma_{2q} M \sigma_{1i}\sigma_{2j} M^{\dagger} \tag{3.3}$$

where σ is the unpolarized differential cross section and the labels from left to right correspond to the spin of the scattered, recoil, beam and target particle respectively. In the CM the labels take the values 0, l, m, n (where 0 means "unpolarized" for the beam or target, or "non measured" for the scattered or recoil particle). In the ls we use the usual sets of orthonormal vectors

$$(\vec{n}, \vec{k}, \vec{s} = [\vec{n} \times \vec{k}]), \qquad (\vec{n}, \vec{k}', \vec{s}' = [\vec{n} \times \vec{k}']), \qquad (\vec{n}, \vec{k}'', \vec{s}'' = [\vec{n} \times \vec{k}'']) \tag{3.4}$$

where $\vec{k}$, $\vec{k}'$ and $\vec{k}''$ are unit vectors in the direction of the initial, scattered and recoil particle ls momenta, respectively.

In terms of the amplitudes (3.1) we calculate the CM observables as

$$\sigma = \tfrac{1}{4}\{|M_0|^2 + M_{1a}M_{1a}^* + M_{2a}M_{2a}^* + M_{ab}M_{ab}^*\} \tag{3.5}$$

$$\sigma A_{oooj} = \tfrac{1}{4}\{2 \operatorname{Re}(M_0 M_{2j}^* + M_{1a}M_{aj}^*) + \varepsilon_{jac} \operatorname{Im}(M_{2a}M_{2c}^* + M_{ba}M_{bc}^*)\} \tag{3.6a}$$

$$\sigma P_{oqoo} = \tfrac{1}{4}\{2 \operatorname{Re}(M_0 M_{2q}^* + M_{1a}M_{aq}^*) - \varepsilon_{qac} \operatorname{Im}(M_{2a}M_{2c}^* + M_{ba}M_{bc}^*)\} \tag{3.6b}$$

$$\sigma A_{ooio} = \tfrac{1}{4}\{2 \operatorname{Re}(M_0 M_{1i}^* + M_{2a}M_{ia}^*) + \varepsilon_{iac} \operatorname{Im}(M_{1a}M_{1c}^* + M_{ab}M_{cb}^*)\} \tag{3.6c}$$

$$\sigma P_{pooo} = \tfrac{1}{4}\{2 \operatorname{Re}(M_0 M_{1p}^* + M_{2a}M_{pa}^*) - \varepsilon_{pac} \operatorname{Im}(M_{1a}M_{1c}^* + M_{ab}M_{cb}^*)\} \tag{3.6d}$$

$$
\begin{aligned}
\sigma A_{ooij} = \tfrac{1}{2}\{&\delta_{ij} \operatorname{Re}(M_{lm}M_{ml}^* + M_{ln}M_{nl}^* + M_{mn}M_{nm}^* - M_{ll}M_{mm}^* - M_{ll}M_{nn}^*\\
&- M_{mm}M_{nn}^*) + \operatorname{Re}[M_0 M_{ij}^* + M_{1i}M_{2j}^* + (M_{ll} + M_{mm} + M_{nn})M_{ji}^*\\
&- M_{jl}M_{li}^* - M_{jm}M_{mi}^* - M_{jn}M_{ni}^*]\\
&+ \varepsilon_{iab} \operatorname{Im} M_{1a}M_{bj}^* + \varepsilon_{jab} \operatorname{Im} M_{2a}M_{ib}^*\}
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
\sigma C_{pqoo} = \tfrac{1}{2}\{&\delta_{pq} \operatorname{Re}(M_{lm}M_{ml}^* + M_{ln}M_{nl}^* + M_{mn}M_{nm}^* - M_{ll}M_{mm}^* - M_{ll}M_{nn}^*\\
&- M_{mm}M_{nn}^*) + \operatorname{Re}[M_0 M_{pq}^* + M_{1p}M_{2q}^* + (M_{ll} + M_{mm} + M_{nn})M_{qp}^*\\
&- M_{ql}M_{lp}^* - M_{qm}M_{mp}^* - M_{qn}M_{np}^*]\\
&- \varepsilon_{pab} \operatorname{Im} M_{1a}M_{bq}^* - \varepsilon_{qab} \operatorname{Im} M_{2a}M_{pb}^*\}
\end{aligned}
\tag{3.8}
$$

$$\begin{aligned}\sigma D_{poio} &= \tfrac{1}{4}\{\delta_{ip}(|M_0|^2 - M_{1a}M^*_{1a} + M_{2a}M^*_{2a} - M_{ab}M^*_{ab})\\ &\quad + 2\,\mathrm{Re}\,(M_{1i}M^*_{1p} + M_{ib}M^*_{pb}) - 2\varepsilon_{pia}\,\mathrm{Im}\,(M_0M^*_{1a} + M_{2b}M^*_{ab})\} \qquad (3.9)\end{aligned}$$

$$\begin{aligned}\sigma D_{oqok} &= \tfrac{1}{4}\{\delta_{qk}(|M_0|^2 + M_{1a}M^*_{1a} - M_{2a}M^*_{2a} - M_{ab}M^*_{ab})\\ &\quad + 2\,\mathrm{Re}\,(M_{2k}M^*_{2q} + M_{ak}M^*_{aq}) - 2\varepsilon_{qka}\,\mathrm{Im}\,(M_0M^*_{2a} + M_{1b}M^*_{ba})\} \qquad (3.10)\end{aligned}$$

$$\begin{aligned}\sigma K_{oqio} &= \tfrac{1}{2}\{\delta_{iq}\,\mathrm{Re}\,(-M_{lm}M^*_{ml} - M_{ln}M^*_{nl} - M_{mn}M^*_{nm} + M_{ll}M^*_{mm} + M_{ll}M^*_{nn}\\ &\quad + M_{mm}M^*_{nn}) + \mathrm{Re}\,(M_0M^*_{iq} + M_{1i}M^*_{2q} + M_{ai}M^*_{qa} - M_{qi}M^*_{aa})\\ &\quad + \varepsilon_{iab}\,\mathrm{Im}\,M_{1a}M^*_{bq} - \varepsilon_{qab}\,\mathrm{Im}\,M_{2a}M^*_{ib}\} \qquad (3.11)\end{aligned}$$

$$\begin{aligned}\sigma K_{pook} &= \tfrac{1}{2}\{\delta_{pk}\,\mathrm{Re}\,(-M_{lm}M^*_{ml} - M_{ln}M^*_{nl} - M_{mn}M^*_{nm} + M_{ll}M^*_{mm} + M_{ll}M^*_{nn}\\ &\quad + M_{mm}M^*_{nn}) + \mathrm{Re}\,(M_0M^*_{pk} + M_{1p}M^*_{2k} + M_{ap}M^*_{ka} - M_{kp}M^*_{aa})\\ &\quad - \varepsilon_{pab}\,\mathrm{Im}\,M_{1a}M^*_{bk} + \varepsilon_{kab}\,\mathrm{Im}\,M_{2a}M^*_{pb}\} \qquad (3.12)\end{aligned}$$

The expressions for the above observables in terms of the invariant amplitudes of equation (2.1) are given in Table 2.

Table 2
CM observables with at most two spin indices expressed in terms of the 16 scattering amplitudes (2.1).

$$\begin{aligned}
2\sigma &= |a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |h|^2 + |q|^2 + |r|^2 + |s|^2\\
&\quad + |t|^2 + |F_1|^2 + |F_2|^2 + |F_3|^2 + |G_1|^2 + |G_2|^2 + |G_3|^2\\
2\sigma A_{oonn} &= |a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 - |h|^2 - |F_1|^2 + |G_1|^2\\
&\quad + 2\,\mathrm{Im}\,(-s^*F_2 + t^*F_3 - q^*G_2 - r^*G_3)\\
2\sigma C_{nnoo} &= |a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 - |h|^2 - |F_1|^2 + |G_1|^2\\
&\quad - 2\,\mathrm{Im}\,(-s^*F_2 + t^*F_3 - q^*G_2 - r^*G_3)\\
2\sigma D_{nono} &= |a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 - |h|^2 + |F_1|^2 - |G_1|^2\\
&\quad - 2\,\mathrm{Re}\,(r^*F_2 + q^*F_3 + t^*G_2 - s^*G_3)\\
2\sigma D_{onon} &= |a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 - |h|^2 + |F_1|^2 - |G_1|^2\\
&\quad + 2\,\mathrm{Re}\,(r^*F_2 + q^*F_3 + t^*G_2 - s^*G_3)\\
2\sigma K_{onno} &= |a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 + |h|^2 - |F_1|^2 - |G_1|^2\\
&\quad - 2\,\mathrm{Im}\,(9^*t - r^*s + F^*_2G_3 + F^*_3G_2)\\
2\sigma K_{noon} &= |a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 + |h|^2 - |F_1|^2 - |G_1|^2\\
&\quad + 2\,\mathrm{Im}\,(q^*t - r^*s + F^*_2G_3 + F^*_3G_2)\\
2\sigma A_{ooon} &= \mathrm{Re}\,(2a^*e + q^*s + r^*t - 2b^*F_1 + t^*F_2 + s^*F_3 + r^*G_2 - q^*G_3 + F^*_2G_2 - F^*_3G_3)\\
&\quad + \mathrm{Im}\,(-2c^*h + q^*r + s^*t + 2d^*G_1 - q^*F_2 + r^*F_3 - s^*G_2 - t^*G_3 - F^*_2F_3 + G^*_2G_3)\\
2\sigma P_{nooo} &= \mathrm{Re}\,(2a^*e + q^*s + r^*t + 2b^*F_1 - t^*F_2 - s^*F_3 - r^*G_2 + q^*G_3 + F^*_2G_2 - F^*_3G_3)\\
&\quad + \mathrm{Im}\,(-2c^*h + q^*r + s^*t - 2d^*G_1 + q^*F_2 - r^*F_3 + s^*G_2 + t^*G_3 - F^*_2F_3 + G^*_2G_3)\\
2\sigma A_{oono} &= \mathrm{Re}\,(2a^*e + q^*s + r^*t + 2b^*F_1 - t^*F_2 - s^*F_3 - r^*G_2 + q^*G_3 + F^*_2G_2 - F^*_3G_3)\\
&\quad - \mathrm{Im}\,(-2c^*h + q^*r + s^*t - 2d^*G_1 + q^*F_2 - r^*F_3 + s^*G_2 + t^*G_3 - F^*_2F_3 + G^*_2G_3)
\end{aligned}$$

Table 2. *Contd*

$$2\sigma P_{onoo} = \mathrm{Re}\,(2a^*e + q^*s + r^*t - 2b^*F_1 + t^*F_2 + s^*F_3 + r^*G_2 - q^*G_3 + F_2^*G_2 - F_3^*G_3)$$
$$- \mathrm{Im}\,(-2c^*h + q^*r + s^*t + 2d^*G_1 - q^*F_2 + r^*F_3 - s^*G_2 - t^*G_3 - F_2^*F_3 + G_2^*G_3)$$

$$2\sigma A_{oool} = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)q - (a^* + b^* - c^* + d^*)F_3$$
$$+ e^*(s + G_3) - h^*(r + F_2) + r^*G_1 + s^*F_1 + F_1^*G_3 + F_2^*G_1]$$
$$+ \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)t + (a^* - b^* - c^* - d^*)G_2$$
$$+ e^*(-r + F_2) + h^*(-s + G_3) - r^*F_1 + s^*G_1 - F_1^*F_2 + G_1^*G_3]$$

$$2\sigma P_{looo} = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)q + (a^* + b^* - c^* - d^*)F_3$$
$$+ e^*(s - G_3) - h^*(r - F_2) - r^*G_1 - s^*F_1 + F_1^*G_3 + F_2^*G_1]$$
$$+ \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)t - (a^* - b^* - c^* - d^*)G_2$$
$$- e^*(r + F_2) - h^*(s + G_3) + r^*F_1 - s^*G_1 - F_1^*F_2 + G_1^*G_3]$$

$$2\sigma A_{oolo} = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)q + (a^* + b^* - c^* - d^*)F_3$$
$$+ e^*(s - G_3) - h^*(r - F_2) - r^*G_1 - s^*F_1 + F_1^*G_3 + F_2^*G_1]$$
$$- \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)t - (a^* - b^* - c^* - d^*)G_2$$
$$- e^*(r + F_2) - h^*(s + G_3) + r^*F_1 - s^*G_1 - F_1^*F_2 + G_1^*G_3]$$

$$2\sigma P_{oloo} = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)q - (a^* + b^* - c^* + d^*)F_3$$
$$+ e^*(s + G_3) - h^*(r + F_2) + r^*G_1 + s^*F_1 + F_1^*G_3 + F_2^*G_1]$$
$$- \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)t + (a^* - b^* - c^* - d^*)G_2$$
$$+ e^*(-r + F_2) + h^*(-s + G_3) - r^*F_1 + s^*G_1 - F_1^*F_2 + G_1^*G_3]$$

$$2\sigma A_{ooom} = \mathrm{Re}\,[(-a^* - b^* + c^* + d^*)r + (a^* + b^* + c^* + d^*)F_2$$
$$+ e^*(-t + G_2) + h^*(q + F_3) + q^*G_1 - t^*F_1 + F_1^*G_2 + F_3^*G_1]$$
$$+ \mathrm{Im}\,[(-a^* + b^* + c^* - d^*)s + (-a^* + b^* - c^* + d^*)G_3$$
$$+ e^*(-q + F_3) - h^*(t + G_2) - q^*F_1 - t^*G_1 - F_1^*F_3 + G_1^*G_2]$$

$$2\sigma P_{mooo} = \mathrm{Re}\,[(a^* + b^* - c^* - d^*)r + (a^* + b^* + c^* + d^*)F_2$$
$$+ e^*(t + G_2) + h^*(-q + F_3) + q^*G_1 - t^*F_1 - F_1^*G_2 - F_3^*G_1]$$
$$+ \mathrm{Im}\,[(a^* - b^* - c^* + d^*)s + (-a^* + b^* - c^* + d^*)G_3$$
$$+ e^*(q + F_3) + h^*(t - G_2) - q^*F_1 - t^*G_1 + F_1^*F_3 - G_1^*G_2]$$

$$2\sigma P_{omoo} = \mathrm{Re}\,[(-a^* - b^* + c^* + d^*)r + (a^* + b^* + c^* + d^*)F_2$$
$$+ e^*(-t + G_2) + h^*(q + F_3) + q^*G_1 - t^*F_1 + F_1^*G_2 + F_3^*G_1]$$
$$- \mathrm{Im}\,[(-a^* + b^* + c^* - d^*)s + (-a^* + b^* - c^* + d^*)G_3$$
$$+ e^*(-q + F_3) - h^*(t + G_2) - q^*F_1 - t^*G_1 - F_1^*F_3 + G_1^*G_2]$$

$$2\sigma A_{oomo} = \mathrm{Re}\,[(a^* + b^* - c^* - d^*)r + (a^* + b^* + c^* + d^*)F_2$$
$$+ e^*(t + G_2) + h^*(-q + F_3) + q^*G_1 - t^*F_1 - F_1^*G_2 - F_3^*G_1]$$
$$- \mathrm{Im}\,[(a^* - b^* - c^* + d^*)s + (-a^* + b^* - c^* + d^*)G_3$$
$$+ e^*(q + F_3) + h^*(t - G_2) - q^*F_1 - t^*G_1 + F_1^*F_3 - G_1^*G_2]$$

$$2\sigma A_{ooll} = 2\,\mathrm{Re}\,(b^*c - a^*d) + 2\,\mathrm{Im}\,(h^*F_1 + e^*G_1 + s^*F_2 - r^*G_3)$$
$$+ |q|^2 - |t|^2 - |F_3|^2 + |G_2|^2$$

$$2\sigma A_{oomm} = 2\,\mathrm{Re}\,(b^*c + a^*d) + 2\,\mathrm{Im}\,(h^*F_1 - e^*G_1 + t^*F_3 + q^*G_2)$$
$$- |r|^2 + |s|^2 + |F_2|^2 - |G_3|^2$$

Table 2. *Contd.*

$$2\sigma A_{ooml} = \mathrm{Re}\,(2a^*G_1 - 2b^*h + r^*q + s^*t + q^*F_2 - r^*F_3 - s^*G_2 - t^*G_3 - F_2^*F_3 + G_2^*G_3)$$
$$+ \mathrm{Im}\,(-2d^*e + 2c^*F_1 + q^*s + r^*t + t^*F_2 - s^*F_3 + r^*G_2 + q^*G_3 - F_2^*G_2 + F_3^*G_3)$$

$$2\sigma A_{oolm} = \mathrm{Re}\,(2a^*G_1 + 2b^*h - r^*q - s^*t + q^*F_2 - r^*F_3 - s^*G_2 - t^*G_3 + F_2^*F_3 - G_2^*G_3)$$
$$+ \mathrm{Im}\,(-2d^*e - 2c^*F_1 + q^*s + r^*t - t^*F_2 + s^*F_3 - r^*G_2 - q^*G_3 - F_2^*G_2 + F_3^*G_3)$$

$$2\sigma A_{ooln} = \mathrm{Re}\,[(a^* + b^* + c^* + d^*)s + (-a^* - b^* + c^* + d^*)G_3$$
$$+ e^*(q + F_3) + h^*(-t + G_2) - q^*F_1 + t^*G_1 - F_1^*F_3 - G_1^*G_2]$$
$$+ \mathrm{Im}\,[(a^* - b^* + c^* - d^*)r + (a^* - b^* - c^* + d^*)F_2$$
$$+ e^*(t + G_2) + h^*(q - F_3) - t^*F_1 - q^*G_1 + F_1^*G_2 + F_3^*G_1]$$

$$2\sigma A_{oonl} = \mathrm{Re}\,[(a^* + b^* + c^* + d^*)s + (a^* + b^* - c^* - d^*)G_3$$
$$+ e^*(q - F_3) - h^*(t + G_2) + q^*F_1 - t^*G_1 - F_1^*F_3 - G_1^*G_2]$$
$$+ \mathrm{Im}\,[(-a^* + b^* - c^* + d^*)r + (a^* - b^* - c^* + d^*)F_2$$
$$+ e^*(-t + G_2) - h^*(q + F_3) - t^*F_1 - q^*G_1 - F_1^*G_2 - F_3^*G_1]$$

$$2\sigma A_{oomn} = \mathrm{Re}\,[(a^* + b^* - c^* + d^*)t + (a^* + b^* + c^* - d^*)G_2$$
$$+ e^*(r + F_2) - h^*(s + G_3) - r^*F_1 - s^*G_1 - F_1^*F_2 - G_1^*G_3]$$
$$+ \mathrm{Im}\,[(-a^* + b^* + c^* + d^*)q + (-a^* + b^* - c^* - d^*)F_3$$
$$+ e^*(-s + G_3) + h^*(-r + F_2) + s^*F_1 - r^*G_1 + F_1^*G_3 + F_2^*G_1]$$

$$2\sigma A_{oonm} = \mathrm{Re}\,[(-a^* - b^* + c^* - d^*)t + (a^* + b^* + c^* - d^*)G_2$$
$$+ e^*(-r + F_2) + h^*(s - G_3) - r^*F_1 - s^*G_1 + F_1^*F_2 + G_1^*G_3]$$
$$+ \mathrm{Im}\,[(-a^* + b^* + c^* + d^*)q + (a^* - b^* + c^* + d^*)F_3$$
$$- e^*(s + G_3) - h^*(r + F_2) - s^*F_1 + r^*G_1 + F_1^*G_3 + F_2^*G_1]$$

$$C_{rsoo} = A_{oors} \text{ where one replaces } \mathrm{Re}\,[\;] \Rightarrow \mathrm{Re}\,[\;] \text{ and } \mathrm{Im}\,[\;] \Rightarrow -\mathrm{Im}\,[\;]$$

$$2\sigma D_{lolo} = 2\,\mathrm{Re}\,(a^*b - c^*d - e^*F_1 + h^*G_1 - r^*F_2 - s^*G_3)$$
$$- |t|^2 + |q|^2 + |F_3|^2 - |G_2|^2$$

$$2\sigma D_{momo} = 2\,\mathrm{Re}\,(a^*b + c^*d - e^*F_1 - h^*G_1 - q^*F_3 + t^*G_2)$$
$$- |s|^2 + |r|^2 + |F_2|^2 - |G_3|^2$$

$$2\sigma D_{lomo} = \mathrm{Re}\,[2c^*G_1 + 2d^*h + s^*t + r^*q + q^*F_2$$
$$+ r^*F_3 + s^*G_2 - t^*G_3 + F_2^*F_3 - G_2^*G_3]$$
$$+ \mathrm{Im}\,[2b^*e + 2a^*F_1 + q^*s + r^*t + t^*F_2$$
$$+ s^*F_3 - r^*G_2 + q^*G_3 + F_2^*G_2 - F_3^*G_3]$$

$$2\sigma D_{molo} = \mathrm{Re}\,[2c^*G_1 + 2d^*h + s^*t + r^*q + q^*F_2$$
$$+ r^*F_3 + s^*G_2 - t^*G_3 + F_2^*F_3 - G_2^*G_3]$$
$$- \mathrm{Im}\,[2b^*e + 2a^*F_1 + q^*s + r^*t + t^*F_2$$
$$+ s^*F_3 - r^*G_2 + q^*G_3 + F_2^*G_2 - F_3^*G_3]$$

$$2\sigma D_{lono} = \mathrm{Re}\,[(a^* - b^* + c^* - d^*)s + (-a^* + b^* + c^* - d^*)G_3$$
$$+ e^*(q + F_3) + h^*(-t + G_2) + q^*F_1 - t^*G_1 + F_1^*F_3 + G_1^*G_3]$$
$$+ \mathrm{Im}\,[-(a^* + b^* + c^* + d^*)r + (-a^* - b^* + c^* + d^*)F_2$$
$$- e^*(t + G_2) - h^*(q - F_3) - t^*F_1 - q^*G_1 + F_1^*G_2 + F_3^*G_1]$$

Table 2. *Contd.*

$$2\sigma D_{nolo} = \text{Re}\,[(a^* - b^* + c^* - d^*)s + (-a^* + b^* + c^* - d^*)G_3 + e^*(q + F_3) + h^*(-t + G_2) + q^*F_1 - t^*G_1 + F_1^*F_3 + G_1^*G_3] - \text{Im}\,[-(a^* + b^* + c^* + d^*)r + (-a^* - b^* + c^* + d^*)F_2 - e^*(t + G_2) - h^*(q - F_3) - t^*F_1 - q^*G_1 + F_1^*G_2 + F_3^*G_1]$$

$$2\sigma D_{mono} = \text{Re}\,[(a^* - b^* - c^* - d^*)t + (a^* - b^* + c^* - d^*)G_2 + e^*(r + F_2) - h^*(s + G_2) + r^*F_1 + s^*G_1 + F_1^*F_2 + G_1^*G_3] + \text{Im}\,[(a^* + b^* - c^* + d^*)q + (a^* + b^* + c^* - d^*)F_3 + e^*(s - G_3) + h^*(r - F_2) + s^*F_1 - r^*G_1 + F_2^*G_1 - F_1^*G_3]$$

$$2\sigma D_{nomo} = \text{Re}\,[(a^* - b^* - c^* - d^*)t + (a^* - b^* + c^* - d^*)G_2 + e^*(r + F_2) - h^*(s + G_2) + r^*F_1 + s^*G_1 + F_1^*F_2 + G_1^*G_3] - \text{Im}\,[(a^* + b^* - c^* + d^*)q + (a^* + b^* + c^* - d^*)F_3 + e^*(s - G_3) + h^*(r - F_2) + s^*F_1 - r^*G_1 + F_2^*G_1 - F_1^*G_3]$$

$$2\sigma D_{olol} = 2\,\text{Re}\,(a^*b - c^*d + e^*F_1 - h^*G_1 + r^*F_2 + s^*G_3) - |t|^2 + |q|^2 + |F_3|^2 - |G_2|^2$$

$$2\sigma D_{omom} = 2\,\text{Re}\,(a^*b + c^*d + e^*F_1 + h^*G_1 + q^*F_3 - t^*G_2) - |s|^2 + |r|^2 + |F_2|^2 - |G_3|^2$$

$$2\sigma D_{olom} = \text{Re}\,[2c^*G_1 - 2d^*h - s^*t - r^*q + q^*F_2 + r^*F_3 + s^*G_2 - t^*G_3 - F_2^*F_3 + G_2^*G_3] + \text{Im}\,[2b^*e - 2a^*F_1 + q^*s + r^*t - t^*F_2 - s^*F_3 - r^*G_2 - q^*G_3 + F_2^*G_2 + F_3^*G_3]$$

$$2\sigma D_{omol} = \text{Re}\,[2c^*G_1 - 2d^*h - s^*t - r^*q + q^*F_2 + r^*F_3 + s^*G_2 - t^*G_3 - F_2^*F_3 + G_2^*G_3] - \text{Im}\,[2b^*e - 2a^*F_1 + q^*s + r^*t - t^*F_2 - s^*F_3 - r^*G_2 - q^*G_3 + F_2^*G_2 + F_3^*G_3]$$

$$2\sigma D_{olon} = \text{Re}\,[(a^* - b^* + c^* - d^*)s + (a^* - b^* - c^* + d^*)G_3 + e^*(q - F_3) - h^*(t + G_2) - q^*F_1 + t^*G_1 + F_1^*F_3 - G_1^*G_3] + \text{Im}\,[(a^* + b^* + c^* + d^*)r + (-a^* - b^* + c^* + d^*)F_2 + e^*(t - G_2) + h^*(q + F_3) - q^*G_1 - t^*F_1 - F_1^*G_2 - F_3^*G_1]$$

$$2\sigma D_{onol} = \text{Re}\,[(a^* - b^* + c^* - d^*)s + (a^* - b^* - c^* + d^*)G_3 + e^*(q - F_3) - h^*(t + G_2) - q^*F_1 + t^*G_1 + F_1^*F_3 - G_1^*G_3] - \text{Im}\,[(a^* + b^* + c^* + d^*)r + (-a^* - b^* + c^* + d^*)F_2 + e^*(t - G_2) + h^*(q + F_3) - q^*G_1 - t^*F_1 - F_1^*G_2 - F_3^*G_1]$$

$$2\sigma D_{omon} = \text{Re}\,[(-a^* + b^* + c^* + d^*)t + (a^* - b^* + c^* + d^*)G_2 + e^*(-r + F_2) + h^*(s - G_3) + r^*F_1 + s^*G_1 - F_1^*F_2 - G_1^*G_3] + \text{Im}\,[(a^* + b^* - c^* + d^*)q + (-a^* - b^* - c^* + d^*)F_3 + e^*(s + G_3) + h^*(r + F_2) + r^*G_1 - s^*F_1 + F_1^*G_3 + F_2^*G_1]$$

$$2\sigma D_{onom} = \text{Re}\,[(-a^* + b^* + c^* + d^*)t + (a^* - b^* + c^* + d^*)G_2 + e^*(-r + F_2) + h^*(s - G_3) + r^*F_1 + s^*G_1 - F_1^*F_2 - G_1^*G_3]$$

Table 2. *Contd.*

$$
\begin{aligned}
&\quad - \mathrm{Im}\,[(a^* + b^* - c^* + d^*)q + (-a^* - b^* - c^* + d^*)F_3 \\
&\quad + e^*(s + G_3) + h^*(r + F_2) + r^*G_1 - s^*F_1 + F_1^*G_3 + F_2^*G_1] \\
2\sigma K_{ollo} &= 2\,\mathrm{Re}\,(a^*c - b^*d) + 2\,\mathrm{Im}\,(-e^*h + F_1^*G_1 + r^*s - F_2^*G_3) \\
&\quad + |t|^2 + |q|^2 - |F_3|^2 - |G_2|^2 \\
2\sigma K_{ommo} &= 2\,\mathrm{Re}\,(a^*c + b^*d) + 2\,\mathrm{Im}\,(-e^*h - F_1^*G_1 - q^*t + F_3^*G_2) \\
&\quad - |s|^2 - |r|^2 + |F_2|^2 + |G_3|^2 \\
2\sigma K_{olmo} &= \mathrm{Re}\,(-2a^*h + 2b^*G_1 + r^*q - s^*t + q^*F_2 \\
&\quad - r^*F_3 + s^*G_2 + t^*G_3 - F_2^*F_3 - G_2^*G_3) \\
&\quad + \mathrm{Im}\,(2c^*e - 2d^*F_1 - s^*q + t^*r - t^*F_2 \\
&\quad - s^*F_3 - r^*G_2 + q^*G_3 + F_2^*G_2 + F_3^*G_3) \\
2\sigma K_{omlo} &= \mathrm{Re}\,(2a^*h + 2b^*G_1 - r^*q + s^*t + q^*F_2 \\
&\quad - r^*F_3 + s^*G_2 + t^*G_3 + F_2^*F_3 + G_2^*G_3) \\
&\quad + \mathrm{Im}\,(-2c^*e - 2d^*F_1 + s^*q - t^*r - t^*F_2 \\
&\quad - s^*F_3 - r^*G_2 + q^*G_3 - F_2^*G_2 - F_3^*G_3) \\
2\sigma K_{onlo} &= \mathrm{Re}\,[(a^* + b^* - c^* - d^*)s - (a^* + b^* + c^* + d^*)G_3 \\
&\quad + e^*(q + F_3) + h^*(t - G_2) - q^*F_1 - t^*G_1 - F_1^*F_3 + G_1^*G_2] \\
&\quad + \mathrm{Im}\,[(a^* - b^* - c^* + d^*)r + (a^* - b^* + c^* - d^*)F_2 \\
&\quad + e^*(t + G_2) - h^*(q - F_3) - t^*F_1 + q^*G_1 + F_1^*G_2 - F_3^*G_1] \\
2\sigma K_{olno} &= \mathrm{Re}\,[(a^* + b^* - c^* - d^*)s + (a^* + b^* + c^* + d^*)G_3 \\
&\quad + e^*(q - F_3) + h^*(t + G_2) + q^*F_1 + t^*G_1 - F_1^*F_3 + G_1^*G_2] \\
&\quad + \mathrm{Im}\,[(a^* - b^* - c^* + d^*)r - (a^* - b^* + c^* - d^*)F_2 \\
&\quad + e^*(t - G_2) - h^*(q + F_3) + t^*F_1 - q^*G_1 - F_3^*G_1 + F_1^*G_2] \\
2\sigma K_{onmo} &= \mathrm{Re}\,[(a^* + b^* + c^* - d^*)t + (a^* + b^* - c^* + d^*)G_2 \\
&\quad + e^*(r + F_2) + h^*(s + G_3) - r^*F_1 + s^*G_1 - F_1^*F_2 + G_1^*G_2] \\
&\quad + \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)q - (a^* - b^* - c^* - d^*)F_3 \\
&\quad + e^*(-s + G_3) + h^*(r - F_2) + s^*F_1 + r^*G_1 + F_1^*G_3 - F_2^*G_1] \\
2\sigma K_{omno} &= \mathrm{Re}\,[(-a^* - b^* - c^* + d^*)t + (a^* + b^* - c^* + d^*)G_2 \\
&\quad + e^*(-r + F_2) - h^*(s - G_3) - r^*F_1 + s^*G_1 + F_1^*F_2 - G_1^*G_3] \\
&\quad + \mathrm{Im}\,[(a^* - b^* + c^* + d^*)q + (-a^* + b^* + c^* + d^*)F_3 \\
&\quad + e^*(s + G_3) - h^*(r + F_2) + s^*F_1 + r^*G_1 - F_1^*G_3 + F_2^*G_1] \\
K_{iooq} &= K_{oqio} \text{ where one replaces } \mathrm{Re}\,[\ ] \Rightarrow \mathrm{Re}\,[\ ] \text{ and } \mathrm{Im}\,[\ ] \Rightarrow -\mathrm{Im}\,[\ ]
\end{aligned}
$$

4. Experimental tests of the *CPT* theorem in elastic antiproton-proton scattering

Using Table 2 we shall now extract the simplest experimental quantities that provide tests of the *CPT* theorem. In the expressions in terms of invariant amplitudes we shall keep only the leading terms. These correspond to the interference (if any) between the *CPT* violating amplitudes F_i and G_i and the "large" amplitudes a, b, c, d and e.

In the CM we have, in this approximation,

$$\sigma(A_{ooon} - P_{nooo}) = -2\,\mathrm{Re}\, b^*F_1 + 2\,\mathrm{Im}\, d^*G_1 \tag{4.1a}$$

$$\sigma(A_{oono} - P_{onoo}) = 2\,\mathrm{Re}\, b^*F_1 + 2\,\mathrm{Im}\, d^*G_1 \tag{4.1b}$$

$$\sigma(A_{oool} - P_{looo}) = \mathrm{Re}\,[(-a^* - b^* + c^* - d^*)F_3 + e^*G_3] + \mathrm{Im}\,[(a^* - b^* - c^* - d^*)G_2 + e^*F_2] \tag{4.2a}$$

$$\sigma(A_{oolo} - P_{oloo}) = \mathrm{Re}\,[(a^* + b^* - c^* + d^*)F_3 - e^*G_3] + \mathrm{Im}\,[(a^* - b^* - c^* - d^*)G_2 + e^*F_2] \tag{4.2b}$$

$$\sigma(A_{ooom} + P_{mooo}) = \mathrm{Re}\,[(a^* + b^* + c^* + d^*)F_2 + e^*G_2] + \mathrm{Im}\,[(-a^* + b^* - c^* + d^*)G_3 + e^*F_3] \tag{4.2c}$$

$$\sigma(A_{oomo} + P_{omoo}) = \mathrm{Re}\,[(a^* + b^* + c^* + d^*)F_2 + e^*G_2] + \mathrm{Im}\,[(a^* - b^* + c^* - d^*)G_3 - e^*F_3] \tag{4.2d}$$

$$\sigma(A_{ooll} - C_{lloo}) = 2\,\mathrm{Im}\, e^*G_1 \tag{4.3a}$$

$$\sigma(A_{oomm} - C_{mmoo}) = -2\,\mathrm{Im}\, e^*G_1 \tag{4.3b}$$

$$\sigma(A_{ooml} - C_{lmoo}) = 2\,\mathrm{Re}\, a^*G_1 + 2\,\mathrm{Im}\, c^*F_1 \tag{4.3c}$$

$$\sigma(A_{oolm} + C_{mloo}) = 2\,\mathrm{Re}\, a^*G_1 - 2\,\mathrm{Im}\, c^*F_1 \tag{4.3d}$$

$$\sigma(A_{ooln} - C_{nloo}) = \mathrm{Re}\,[(-a^* - b^* + c^* + d^*)G_3 + e^*F_3] + \mathrm{Im}\,[(a^* - b^* - c^* + d^*)F_2 + e^*G_2] \tag{4.3e}$$

$$\sigma(A_{oonl} - C_{lnoo}) = \mathrm{Re}\,[(a^* + b^* - c^* - d^*)G_3 - e^*F_3] + \mathrm{Im}\,[(a^* - b^* - c^* + d^*)F_2 + e^*G_2] \tag{4.3f}$$

$$\sigma(A_{oomn} + C_{nmoo}) = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)G_2 + e^*F_2] + \mathrm{Im}\,[(-a^* + b^* - c^* - d^*)F_3 + e^*G_3] \tag{4.3g}$$

$$\sigma(A_{oonm} + C_{mnoo}) = \mathrm{Re}\,[(a^* + b^* + c^* - d^*)G_2 + e^*F_2] + \mathrm{Im}\,[(a^* - b^* + c^* + d^*)F_3 - e^*G_3] \tag{4.3h}$$

$$\sigma(D_{lolo} - D_{olol}) = -2\,\mathrm{Re}\, e^*F_1 \tag{4.4a}$$

$$\sigma(D_{momo} - D_{omom}) = -2\,\mathrm{Re}\, e^*F_1 \tag{4.4b}$$

$$\sigma(D_{lomo} + D_{omol}) = 2\,\mathrm{Re}\, c^*G_1 + 2\,\mathrm{Im}\, a^*F_1 \tag{4.4c}$$

$$\sigma(D_{molo} + D_{olom}) = 2\,\mathrm{Re}\, c^*G_1 - 2\,\mathrm{Im}\, a^*F_1 \tag{4.4d}$$

$$\sigma(D_{lono} - D_{onol}) = \text{Re}\,[(-a^* + b^* + c^* - d^*)G_3 + e^*F_3] + \text{Im}\,[(-a^* - b^* + c^* + d^*)F_2 - e^*G_2] \tag{4.4e}$$

$$\sigma(D_{nolo} - D_{olon}) = \text{Re}\,[(-a^* + b^* + c^* - d^*)G_3 + e^*F_3] + \text{Im}\,[(a^* + b^* - c^* - d^*)F_2 + e^*G_2] \tag{4.4f}$$

$$\sigma(D_{mono} + D_{onom}) = \text{Re}\,[(a^* - b^* + c^* + d^*)G_2 + e^*F_2] + \text{Im}\,[(a^* + b^* + c^* - d^*)F_3 - e^*G_3] \tag{4.4g}$$

$$\sigma(D_{nomo} + D_{omon}) = \text{Re}\,[(a^* - b^* + c^* + d^*)G_2 + e^*F_2] + \text{Im}\,[(-a^* - b^* - c^* + d^*)F_3 + e^*G_3] \tag{4.4h}$$

$$\sigma(K_{olmo} + K_{omlo}) = 2\,\text{Re}\, b^*G_1 - 2\,\text{Im}\, d^*F_1 \tag{4.5a}$$

$$\sigma(K_{loom} + K_{mool}) = 2\,\text{Re}\, b^*G_1 + 2\,\text{Im}\, d^*F_1 \tag{4.5b}$$

$$\sigma(K_{onlo} - K_{olno}) = \text{Re}\,[-(a^* + b^* + c^* + d^*)G_3 + e^*F_3] + \text{Im}\,[(a^* - b^* + c^* - d^*)F_2 + e^*G_2] \tag{4.5c}$$

$$\sigma(K_{loon} - K_{nool}) = \text{Re}\,[-(a^* + b^* + c^* + d^*)G_3 + e^*F_3] + \text{Im}\,[(-a^* + b^* - c^* + d^*)F_2 - e^*G_2] \tag{4.5d}$$

$$\sigma(K_{onmo} + K_{omno}) = \text{Re}\,[(a^* + b^* - c^* + d^*)G_2 + e^*F_2] + \text{Im}\,[(-a^* + b^* + c^* + d^*)F_3 + e^*G_3] \tag{4.5e}$$

$$\sigma(K_{moon} + K_{noom}) = \text{Re}\,[(a^* + b^* - c^* + d^*)G_2 + e^*F_2] + \text{Im}\,[(a^* - b^* - c^* - d^*)F_3 - e^*G_3] \tag{4.5f}$$

Notice that the normal components of the two index polarization tensors do not provide sensitive tests of *CPT* invariance. Indeed, from Table 2 we see that the differences

$$\sigma(A_{oonn} - C_{nnoo}), \qquad \sigma(D_{nono} - D_{onon})$$

depend on the interference between the *CPT* violating amplitudes F_2, F_3, G_2, G_3 and the small parity violating amplitudes s, t, q and r. The differences

$$\sigma(K_{onno} - K_{noon}), \qquad \sigma(K_{ollo} - K_{lool}), \qquad \sigma(K_{ommo} - K_{moom})$$

on the other hand, do not vanish when we set $F_i = G_i = 0$ $(i = 1, 2, 3)$.

Finally, let us transform the CM relations (4.1) to (4.5) into the ls. Using Fig. 1 of Ref. I we see that we must perform the rotations

$$\begin{aligned} \vec{l} &= \vec{k}'_{R_1} \cos\alpha - \vec{s}'_{R_1} \sin\alpha = -\vec{k}''_{R_2} \cos\beta + \vec{s}''_{R_2} \sin\beta = \vec{k} \cos\theta/2 + \vec{s} \sin\theta/2 \\ \vec{m} &= \vec{k}'_{R_1} \sin\alpha + \vec{s}'_{R_1} \cos\alpha = -\vec{k}''_{R_2} \sin\beta - \vec{s}''_{R_2} \cos\beta = -\vec{k} \sin\theta/2 + \vec{s} \cos\theta/2 \end{aligned} \tag{4.6}$$

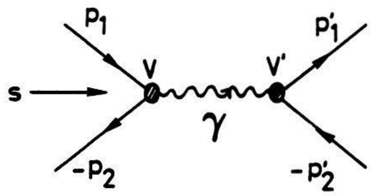

Figure 1
Annihilation and creation of a $\bar{N}N$ pair via a virtual photon γ.

where θ is the CM scattering angle and α and β are related to the relativistic spin rotation for the scattered and recoil particles, respectively:

$$\alpha = \tfrac{1}{2}\theta - \theta_1, \qquad \beta = \tfrac{1}{2}\theta + \theta_2 \tag{4.7}$$

where θ_1 and θ_2 are the ls scattering and recoil angles, respectively.

Relations (4.1) are also valid as the are in the ls. For all other quantities we shall express the left hand sides of (4.2) to (4.5) in term of ls quantities:

$$(A_{oool} - P_{looo}) = A_{oook} \cos\theta/2 + A_{ooos} \sin\theta/2 - P_{k'ooo} \cos\alpha + P_{s'ooo} \sin\alpha \tag{4.8a}$$

$$(A_{ooom} + P_{mooo}) = -A_{oook} \sin\theta/2 + A_{ooos} \cos\theta/2 + P_{k'ooo} \sin\alpha + P_{s'ooo} \cos\alpha \tag{4.8b}$$

$$(A_{oolo} - P_{oloo}) = A_{ooko} \cos\theta/2 + A_{ooso} \sin\theta/2 + P_{ok''oo} \cos\beta - P_{os''oo} \sin\beta \tag{4.8c}$$

$$(A_{oomo} + P_{omoo}) = -A_{ooko} \sin\theta/2 + A_{ooso} \cos\theta/2 - P_{ok''oo} \sin\beta - P_{os''oo} \cos\beta \tag{4.8d}$$

$$(A_{ooll} - C_{lloo}) = A_{ookk} \cos^2\theta/2 + \tfrac{1}{2}(A_{ooks} + A_{oosk}) \sin\theta + A_{ooss} \sin^2\theta/2 + C_{k'k''oo} \cos\alpha \cos\beta - C_{k's''oo} \cos\alpha \sin\beta - C_{s'k''oo} \sin\alpha \cos\beta + C_{s's''oo} \sin\alpha \sin\beta \tag{4.9a}$$

$$(A_{oomm} - C_{mmoo}) = A_{ookk} \sin^2\theta/2 - \tfrac{1}{2}(A_{ooks} + A_{oosk}) \sin\theta + A_{ooss} \cos^2\theta/2 + C_{k'k''oo} \sin\alpha \sin\beta + C_{k's''oo} \sin\alpha \cos\beta + C_{s'k''oo} \cos\alpha \sin\beta + C_{s's''oo} \cos\alpha \cos\beta \tag{4.9b}$$

$$(A_{ooml} - C_{lmoo}) = -\tfrac{1}{2}A_{ookk} \sin\theta - A_{ooks} \sin^2\theta/2 + A_{oosk} \cos^2\theta/2 + \tfrac{1}{2}A_{ooss} \sin\theta + C_{k'k''oo} \cos\alpha \sin\beta + C_{k's''oo} \cos\alpha \cos\beta - C_{s'k''oo} \sin\alpha \sin\beta - C_{s's''oo} \sin\alpha \cos\beta \tag{4.9c}$$

$$(A_{oolm} + C_{mloo}) = -\tfrac{1}{2}A_{ookk} \sin\theta + A_{ooks} \cos^2\theta/2 - A_{oosk} \sin^2\theta/2 + \tfrac{1}{2}A_{ooss} \sin\theta - C_{k'k''oo} \sin\alpha \cos\beta + C_{k's''oo} \sin\alpha \sin\beta - C_{s'k''oo} \cos\alpha \cos\beta + C_{s's''oo} \cos\alpha \sin\beta \tag{4.9d}$$

$$(A_{ooln} - C_{nloo}) = A_{ookn} \cos\theta/2 + A_{oosn} \sin\theta/2 + C_{nk''oo} \cos\beta - C_{ns''oo} \sin\beta \tag{4.9e}$$

$$(A_{oomn} + C_{nmoo}) = -A_{ookn} \sin\theta/2 + A_{oosn} \cos\theta/2 - C_{nk''oo} \sin\beta - C_{ns''oo} \cos\beta \tag{4.9f}$$

$$(A_{oonl} - C_{lnoo}) = A_{oonk} \cos\theta/2 + A_{oons} \sin\theta/2 - C_{k'noo} \cos\alpha + C_{s'noo} \sin\alpha \tag{4.9g}$$

$$(A_{oonm} + C_{mnoo}) = -A_{oonk} \sin\theta/2 + A_{oons} \cos\theta/2 + C_{k'noo} \sin\alpha + C_{s'noo} \cos\alpha \tag{4.9h}$$

$$\begin{aligned}(D_{lolo} - D_{olol}) = {} & D_{k'oko} \cos\alpha \cos\theta/2 + D_{k'oso} \cos\alpha \sin\theta/2 \\ & - D_{s'oko} \sin\alpha \cos\theta/2 - D_{s'oso} \sin\alpha \sin\theta/2 \\ & + D_{ok''ok} \cos\beta \cos\theta/2 + D_{ok''os} \cos\beta \sin\theta/2 \\ & - D_{os''ok} \sin\beta \cos\theta/2 - D_{os''os} \sin\beta \sin\theta/2\end{aligned} \tag{4.10a}$$

$$\begin{aligned}(D_{momo} - D_{omom}) = {} & -D_{k'oko} \sin\alpha \sin\theta/2 + D_{k'oso} \sin\alpha \cos\theta/2 \\ & - D_{s'oko} \cos\alpha \sin\theta/2 + D_{s'oso} \cos\alpha \cos\theta/2 \\ & - D_{ok''ok} \sin\beta \sin\theta/2 + D_{ok''os} \sin\beta \cos\theta/2 \\ & - D_{os''ok} \cos\beta \sin\theta/2 + D_{os''os} \cos\beta \cos\theta/2\end{aligned} \tag{4.10b}$$

$$\begin{aligned}(D_{lomo} + D_{omol}) = {} & -D_{k'oko} \cos\alpha \sin\theta/2 + D_{k'oso} \cos\alpha \cos\theta/2 \\ & + D_{s'oko} \sin\alpha \sin\theta/2 - D_{s'oso} \sin\alpha \cos\theta/2 \\ & - D_{ok''ok} \sin\beta \cos\theta/2 - D_{ok''os} \sin\beta \sin\theta/2 \\ & - D_{os''ok} \cos\beta \cos\theta/2 - D_{os''os} \cos\beta \sin\theta/2\end{aligned} \tag{4.10c}$$

$$\begin{aligned}(D_{molo} + D_{olom}) = {} & D_{k'oko} \sin\alpha \cos\theta/2 + D_{k'oso} \sin\alpha \sin\theta/2 \\ & + D_{s'oko} \cos\alpha \cos\theta/2 + D_{s'oso} \cos\alpha \sin\theta/2 \\ & + D_{ok''ok} \cos\beta \sin\theta/2 - D_{ok''os} \cos\beta \cos\theta/2 \\ & - D_{os''ok} \sin\beta \sin\theta/2 + D_{os''os} \sin\beta \cos\theta/2\end{aligned} \tag{4.10d}$$

$$(D_{lono} - D_{onol}) = D_{k'ono} \cos\alpha - D_{s'ono} \sin\alpha - D_{onok} \cos\theta/2 - D_{onos} \sin\theta/2 \tag{4.10e}$$

$$(D_{mono} + D_{onom}) = D_{k'ono} \sin\alpha + D_{s'ono} \cos\alpha - D_{onok} \sin\theta/2 + D_{onos} \cos\theta/2 \tag{4.10f}$$

$$(D_{nolo} - D_{olon}) = D_{noko} \cos\theta/2 + D_{noso} \sin\theta/2 + D_{ok''on} \cos\beta - D_{os''on} \sin\beta \tag{4.10g}$$

$$(D_{nomo} + D_{omon}) = -D_{noko} \sin\theta/2 + D_{noso} \cos\theta/2 - D_{ok''on} \sin\beta - D_{os''on} \cos\beta \tag{4.10h}$$

$$(K_{olmo} + K_{omlo}) = (K_{os''so} - K_{ok''ko}) \sin\theta_2 - (K_{ok''so} + K_{os''ko}) \cos\theta_2 \tag{4.11a}$$

$$(K_{loom} + K_{mool}) = (K_{s'oos} - K_{k'ook}) \sin\theta_1 + (K_{k'oos} + K_{s'ook}) \cos\theta_1 \tag{4.11b}$$

$$(K_{onlo} - K_{olno}) = K_{onko} \cos\theta/2 + K_{onso} \sin\theta/2 + K_{ok''no} \cos\beta - K_{os''no} \sin\beta \tag{4.11c}$$

$$(K_{onmo} + K_{omno}) = -K_{onko} \sin\theta/2 + K_{onso} \cos\theta/2 - K_{ok''no} \sin\beta - K_{os''no} \cos\beta \tag{4.11d}$$

$$(K_{loon} - K_{nool}) = K_{k'oon} \cos\alpha - K_{s'oon} \sin\alpha - K_{nook} \cos\theta/2 - K_{noos} \sin\theta/2 \tag{4.11e}$$

$$(K_{moon} + K_{noom}) = K_{k'oon} \sin\alpha + K_{s'oon} \cos\alpha - K_{nook} \sin\theta/2 + K_{noos} \cos\theta/2. \tag{4.11f}$$

Among the *CPT* tests listed above the most realistic ones are probably (4.1a) and (4.1b). They are sensitive to the amplitude F_1 that violates C invariance, while satisfying P and T invariance, and the amplitude G_1, violating T invariance, but satisfying P and C (see Table 1). A further interesting possibility are the tests (4.11a, b), sensitive to the same *CPT* violating amplitudes. Indeed, we have

$$\begin{aligned}
&\sigma[(K_{s'oos} - K_{k'ook}) \sin\theta_1 + (K_{s'ook} + K_{k'oos}) \cos\theta_1] \\
&\quad = 2\,\mathrm{Re}\,(b^*G_1 + d^*F_1) \\
&\sigma[(K_{os''so} - K_{ok''ko}) \sin\theta_2 - (K_{ok''so} + K_{os''ko}) \cos\theta_2] \\
&\quad = 2\,\mathrm{Re}\,(b^*G_1 - d^*F_1).
\end{aligned} \tag{4.12}$$

Interestingly, each of the relations (4.12) for the polarization transfer tensors for a polarized beam or a polarized target can be tested while performing only two scattering experiments rather than four. The corresponding experimental setup is identical to the one discussed for the same quantities in Ref. III in the context of possible tests of TRI violations in proton-proton scattering.

The other experiments sensitive to the parity conserving, *CPT* violating amplitudes F_1 and G_1 are more complicated.

It should be noted that in the considered approximation we have

$$\sigma[A_{ookk} + A_{ooss} + (C_{k'k''oo} + C_{s's''oo}) \cos(\theta_1 + \theta_2) - (C_{k's''oo} - C_{s'k''oo}) \sin(\theta_1 + \theta_2)] = 0 \tag{4.13a}$$

$$\sigma[(D_{k'oko} - D_{s'oso}) \cos \theta_1 + (D_{k'oso} + D_{s'oko}) \sin \theta_1 + (D_{ok''ok} - D_{os''os}) \cos \theta_2 - (D_{ok''os} + D_{os''ok}) \sin \theta_2] = 0 \tag{4.13b}$$

(since these quantities depend on the interference between the F_i, G_i and the other small amplitudes h, q, r, s and t). On the other hand, from equations (4.9a) and (4.10a) we have

$$\sigma[(A_{ookk} - A_{ooss}) \cos \theta + (A_{ooks} + A_{oosk}) \sin \theta + (C_{k'k''oo} - C_{s's''oo}) \cos (\theta - \theta_1 + \theta_2) - (C_{k's''oo} + C_{s'k''oo}) \sin (\theta - \theta_1 + \theta_2)] = 4 \operatorname{Im} e^* G_1 \tag{4.14a}$$

$$\sigma[(D_{k'oko} + D_{s'oso}) \cos (\theta - \theta_1) + (D_{k'oso} - D_{s'oko}) \sin (\theta - \theta_1) + (D_{ok''ok} + D_{os''os}) \cos (\theta + \theta_2) + (D_{ok''os} - D_{os''ok}) \sin (\theta + \theta_2)] = -4 \operatorname{Re} e^* F_1 . \tag{4.14b}$$

The simplest test of CPT violation due to the amplitudes F_2, G_2, that violate all three symmetries C, P and T, and F_3, G_3 that violate only parity P, are clearly the polarization-asymmetry experiments (4.8), with spin components in the scattering plane. The experiments (4.9e, f, g, h), (4.10e, f, g, h) and (4.11c, d, e, f) are also sensitive to the amplitudes F_2, G_2 and F_3, G_3.

5. Electromagnetic corrections

The one-photon exchange in the antinucleon-nucleon channel gives rise to the contribution of two processes. The first one is the direct exchange diagram with scattering matrix $\bar{M}_D(s, t)$ (the Coulomb term) and the second one is the annihilation and creation of a $\bar{N}N$ pair via a virtual photon and described by the scattering matrix $\bar{M}_A(s, u)$ obtained from $\bar{M}_D$ after interchanging the final states, for identical particles. These matrices can be obtained from their equivalent ones defined in the nucleon-nucleon sector. Indeed, by using hermiticity of the scattering matrix one can show that, for instance in the $\bar{p}p$ elastic case in the direct process, the current at the $\bar{p}\gamma\bar{p}$ vertex is equal to that at the $p\gamma p$ vertex so that $\bar{M}_D(s, t) = -M_D(s, t)$ because the charge of the antiproton is $-e$. On the other hand, we know from Section 2 on crossing that $\bar{M}_A(s, u)$ for the crossed channel can be obtained from $M_D(s, t)$ by applying the crossing matrix M_C (2.6). Nevertheless a straightforward calculation of the annihilation and creation diagram shown in Fig. 1 helps to fix the conventions of the formalism.

For the $\bar{p}p$ or $\bar{n}n$ elastic scattering we write the contribution of the Fig. 1 diagram to the scattering matrix as:

$$\bar{M}_A = N(s)\{\bar{v}(-p_2)[F_1\gamma^\mu - iF_2\sigma^{\mu\beta}q_{A\beta}]u(p_1)\} \times \{\bar{u}(p_1')[F_1\gamma_\mu' + iF_2\sigma_{\mu\alpha}'q_A'^\alpha]v(-p_2')\} \tag{5.1}$$

where the primes in the rightmost current refer to the vertex V' in the figure and $N(s)$ is a normalization factor to be specified below. The physical states and momenta are defined as:

$$p_1 = (E, \vec{p}), \quad \bar{p}_2 \equiv -p_2 = (E, -\vec{p}) \ldots \text{initial states,}$$

$$p_1' = (E, \vec{p}\,'), \quad \bar{p}_2' \equiv -p_2' = (E, -\vec{p}\,') \ldots \text{final states,}$$

$$q_A = p_1 + \bar{p}_2 \equiv p_1 - p_2 = (2E, 0) = q_A',$$

$$t_A = q_A^2 = 4E^2 = s_D > 0 \ldots \text{timelike region,}$$

s_D being the total energy in the direct channel and t_A the "momentum transfer" of the virtual photon in the crossed channel. The Dirac spinors are normalized to $+1$ (-1) for the particles (antiparticles) and are taken to be:

$$u(p_1) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}_1 \cdot \vec{p}}{E+m} \end{pmatrix} \psi,$$
$$v(-p_2') = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} -\dfrac{\vec{\sigma}_2 \cdot \vec{p}\,'}{E+m} \\ 1 \end{pmatrix} \chi, \tag{5.2}$$

with Pauli spinors ψ and χ. The nucleon form factors F_1 and F_2 (not to be confused with the amplitudes of the same name in eq. (2.1)) in the timelike region $t_A > 0$ are derived from a generalized vector-meson-dominance model with an ansatz given in Ref. [30], and satisfy the usual assumptions like asymptotic behaviour, normalization, vector coupling of the photon, etc. We choose their representation:

$$F_1^{\substack{p\\n}}(s) = \tfrac{1}{2}\left[\left(\frac{1}{1-q^2/m_\omega^2}\right)\left(\frac{1}{1-q^2/m_{\omega'}^2}\right) \pm \left(\frac{1}{1-q^2/m_\rho^2}\right)\left(\frac{1}{1-q^2/m_{\rho'}^2}\right)\right]$$

$$F_2^{\substack{p\\n}}(s) = \kappa_p \left[-0.0335\left(\frac{1}{1-q^2/m_\omega^2}\right)\left(\frac{1}{1-q^2/m_{\omega'}^2}\right)\left(\frac{1}{1-q^2/m_{\omega''}^2}\right)\right.$$
$$\left. \pm 1.0335\left(\frac{1}{1-q^2/m_\rho^2}\right)\left(\frac{1}{1-q^2/m_{\rho'}^2}\right)\left(\frac{1}{1-q^2/m_{\rho''}^2}\right)\right] \tag{5.3}$$

$$m_\rho = 0.77 \text{ GeV}, \qquad m_{\rho'} = 1.26 \text{ GeV}, \qquad m_{\rho''} = 1.61 \text{ GeV}$$

$$m_\omega = 0.78 \text{ GeV}, \qquad m_{\omega'} = 1.27 \text{ GeV}, \qquad m_{\omega''} = 1.62 \text{ GeV}$$

where the superscripts "p" and "n" stand for proton and neutron, respectively (the $+$ sign has to be taken with the superscript p and the $-$ sign with the superscript n), ω and ρ are the vector mesons and ω', ω'', ρ', ρ'' their corresponding Regge recurrencies. The anomalous magnetic moment of the proton is $\kappa_p = 1.7928456$. Note that contrary to their representation in the spacelike region where $t_D = -q^2 < 0$, the nucleon form factors here do not have any angular dependence as they are functions of s only.

The calculation of the *exact* annihilation and creation amplitudes from equation (5.1), after a Fierz transformation and the reduction to the two-component form, gives the following results:

$$\bar{a}_A = [-\alpha/(2m^2 s\sqrt{s}(\sqrt{s}+2m)^2)]\{[ut - \tfrac{1}{4}(\sqrt{s}+2m)^2(s+4m^2)](4m^2F_1^2 + sF_2^2) + [ut(s+4m^2) - (\sqrt{s}+2m)^2(ut+4sm^2)]F_1F_2\} \tag{5.4a}$$

$$\bar{b}_A = \bar{c}_A = [\alpha/2\sqrt{s}](F_1+F_2)^2 \tag{5.4b}$$

$$\bar{d}_A = [-\alpha/(8m^2 s\sqrt{s})](t-u)(4m^2F_1^2 - sF_2^2) \tag{5.4c}$$

$$\bar{e}_A = [-i\alpha/(4m^2 s(\sqrt{s}+2m)^2)](\sqrt{ut/s})(t-u)(2mF_1 - \sqrt{s}F_2)^2 \tag{5.4d}$$

where $\alpha = e^2/4\pi$ is the fine structure constant, $s+t+u = 4m^2$ and $i = \sqrt{-1}$. The amplitudes are normalized in such a way that the differential cross section

$$d\sigma/d\Omega = \tfrac{1}{2}\{|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2\}$$

is in millibarns.

In order to estimate the relative importance of this "annihilation-and-creation" diagram, we compare quantitatively these amplitudes with the *exact* direct Coulomb ones in the $\bar{N}N$ sector for all angles and several laboratory kinetic energies T_{kin}. The direct amplitudes for all particles with equal mass can be derived from Ref. [31] and are given by:

$$\bar{a}_D = [\alpha/(4m^2 t\sqrt{s}(\sqrt{s}+2m)^2)]\{[ut - \tfrac{1}{2}(\sqrt{s}+2m)^2(s+2t-u)](4m^2F_1^2 + tF_2^2) - 8mt[(s+t)\sqrt{s} + m(2s+t-u)]F_1F_2\} \tag{5.5a}$$

$$\bar{b}_D = [-\alpha/(8m^2 t\sqrt{s})](s-u)(4m^2F_1^2 - tF_2^2) \tag{5.5b}$$

$$\bar{c}_D = -\bar{d}_D = [-\alpha/2\sqrt{s}](F_1+F_2)^2 \tag{5.5c}$$

$$\bar{e}_D = [-i\alpha/(4m^2\sqrt{s}(\sqrt{s}+2m)^2)](\sqrt{u/t})\{(4m^2F_1^2 + tF_2^2) \times (3s+2t+u+6m\sqrt{s}) + 4m[(2s+3t+u)\sqrt{s} + 4m(s+t)]F_1F_2\} \tag{5.5d}$$

with CM particle momentum

$$k^2 = \tfrac{1}{4}(s-4m^2) = \tfrac{1}{2}mT_{kin}.$$

We remind the reader that $\bar{A}_A = M_C A_D$ with $A_D = -\bar{A}_D$. The corresponding curves for both set of amplitudes (5.4) and (5.5) in the $\bar{p}p \Rightarrow \bar{p}p$ case are shown in Fig. 2 for $T_{kin} = 500$ MeV. One can see that because of the t-pole term in $\bar{M}_D$, this process remains several orders of magnitude bigger than $\bar{M}_A$ even at large angles. The kinematical dependence is such that the "annihilation" contribution is always negligible, except in the region $T_{kin} \gtrsim 500$ MeV for backward angles $\theta \gtrsim 150°$, where the value of only one "annihilation" amplitude $\bar{a}_A$ is greater than the direct one-photon exchange one $\bar{a}_D$. Such specific kinematical domains should be explored when searching explicitly for one-virtual-photon annihilation effects in the $\bar{N}N$ polarization observables. The $\bar{n}n \Rightarrow \bar{n}n$ one-virtual-photon annihilation amplitudes at $T_{kin} = 500$ MeV

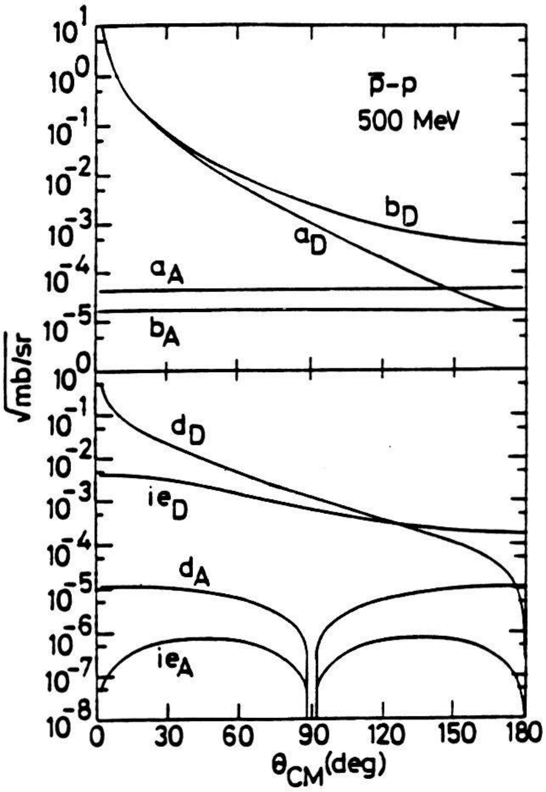

Figure 2
$\bar{p}p$ electromagnetic amplitudes from eqs. (5.4) and (5.5) at $T_{kin} = 500$ MeV.

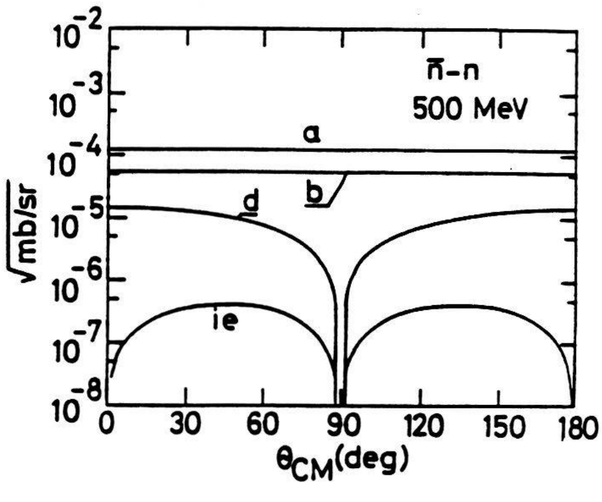

Figure 3
$\bar{n}n$ amplitudes (eq. (5.4)) corresponding to Fig. 1 at $T_{kin} = 500$ MeV.

are displayed in Fig. 3. Finally, the corresponding values for the charge-exchange process $\bar{p}p \Rightarrow \bar{n}n$ are of the same order of magnitude and remain small compared to the coulombian values (they are not shown here).

6. One-pion exchange amplitudes

As in the NN scattering case the longest range or the most peripheral interaction in $\bar{N}N$ scattering is given by the one pion exchange (OPE). It was realized long ago [32a] that the OPE amplitudes in the two processes are related by G parity [32b], the $\bar{N}N$ potential having the opposite sign of the NN potential. The expressions of the NN OPE amplitudes can be found e.g. in Ref. [33] and, taking into account the mass differences for np scattering, in II. However in view of their importance in phase shift analyses and for completeness we shall derive here the $\bar{N}N$ OPE amplitudes. For all $\bar{N}N$ reactions (2.3), the OPE amplitudes for pseudoscalar coupling is

$$M_{\mathrm{OPE}} = \frac{1}{8\pi\sqrt{s}} \frac{g_1 g_2}{(t-\mu^2)} \bar{u}(p_1')\gamma_5^{(1)} u(p_1) \bar{v}(-p_2)\gamma_5^{(2)} v(-p_2'). \tag{6.1}$$

Reducing (6.1) to its two component form in the CM for the processes (2.3a), (2.3b), (2.3c) and (2.3d), where only neutral pion exchange is allowed, we obtain

$$a_{\mathrm{OPE}} = b_{\mathrm{OPE}} = e_{\mathrm{OPE}} = f_{\mathrm{OPE}} = g_{\mathrm{OPE}} = h_{\mathrm{OPE}} = 0 \tag{6.2}$$

$$c_{\mathrm{OPE}} = d_{\mathrm{OPE}} = \frac{g_1 g_2}{8\pi\sqrt{s}} \frac{t}{(t-\mu_{\pi^0}^2)}$$

with

$$t = -2k^2(1-\cos\theta) \tag{6.3}$$

and μ_{π^0} is the neutral pion mass (we use the representation of Ref. III).

For the reaction (2.3a), $\bar{p}p \Rightarrow \bar{p}p$, one has

$$g_1 = g_{pp\pi^0}, \qquad g_2 = g_{\bar{p}\bar{p}\pi^0}. \tag{6.4}$$

For the reaction (2.3b), $\bar{n}n \Rightarrow \bar{n}n$,

$$g_1 = g_{nn\pi^0}, \qquad g_2 = g_{\bar{n}\bar{n}\pi^0}. \tag{6.5}$$

For the reaction (2.3c), $\bar{n}p \Rightarrow \bar{n}p$,

$$g_1 = g_{pp\pi^0}, \qquad g_2 = g_{\bar{n}\bar{n}\pi^0}. \tag{6.6}$$

For the reaction (2.3d), $\bar{p}n \Rightarrow \bar{p}n$,

$$g_1 = g_{nn\pi^0}, \qquad g_2 = g_{\bar{p}\bar{p}\pi^0}. \tag{6.7}$$

For the inelastic reactions (2.3f), $\bar{p}p \Rightarrow \bar{n}n$ and (2.3e), $\bar{n}n \Rightarrow \bar{p}p$, where there is either π^+ or π^- exchange, one has

$$a_{\mathrm{OPE}} = b_{\mathrm{OPE}} = e_{\mathrm{OPE}} = f_{\mathrm{OPE}} = h_{\mathrm{OPE}} = 0$$

$$c_{\mathrm{OPE}} = \frac{g_1 g_2}{8\pi\sqrt{s}} \frac{t}{(t-\mu_{\pi^+}^2)} \tag{6.8}$$

$$d_{\mathrm{OPE}} = \frac{g_1 g_2}{8\pi\sqrt{s}} \frac{1}{(t-\mu_{\pi^+}^2)} \{t + (1+\cos\theta)[k - \sqrt{k^2 + m_p^2 - m_n^2}]^2\}$$

$$g_{\mathrm{OPE}} = \frac{g_1 g_2}{8\pi\sqrt{s}} \frac{m_n^2 - m_p^2}{(t-\mu_{\pi^+}^2)} |\sin\theta|$$

with

$$t = -(2k^2 + m_p^2 - m_n^2) + 2k\sqrt{k^2 + m_p^2 - m_n^2}\cos\theta \tag{6.9}$$

$$g_1 = g_{pn\pi^+}, \qquad g_2 = g_{\bar{p}\bar{n}\pi^+} \tag{6.10}$$

and μ_{π^+} is the charged pion mass.

A non-zero value of the amplitude $g(s, t)$ results from the fact that these reactions (2.3e) and (2.3f), with $m_p \neq m_n$, are inelastic. If isospin invariance is assumed, we have

$$g_{pp\pi^0} = -g_{nn\pi^0} = \frac{1}{\sqrt{2}} g_{np\pi^+} \equiv g_0 \tag{6.11}$$

and

$$g_{\bar{p}\bar{p}\pi^0} = -g_{\bar{n}\bar{n}\pi^0} = \frac{1}{\sqrt{2}} g_{\bar{n}\bar{p}\pi^+} \equiv g_0 \tag{6.12}$$

with $\mu_{\pi^0} = \mu_{\pi^+}$ and $m_n = m_p$. In NN scattering the value from Ref. [34], usually used, is

$$g_0^2/4\pi = 14.4. \tag{6.13}$$

The knowledge of the OPE partial wave amplitudes is very useful for phase shift analysis. From formulae (6.2), (6.8) and equations (2.9) of Ref. III one can obtain the OPE singlet-triplet amplitudes and then perform an inversion of the partial-wave expansions (see following Section equations (7.1)), to find:

$$\begin{aligned}
R^J_{J0,J0}(\text{OPE}) &= -2F_N ik\left[\delta_{J0} + \left(\frac{1+\lambda^2}{2\lambda} - z\right) Q_J(z)\right] \\
R^J_{S,1}(\text{OPE}) &= R^J_{1,S}(\text{OPE}) = 0 \\
R^J_{J1,J1}(\text{OPE}) &= -2F_N ik\left[-\frac{1+\lambda^2}{2\lambda} Q_J(z) + \frac{J}{2J+1} Q_{J+1}(z) + \frac{J+1}{2J+1} Q_{J-1}(z)\right] \\
R^J_{J-11,J-11}(\text{OPE}) &= \frac{2F_N ik}{2J+1}\left[\frac{1+\lambda^2}{2\lambda} Q_{J-1}(z) - Q_J(z)\right] \\
R^J_{J+11,J+11}(\text{OPE}) &= \frac{2F_N ik}{2J+1}\left[Q_J(z) - \frac{1+\lambda^2}{2\lambda} Q_{J+1}(z)\right] \\
R^J_{J-11,J+11}(\text{OPE}) &= 2F_N ik \frac{\sqrt{J(J+1)}}{2J+1}\left[2Q_J(z) - \lambda Q_{J+1}(z) - \frac{1}{\lambda} Q_{J-1}(z)\right] \\
R^J_{J+11,J-11}(\text{OPE}) &= 2F_N ik \frac{\sqrt{J(J+1)}}{2J+1}\left[2Q_J(z) - \frac{1}{\lambda} Q_{J+1}(z) - \lambda Q_{J-1}(z)\right].
\end{aligned} \tag{6.14}$$

The Q_J are the Legendre functions of the second kind. For the reactions (2.3a) to (2.3d) one has (see also equations (6.2) to (6.7)):

$$F_N = \frac{g_1 g_2}{8\pi\sqrt{s}}, \quad \lambda = 1, \quad z = 1 + \frac{\mu^2_{\pi^0}}{2k^2}. \tag{6.15}$$

In this case $R^J_{J-11,J+11}(\text{OPE}) = R^J_{J+11,J-11}(\text{OPE})$. For the reactions (2.3e) and (2.3f) one has (see equations (6.8) to (6.10)):

$$F_N = \frac{g_{pn\pi^+} g_{\bar{p}\bar{n}\pi^+}}{8\pi\sqrt{s}}, \quad \lambda = \sqrt{k^2 + m_p^2 - m_n^2}/k, \quad z = \{k^2(1+\lambda^2) + \mu^2_{\pi^+}\}/2\lambda k^2. \tag{6.16}$$

7. Phase shift analysis

The partial wave expansion of the $\bar{N}N$ amplitudes is similar to that of NN amplitudes. The scattering matrix can be written in terms of different sets of amplitudes such as e.g. the invariant amplitudes $a, b, c, d, e \ldots$ in equation (2.1), the singlet-triplet $M_{sm_s,s'm'_s}$ [35] or the helicity Φ_i [36], representations. All the relevant formulae for the NN case can be found in I, II, III and Refs. [37, 38].

In contrast with the NN reactions the Pauli principle does not constrain the $\bar{N}N$ amplitudes. Consequently for a given $\bar{N}N$ isospin, there is no restriction on the parity of the partial waves, they all contribute. We consider, as in III, eight amplitudes and give, in their notation, the partial wave expansion of the singlet-triplet amplitudes. Starting from equation (2.12) of Ref. III, for a given isospin, one obtains:

$$M_{SS} = \frac{1}{2ik} \sum_{J=0}^{\infty} (2J+1) R^J_{J0,J0} P_J(\cos\theta)$$

$$M_{00} = \frac{1}{2ik} \sum_{J=0}^{\infty} (J+1) R^J_{J+11,J+11} P_{J+1}(\cos\theta) + J R^J_{J-11,J-11} P_{J-1}(\cos\theta)$$
$$+ \sqrt{J(J+1)}[R^J_{J+11,J-11} P_{J+1}(\cos\theta) + R^J_{J-11,J+11} P_{J-1}(\cos\theta)]$$

$$M_{11} = \frac{1}{4ik} \sum_{J=1}^{\infty} (2J+1) R^J_{J1,J1} P_J(\cos\theta) + J R^J_{J+11,J+11} P_{J+1}(\cos\theta)$$
$$+ (J+1) R^J_{J-11,J-11} P_{J-1}(\cos\theta)$$
$$- \sqrt{J(J+1)}[R^J_{J+11,J-11} P_{J+1}(\cos\theta) + R^J_{J-11,J+11} P_{J-1}(\cos\theta)]$$

$$M_{1-1} = \frac{1}{4ik} \sum_{J=1}^{\infty} \frac{1}{J(J+1)} \{-(2J+1) R^J_{J1,J1} P^2_J(\cos\theta) + J R^J_{J+11,J+11} P^2_{J+1}(\cos\theta)$$
$$+ (J+1) R^J_{J-11,J-11} P^2_{J-1}(\cos\theta)$$
$$- \sqrt{J(J+1)}[R^J_{J+11,J-11} P^2_{J+1}(\cos\theta) + R^J_{J-11,J+11} P^2_{J-1}(\cos\theta)]\}$$

$$M_{10} = \frac{1}{2\sqrt{2}ik} \sum_{J=0}^{\infty} - R^J_{J+11,J+11} P^1_{J+1}(\cos\theta) + R^J_{J-11,J-11} P^1_{J-1}(\cos\theta)$$
$$- \sqrt{\frac{J}{J+1}} R^J_{J+11,J-11} P^1_{J+1}(\cos\theta) + \sqrt{\frac{J+1}{J}} R^J_{J-11,J+11} P^1_{J-1}(\cos\theta) \tag{7.1}$$

$$M_{01} = \frac{1}{2\sqrt{2}ik} \sum_{J=1}^{\infty} \frac{1}{J(J+1)} [(2J+1)R^J_{J1,J1} P^1_J(\cos\theta) + J^2 R^J_{J+11,J+11} P^1_{J+1}(\cos\theta)$$
$$- (J+1)^2 R^J_{J-11,J-11} P^1_{J-1}(\cos\theta)] - \sqrt{\frac{J}{J+1}} R^J_{J+11,J-11} P^1_{J+1}(\cos\theta)$$
$$+ \sqrt{\frac{J+1}{J}} R^J_{J-11,J+11} P^1_{J-1}(\cos\theta)$$
$$M_{S1} = \frac{1}{2\sqrt{2}ik} \sum_{J=1}^{\infty} \frac{2J+1}{\sqrt{J(J+1)}} R^J_{J0,J1} P^1_J(\cos\theta)$$
$$M_{1S} = -\frac{1}{2\sqrt{2}ik} \sum_{J=1}^{\infty} \frac{2J+1}{\sqrt{J(J+1)}} R^J_{J1,J0} P^1_J(\cos\theta).$$

Here $R^0_{-11,-11} = R^0_{-11,11} = 0$.

In the equations (7.1) the P_J are the Legendre polynomials and the P^m_J, $m = 1, 2$, the associated Legendre functions of the first kind.

The eight corresponding invariant amplitudes in equation (2.1) are related to the previous singlet-triplet amplitudes by (see equation (2.9) of Ref. III):

$$\begin{aligned}
a &= \tfrac{1}{2}(M_{00} + M_{11} - M_{1-1}) \\
b &= \tfrac{1}{2}(M_{SS} + M_{11} + M_{1-1}) \\
c &= \tfrac{1}{2}(-M_{SS} + M_{11} + M_{1-1}) \\
d &= \frac{\cos\theta}{2}(M_{00} - M_{11} + M_{1-1}) - \frac{\sin\theta}{\sqrt{2}}(M_{10} + M_{01}) \\
e &= \frac{i}{\sqrt{2}}(M_{10} - M_{01}) \\
f &= \frac{i}{\sqrt{2}}(M_{S1} - M_{1S}) \\
g &= \frac{\sin\theta}{2}(M_{00} - M_{11} + M_{1-1}) + \frac{\cos\theta}{\sqrt{2}}(M_{10} + M_{01}) \\
h &= \frac{1}{\sqrt{2}}(M_{1S} + M_{S1}).
\end{aligned} \tag{7.2}$$

For the singlet and uncoupled triplet states one defines [3]

$$R = \begin{pmatrix} R^J_{J0,J0} & R^J_{S,1} \\ R^J_{1,S} & R^J_{J1,J1} \end{pmatrix} \tag{7.3}$$

whereas for the coupled triplet states:

$$R = \begin{pmatrix} R^J_{J-11,J-11} & R^J_{J-11,J+11} \\ R^J_{J+11,J-11} & R^J_{J+11,J+11} \end{pmatrix}. \tag{7.4}$$

The matrix of partial wave amplitudes R is related to the S matrix by $R = S - 1$. The R^J are complex numbers and the parametrization of the 2×2

matrices (7.3) and (7.4) is somewhat arbitrary. It could be chosen such that, when the annihilation is set equal to zero, it reduces to the nuclear bar phase shift parametrization [35]. One possible choice is similar to that of III:

$$\begin{aligned}
R^J_{J0,J0} &= \cos 2\gamma_J\, e^{2i\delta_J} - 1 \\
R^J_{J1,J1} &= \cos 2\gamma_J\, e^{2i\delta_{JJ}} - 1 \\
R^J_{S,1} &= i \sin 2\gamma_J\, e^{i(\delta_J + \delta_{JJ} + \phi_{S,1})} \\
R^J_{J\mp 11,J\mp 11} &= \cos 2\varepsilon_J\, e^{2i\delta_{J\mp 1,J}} - 1 \\
R^J_{J\mp 11,J\pm 11} &= i \sin 2\varepsilon_J\, e^{i(\delta_{J-1,J} + \delta_{J+1,J} + \eta_{J\mp 1,J\pm 1})}.
\end{aligned} \tag{7.5}$$

The presence of the annihilation channels makes the δ complex with $\operatorname{Im}\delta \geqslant 0$ and introduces the real parameters ϕ and η. This parametrization then satisfies the unitarity conditions.

Another choice has been advocated in Refs. [39, 40] and reads

$$S = \exp(i\delta)\exp(i\varepsilon\sigma_x)V\exp(i\varepsilon\sigma_x)\exp(i\delta). \tag{7.6}$$

Here δ is a real diagonal matrix with elements $\delta_{\pm}$,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is a Pauli matrix and V is a real 2×2 matrix. The departure of V from the unit matrix is a consequence of absorption. When S is symmetric, V is also symmetric and can be parametrized by a diagonal matrix via a rotation [40]:

$$V = \exp(-i\omega\sigma_y)\begin{pmatrix} \cos 2(\Gamma + \Gamma') & 0 \\ 0 & \cos 2(\Gamma - \Gamma') \end{pmatrix}\exp(i\omega\sigma_y) \tag{7.7}$$

where

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

is another Pauli matrix. Such a parametrization (equations (7.6) and (7.7)) can be used for both of the cases corresponding to equations (7.3) and (7.4).

For a given energy and isospin channel, the number of free parameters to determine is four times larger than in the NN case when below the pion production threshold (above it this ratio drops to two). If one restricts the analysis to a maximum $J = 6$, there are, assuming CPT invariance, 64 free fitting parameters. However the non absorptive peripheral (J large) part of the $\bar{N}N$ interaction is constrained by the OPE amplitudes which were given in the previous section. Furthermore if the strong annihilation is of short range the number of free parameters will be also decreased. Once these parameters are determined the physical scattering amplitudes are given by equations (7.1) and (7.2).

A phase shift analysis using the basis of invariant amplitudes (7.2) has already been done for the NN channel [41]. The question of removing discrete ambiguities in the NN phase shift analysis [42] also arises in the $\bar{N}N$ one [43].

8. Conclusions

During the past years we have learned much from antimatter physics with the better quality $\bar{p}$ beams at the AGS, KEK, BNL and LEAR facilities. Thanks to the LEAR experiments we now know that the $\bar{p}p$ total cross section does not exhibit any evidence for baryonium [44]. The low energy antinucleon-nucleon scattering experiments have shown that while the $\bar{p}p$ elastic (equation (2.3a)) and charge exchange (equation (2.3f)) differential cross sections agree with most potential model predictions, the agreement for the analyzing power A_{ooon} is rather poor (see Refs [45, 46] for $\bar{p}p$ elastic scattering and [47] for charge exchange). The energy behaviour of the forward slope of $d\sigma/dt$ for these two reactions is given to within 10% by the one-pion exchange described in Section 6 [48].

As in the NN sector, spin correlations are required to disentangle the $\bar{N}N$ interaction and to discriminate between phenomenological potential models. An alternative proposal to Ref. [9] to polarize the $\bar{p}$ beam is suggested by the FILTEX collaboration [49]. A measurement of A_{oono}, A_{ooon} (equation (4.1)), A_{oonn} and A_{ooss} (equations (4.9a) to (4.9d)) above 250 MeV/c would then be immediate. A test of CP invariance in the $\bar{p}p$ elastic channel is the equality $A_{oono} = A_{ooon}$ which could also be readily tested. In working with potential models, the more spin-dependent observables we have, the more difficult it is to reproduce all of them by adjusting only the existing short-distance core parameters, so that in principle more information on heavier-meson exchanges and/or quark-antiquark dynamics can be obtained. On the other hand from a model-independent point of view, the final goal of the experimental efforts is to achieve a unique direct statistical reconstruction of the $\bar{N}N$ elastic scattering matrix in terms of complete sets of observables, measured at several angles and energies, as has been performed for pp scattering (see e.g. Ref. [50]).

Although the existing $\bar{N}N$ data don't yet allow an unambiguous determination of the annihilation range in potential analyses [51], it does allow a reasonable determination of the effective-range expansion parameters at low energy [52]. The latter leads to the Argand diagrams for S- and P-waves ($L = 0$ and 1 in Section 7), but so far spin effects are neglected because of the lack of polarization data at low momenta. The $\bar{p}p$ data also indicate the need for a coupled-channel approach to several reactions [53]. The spin dependence of the transition operators and the coupled-channel effects should bring a better description of the laboratory spin-dependent observables (4.1) and (4.8) to (4.11).

The treatment of the exact Coulomb distortions in an amplitude analysis, as introduced in Section 5, cannot be underestimated. Even in the simplest case of spin averaged $\bar{p}p$ data, an unexpected rise in energy of the real-to-imaginary ratio of the forward amplitude (ρ parameter) remains unexplained. A careful comparison between the treatments of the Coulomb-nuclear interference done respectively by experimentalists and theorists had to be made before ruling out this Coulomb effect as the cause of the discrepancy between the data and the model predictions [54, 55]. In general, the long-range Coulomb contributions (equations 5.5) are needed for the electro-hadronic separation at very small angles, that is to say, for the determination of the real part of the forward $\bar{N}N$ nuclear scattering amplitude.

The shortest-range contributions of the electromagnetic corrections given by equations (5.4) are shown here to be small (Figs. 2 and 3). In order to calculate them, we have used a representation of the electromagnetic form factors of the nucleon in the region of timelike momentum transfers given by the generalized vector-dominance model [56]. The behaviour of these form factors close to the threshold is determined to some extent by the nuclear $\bar{N}N$ interaction in the initial state [57]. New precise measurements of the proton form factor in the timelike region (PS170 at LEAR) are available [58a] while the neutron form factor could be measured by the PS201 or the FENICE [58b] collaborations. These new data will constrain the model parameters in equation (5.3) and thus the outcome of the amplitudes $\bar{A}_A$ shown in Figs 2 and 3 (elastic scattering due to virtual annihilation into a photon).

The usual partial-wave expansion of the singlet-triplet amplitudes is generalized from a set of 5 amplitudes (pp case) to a set of 8 amplitudes ($\bar{N}N$ sector with no I or T invariance) in Section 7. So far the phase shift analyses have been restricted to low energies ($T_{kin} \sim 20$ MeV) and dealt with S- and P-waves only. Considering the spin dependence of the scattering matrix one then has the following states in both isospin 0 and 1: $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$. It is found [59] that there is a large admixture of P-waves down to the lowest momenta measured for $\bar{p}p$ scattering. Even in the simplest case (low-energy elastic scattering) a "complete" set of observables has yet to be obtained before determining the preceding 6 complex phase shifts [60]. Once a phase shift analysis including higher-order partial waves is successfully performed, it will be possible to decide whether mesons with specific quantum numbers and mass, observed or anticipated in the $\bar{N}N$ system, correspond to resonant partial waves in the NN system [61].

Direct experimental tests of CPT invariance in the elastic $\bar{p}p$ system are described in Section 4. The simplest spin-dependent observables necessary to do a test are the asymmetry experiments (equations (4.1) and (4.8)). It should be emphasized that this fundamental invariance property, presumably valid for all types of interactions, has a large variety of experimental consequences. Some of them are purely static properties, such as the equality of the proton and antiproton inertial masses. These can be, and indeed have been (PS189 and PS196 at LEAR), tested with great precision (Refs. [62, 63] for the $\bar{p}p$ mass difference). Other consequences of CPT invariance are dynamical ones, such as the absence of CPT violating amplitudes in elastic $\bar{N}N$ scattering, as discussed in Section 4. The interference phenomena that would lead to a violation of the CPT predictions, would not contribute here to a $\bar{p}p$ mass difference. Hence a direct verification of the CPT predictions in spin-dependent experiments would be of great value, even at lower precision than is achieved in the comparison of particle and antiparticle static properties. The advantage of such a verification is that it is model independent.

The present experimental status of the available $\bar{N}N$ data will be treated in a separate paper. Angular distributions of representative measured experimental quantities show that even though some observables are well known over a large energy domain, the existing data set is still far from sufficient for a model-independent description of the $\bar{N}N$ interaction or for a full phase shift analysis. Even at low momenta, the theoretical predictions cannot reproduce all the features of those observables (strong anisotropy due to an intense P-wave, the unexpected behaviour of the ρ parameter, large polarization very close to threshold in some channels, angular dependence of A_{ooon}, etc.). This can be perceived as a lesson in prudence even for the simplest cases.

Challenges for theorists are expected to grow as the $\bar{p}$ beam energy and intensity increase. Indeed, projects already exist for high momenta and luminosity machines such as SUPERLEAR, the KAON facility, the European Hadron Factory and proposals at FERMILAB and JINR-Dubna. These facilities will bring very high fluxes of antiprotons and thus increase the precision of the spin-dependent observables. Such upgrades would make direct tests of discrete symmetries more reliable.

Acknowledgement

We appreciate helpful discussions and comments of J. Ball, R. Bertini, F. Bradamante, R. Hess, R. Kunne, L. Lamanna, C. Lechanoine-Leluc, A. Martin and F. Perrot-Kunne. One of us (PLF) is most indebted to O. Bohigas for his warm hospitality at the IPN-Orsay where part of this work has been done. The same author thanks J. Arvieux for making a stay at the CEN-Saclay possible. The research of P.W. was partly supported by research grants from NSERC of Canada and the Fonds FCAR du Québec.

REFERENCES

[1] J. Bystricky, F. Lehar and P. Winternitz, J. Physique (Paris) *39* (1978) 1.
[2] P. LaFrance and P. Winternitz, J. Physique (Paris) *41* (1980) 1391.
[3] J. Bystricky, F. Lehar and P. Winternitz, J. Physique (Paris) *45* (1984) 207.
[4] P. Lefevre, *Physics at LEAR with low energy antiprotons*, Villars-sur-Ollon, edited by C. Amsler et al., Harwood Academic Publishers (1988) p. 19.
[5] D. J. Simon, D. Dumollard and K.-G. Rensfelt, *Physics with antiprotons at LEAR in the ACOL era*, Tignes, edited by U. Gastaldi et al., Editions Frontières (1985) p. 47.
[6] W. Bruckner, H. Dobbeling, K. Dworschak, H. Kneis, M. Nomachi, S. Paul, B. Povh, R. Ransome, T.-A. Shibata, E. Steffens, M. Treichel and Th. Walcher, *Physics with antiprotons at LEAR in the ACOL era*, Tignes, edited by U. Gastaldi et al., Editions Frontières (1985) p. 245.
[7] C. I. Beard, D. V. Bugg, J. R. Hall, A. S. Clough, R. L. Shypit, R. Birsa, F. Bradamante, S. Dalla Torre-Colautti, A. Martin, A. Penzo, P. Shiavon, F. Tessarotto, A. Villari, S. Degli-Agosti, E. Heer, R. Hess, C. Lechanoine-Leluc, Y. Onel, D. Rapin, K. Bos, J. C. Kluyver, R. A. Kunne and L. Linssen, *Physics at LEAR with low energy antiprotons*, Villars-sur-Ollon, edited by C. Amsler et al., Harwood Academic Publishers (1988) p. 305.
[8] R. Bertini, H. Catz, A. Chaumeaux, B. Fabbro, J.-C. Faivre, H. Fanet, J. Pain, F. Perrot, E. Vercellin, E. Boschitz, W. Gyles, W. List, C. Otterman, R. Haroutunian, J. Y. Grossiord, A. Guichard, J. Arvieux, J. Yonnet, S. Mango, J. A. Konter and B. van den Brandt, *Physics at LEAR with low energy antiprotons*, Villars-sur-Ollon, edited by C. Amsler et al., Harwood Academic Publishers (1988) p. 313.
[9] A. Penzo, *Physics at LEAR with low energy antiprotons*, Villars-sur-Ollon, edited by C. Amsler et al., Harwood Academic Publishers (1988) p. 183.
[10] T. Kamae, Phys. Report *63* (1980) 149; C. Amsler, Adv. Nucl. Phys. *18* (1987) 183.
[11] L. Montanet, G. C. Rossi and G. Veneziano, Nucl. Phys. *A374* (1982) 25c.
[12] M. Lacombe, B. Loiseau, B. Moussallam and R. Vinh Mau, Phys. Rev. *C29* (1984) 1800.
[13] P. H. Timmers, W. A. van der Sanden and J. J. de Swart, Phys. Rev. *D31* (1985) 99; L. N. Bogdanova, O. D. Dalkarov and I. S. Shapiro, Ann. Phys. *84* (1974) 261.
[14] C. B. Dover, T. Gutsche and A. Faessler, Phys. Rev. *C43* (1991) 379.
[15] M. A. Alberg, R. A. Freedman, E. M. Henley, W.-Y. P. Hwang, D. Seckel and L. Wilets, Phys. Rev. *D27* (1983) 536.

[16] J. Côté, M. Lacombe, B. Loiseau, B. Moussallam and R. Vinh Mau, Phys. Rev. Lett. *48* (1982) 1319; M. Pignone, M. Lacombe, B. Loiseau and R. Vinh Mau, Phys. Rev. Lett. *67* (1991) 2423.
[17] P. H. Timmers, W. A. van der Sanden and J. J. de Swart, Phys. Rev. *D29* (1984) 1928.
[18] C. B. Dover and J. M. Richard, Phys. Rev. *C25* (1982) 1952.
[19] T. Mizutani, F. Myhrer and R. Tegen, Phys. Rev. *D32* (1985) 1663.
[20] R. A. Bryan and R. J. N. Phillips, Nucl. Phys. *5B* (1968) 201.
[21] A. D. Martin and T. D. Spearman, *Elementary particle theory*, North Holland, Amsterdam (1970).
[22] E. G. P. Rowe and E. J. Squires, Rep. Prog. Phys. *32* (1969) 273.
[23] P. L. Csonka and M. J. Moravcsik, Phys. Rev. *165* (1968) 1915.
[24] S. M. Bilenkii, Zhetf. Pis'ma *3* (1966) 118; transl. JETP Lett. *3* (1966) 74.
[25] P. L. Csonka, Rev. Mod. Phys. *37* (1965) 177.
[26] K. Gotow and S. Okubo, Phys. Rev. *128* (1962) 1921.
[27] G. Cohen-Tannoudji, A. Morel and H. Navelet, Ann. Phys. *46* (1968) 239.
[28] E. Abers and V. L. Teplitz, Phys. Rev. *158* (1967) 1365.
[29] T. L. Trueman and G. C. Wick, Ann. Phys. *26* (1964) 322.
[30] J. G. Korner, *Physics with antiprotons at LEAR in the ACOL era*, Tignes, edited by U. Gastaldi et al., Editions Frontières (1985) p. 525.
[31] P. LaFrance, C. Lechanoine, F. Lehar, F. Perrot, L. Vinet and P. Winternitz, Nuovo Cimento *64A* (1981) 179.
[32] a) J. Iwadare and S. Hatano, Progr. Theor. Phys. *15* (1956) 185. b) T. D. Lee and C. N. Yang, Il Nuovo Cimento *3* (1956) 749.
[33] P. Cziffra, M. H. MacGregor, M. J. Moravcsik and H. P. Stapp, Phys. Rev. *114* (1959) 880.
[34] M. M. Nagels, J. J. de Swart, H. Nielsen, G. C. Oades, J. L. Petersen, B. Tromborg, G. Gustafson, A. C. Irving, C. Jarlskog, W. Pfeil, H. Pilkuhn, F. Steiner and L. Tauscher, Nucl. Phys. *B109* (1976) 1.
[35] H. P. Stapp, T. J. Ypsilantis and M. Metropolis, Phys. Rev. *105* (1957) 302.
[36] M. Jacob and G. C. Wick, Ann. Phys. (NY) *7* (1959) 404.
[37] N. Hoshizaki, Suppl. Progr. Theor. Phys. *42* (1968) 107; M. J. Moravcsik, J. Pauschenwein and G. R. Goldstein, J. Phys. France *50* (1989) 1167.
[38] J. Raynal, Nucl. Phys. *28* (1961) 220.
[39] R. A. Bryan, Phys. Rev. *C24* (1981) 2659.
[40] D. W. L. Sprung and S. Klarsfeld, Phys. Lett. *B206* (1988) 182.
[41] J. Bystricky, C. Lechanoine-Leluc and F. Lehar, J. Physique (Paris) *48* (1987) 199.
[42] N. P. Klepikov, Sov. Phys. JETP *14* (1962) 846; Sov. Phys. JETP *20* (1965) 505; D. Atkinson, P. W. Johnson, N. Mehta and M. de Roo, Nucl. Phys. *B55* (1973) 125; L. P. Kok, *Few body dynamics*, edited by A. N. Mitra, North-Holland Publishing Company (1976) p. 43.
[43] A. Gersten, *Few body problems in physics*, Vol. II, edited by B. Zeitnitz, Elsevier Science Publishers (1984) p. 123.
[44] C. Amsler, Nucl. Phys. *A508* (1990) 501c and PSI proceedings on *Prospects of hadronic physics at low energy*, Zuoz, Switzerland, PSI Documentation (April 1990), p. 197.
[45] R. Bertini, M. Costa, F. Perrot, H. Catz, A. Chameaux, J. C. Faivre, E. Vercellin, J. Arvieux, J. Yonnet, B. van den Brandt, J. A. Konter, D. R. Gill, S. Mango, G. D. Wait, E. Boschitz, W. Gyles, W. List, C. Otterman, T. Tacik, M. Wessler, E. Descroix, J. Y. Grossiord and A. Guichard, Phys. Lett. *B228* (1989) 531.
[46] R. A. Kunne, C. I. Beard, R. Birsa, K. Bos, F. Bradamante, D. V. Bugg, A. S. Clough, S. Dalla Torre-Colautti, S. Degli-Agosti, J. A. Edgington, R. J. Hall, E. Heer, R. Hess, J. C. Kluyver, C. Lechanoine-Leluc, L. Linssen, A. Martin, T. O. Niinikoski, Y. Onel, A. Penzo, D. Rapin, J. M. Rieubland, A. Rijllart, P. Schiavon, R. L. Shypit, F. Tessarotto, A. Villari and P. Wells, Phys. Lett. *206B* (1988) 557.
[47] R. Birsa, F. Bradamante, S. Dalla Torre-Colautti, M. Giorgi, M. Lamanna, A. Martin, A. Penzo, P. Schiavon, F. Tessarotto, M. P. Macciotta, A. Masoni, G. Puddu, S. Serci, T. Niinikoski, A. Rijllart, A. Amidouch, E. Heer, R. Hess, R. Kunne, C. Lechanoine-Leluc, C. Mascarini, D. Rapin, J. Arvieux, R. Bertini, H. Catz, J. C. Faivre, F. Perrot-Kunne, M. Angello, F. Iazzi, B. Minetti, T. Bressani, E. Chiavassa, N. De Marco, A. Musso and A. Riccotti, Phys. Lett. *B246* (1990) 267; M. Lamanna, Private Communication January 1991.
[48] F. Myhrer, Nucl. Phys. *A508* (1990) 513c; J. Haidenbauer, T. Hippchen and R. Tegen, Phys. Rev. *C44* (1991) 1812.
[49] W. Haeberli, *Physics at LEAR with low energy antiprotons*, Villars-sur-Ollon, edited by C. Amsler et *al.*, Harwood Academic Publishers (1988) p. 195.

[50] C. D. LAC, J. BALL, J. BYSTRICKY, J. DERÉGEL, F. LEHAR, A. DE LESQUEN, L. VAN ROSSUM, J. M. FONTAINE, F. PERROT and P. WINTERNITZ, J. Phys. France *51* (1990) 2689.
[51] B. POVH and T. WALCHER, Comments Nucl. Part. Phys. *16* (1986) 85; J. HAIDENBAUER, T. HIPPCHEN and K. HOLINDE, Nucl. Phys. *A508* (1990) 329c.
[52] H. J. PIRNER, B. KERBIKOV and J. MAHALANABIS, Z. Phys. *A338* (1991) 111.
[53] G. Q. LIU and F. TABAKIN, Phys. Rev. *C41* (1990) 665.
[54] B. MOUSSALLAM, Z. Phys. *A325* (1986) 1.
[55] J. A. NISKANEN, Mod. Phys. Lett. *A3* (1988) 57.
[56] J. G. KORNER and M. KURODA, Phys. Rev. *D16* (1977) 2165.
[57] O. D. DALKHAROV and K. V. PROTASOV, Sov. J. Nucl. Phys. *50* (1989) 1030, transl. Yad. Fiz. *50* (1989) 1661.
[58] a) G. BARDIN et al., Phys. Lett. *B255* (1991) 149. b) A. ANTONELLI et al., *Low energy antiproton physics*, Stockholm, edited by P. Carlson et al., World Scientific Pub. (1991) p. 354.
[59] B. O. KERBIKOV, L. A. KONDRATYUK and M. G. SAPOZHNIKOV, Sov. Phys. Usp. *32* (1989) 739, transl. Usp. Fiz. Nauk. *159* (1989) 3.
[60] R. A. KUNNE, Preprint LNS/Ph/90-18, Saclay 1990.
[61] R. E. SHAMU and P. W. LISOWSKI, Phys. Rev. *D26* (1982) 2540.
[62] G. GABRIELSE, X. FEI, L. A. OROZCO, R. J. TJOELKER, J. HAAS, H. KALINOWSKY, T. A. TRAINOR and W. KELLS, Phys. Rev. Lett. *65* (1990) 1317.
[63] A. COC et al., *Low energy antiproton physics*, Stockholm, edited by P. Carlson et al., World Scientific Pub. (1991) p. 431.