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Vortex Correction to the SCHA in the 2D-XY Model

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Abstract. A self consistent probability distribution $\wp(\phi)$ for bond phase differences in the 2D-XY model is constructed. This distribution is taylored in order to include the effect of vortices in a variational harmonic framework, and it provides us with a correction to the Self Consistent Harmonic Approximation (SCHA). It gives the correct value for the universal jump of the helicity modulus Γ and has the predicted critical behaviour $\Gamma - \Gamma_c \sim \sqrt{1 - t/t_c}$. Furthermore, the absolute value of t_c corresponds to the best Monte-Carlo simulations.

Introduction

2D-XY model in the SCHA have been used by us[1] to study the effect of quantum fluctuations due to charging effects on the critical temperature of HTS. SCHA is the most simple[2] but rather controversial[3] approach to the 2D-XY model. Indeed, in the context of SCHA, the effect of vortices, that are responsible for the B-K-T transition, is underestimated. SCHA consists in replacing the $J(1-cos(\phi))$ potential (ϕ being the phase difference of a bond) by an effective harmonic potential :$\frac{1}{2}K\phi^2$. This self-consistent harmonic potential contains enough anharmonic corrections to provide a correct description of low amplitude phase fluctuations. Unfortunately, it attributes an excessive energetic cost to topological exitations (vortices) causing an overestimation of the B-K-T transition temperature and a too small value (1/4) for the universal jump γ_c/t_c that is expected to be $2/\pi$ from RG analysis($\gamma = K/J$ and $t = k_B T/J$).

Vortex correction to the SCHA.

The simple minded SCHA produces the well-known self-consistent solution for the effective coupling constant (main contribution to the SCHA helicity modulus):

$$\gamma = e^{-X/2} \qquad where \qquad X \equiv <\phi^2>_\gamma$$

The above solution is related to an implicit *local* probability distribution $\wp_o(\phi)$ for the bond angular differences that presents a unique bump centered at $\phi_o = 0$. For a periodic potential the actual distribution should have a bump at each value $\phi_n = 2\pi n$. Let us modelise this distribution by a three-bump function defined by:

$$\wp(\phi) = (1-p)\wp_o(\phi) + \frac{1}{2}p[\wp_o(\phi - 2\pi) + \wp_o(\phi + 2\pi)]$$

where p is the probability for the phase difference to be out of the interval $[-\pi, \pi]$. This new distribution provides us with a correction to the mean square fluctuation, which we now denote by $\tilde{X}$:

$$\tilde{X} = X + 4\pi^2 p$$

The improved effective coupling constant Γ will be:

$$\Gamma = e^{-\tilde{X}/2} = \gamma e^{-2\pi^2 p} \quad .$$

The probability p can be evaluated self-consistently trough the local effective Boltzmann factor as follows:

$$p = 1 - \frac{\int_{\pi}^{\pi} d\phi exp(-\Gamma\phi^2/2t)}{\int_{\infty}^{\infty} d\phi exp(-\Gamma\phi^2/2t)} = 1 - \Phi(\sqrt{\Gamma\pi^2/2t})$$

where Φ is the error function (probability integral).

In Figs.(1) and (2) we illustrate, respectively, the general $\Gamma(t)$ dependence (with and without vortex correction) and the critical behaviour $\Gamma - \Gamma_c \sim \sqrt{1 - t/t_c}$.

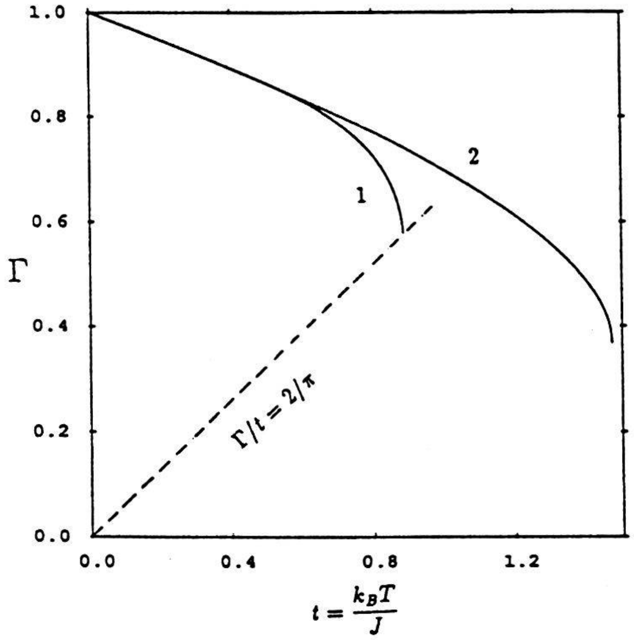

FIG.(1): Helicity modulus vs temperature with(1) and without(2) vortex correction.

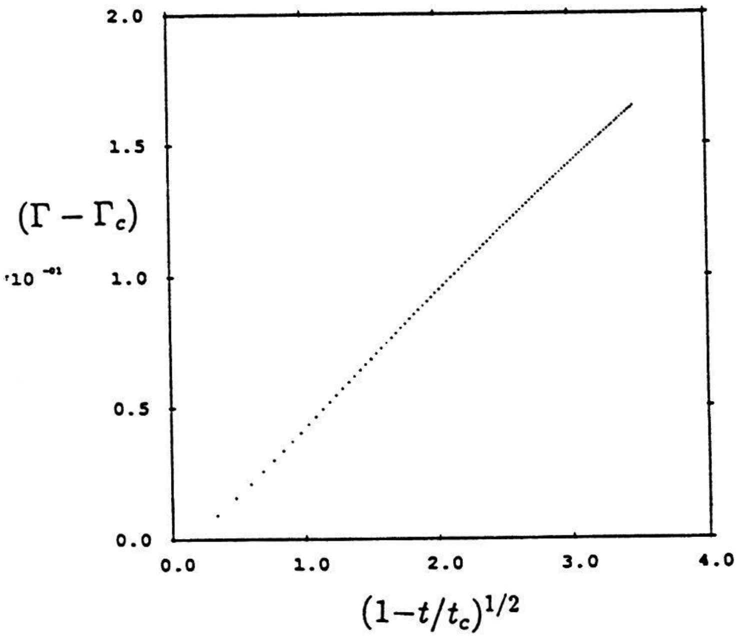

FIG.(2): Critical behaviour of Γ near the transition point.

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