

**Zeitschrift:** Helvetica Physica Acta

**Band:** 65 (1992)

**Heft:** 2-3

**Artikel:** Vortex corrections to the SCHA in the 2D-XY model

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**DOI:** <https://doi.org/10.5169/seals-116500>

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## Vortex Correction to the SCHA in the 2D-XY Model

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**Abstract.** A self consistent probability distribution  $\wp(\phi)$  for bond phase differences in the 2D-XY model is constructed. This distribution is tailored in order to include the effect of vortices in a variational harmonic framework, and it provides us with a correction to the Self Consistent Harmonic Approximation (SCHA). It gives the correct value for the universal jump of the helicity modulus  $\Gamma$  and has the predicted critical behaviour  $\Gamma - \Gamma_c \sim \sqrt{1 - t/t_c}$ . Furthermore, the absolute value of  $t_c$  corresponds to the best Monte-Carlo simulations.

### Introduction

2D-XY model in the SCHA have been used by us[1] to study the effect of quantum fluctuations due to charging effects on the critical temperature of HTS. SCHA is the most simple[2] but rather controversial[3] approach to the 2D-XY model. Indeed, in the context of SCHA, the effect of vortices, that are responsible for the B-K-T transition, is underestimated. SCHA consists in replacing the  $J(1 - \cos(\phi))$  potential ( $\phi$  being the phase difference of a bond) by an effective harmonic potential  $\frac{1}{2}K\phi^2$ . This self-consistent harmonic potential contains enough anharmonic corrections to provide a correct description of low amplitude phase fluctuations. Unfortunately, it attributes an excessive energetic cost to topological excitations (vortices) causing an overestimation of the B-K-T transition temperature and a too small value (1/4) for the universal jump  $\gamma_c/t_c$  that is expected to be  $2/\pi$  from RG analysis ( $\gamma = K/J$  and  $t = k_B T/J$ ).

### Vortex correction to the SCHA.

The simple minded SCHA produces the well-known self-consistent solution for the effective coupling constant (main contribution to the SCHA helicity modulus):

$$\gamma = e^{-X/2} \quad \text{where} \quad X \equiv \langle \phi^2 \rangle_\gamma$$

The above solution is related to an implicit *local* probability distribution  $\wp_o(\phi)$  for the bond angular differences that presents a unique bump centered at  $\phi_o = 0$ . For a periodic potential the actual distribution should have a bump at each value  $\phi_n = 2\pi n$ . Let us modelise this distribution by a three-bump function defined by:

$$\wp(\phi) = (1 - p)\wp_o(\phi) + \frac{1}{2}p[\wp_o(\phi - 2\pi) + \wp_o(\phi + 2\pi)]$$

where  $p$  is the probability for the phase difference to be out of the interval  $[-\pi, \pi]$ . This new distribution provides us with a correction to the mean square fluctuation, which we now denote by  $\tilde{X}$ :

$$\tilde{X} = X + 4\pi^2 p$$

The improved effective coupling constant  $\Gamma$  will be:

$$\Gamma = e^{-\tilde{X}/2} = \gamma e^{-2\pi^2 p} .$$

The probability  $p$  can be evaluated self-consistently through the local effective Boltzmann factor as follows:

$$p = 1 - \frac{\int_{-\infty}^{\infty} d\phi \exp(-\Gamma\phi^2/2t)}{\int_{-\infty}^{\infty} d\phi \exp(-\Gamma\phi^2/2t)} = 1 - \Phi(\sqrt{\Gamma\pi^2/2t})$$

where  $\Phi$  is the error function (probability integral).

In Figs.(1) and (2) we illustrate, respectively, the general  $\Gamma(t)$  dependence (with and without vortex correction) and the critical behaviour  $\Gamma - \Gamma_c \sim \sqrt{1 - t/t_c}$ .

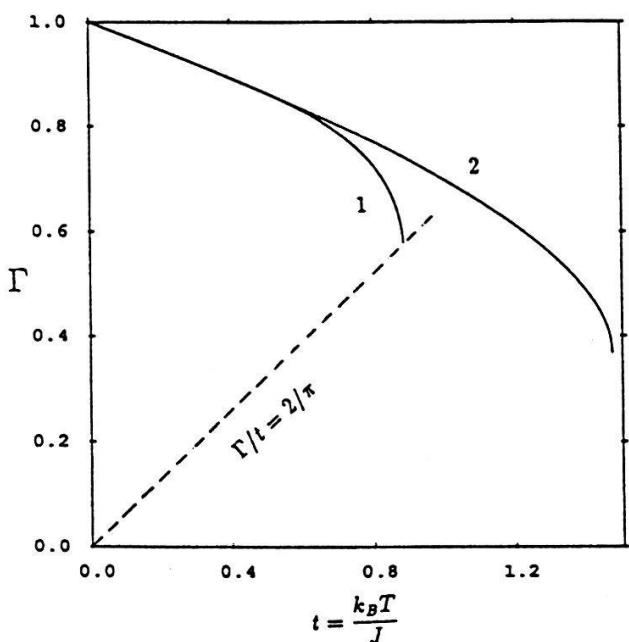


FIG.(1): Helicity modulus vs temperature with(1) and without(2) vortex correction.

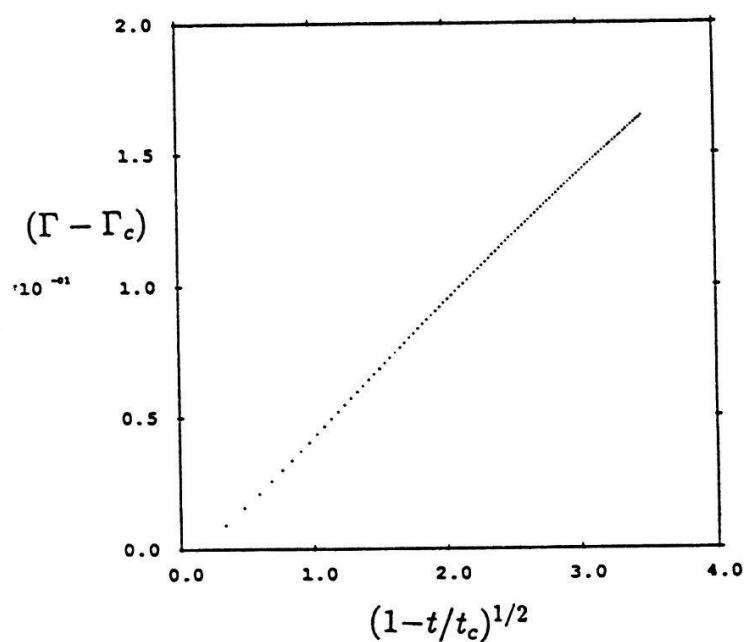


FIG.(2): Critical behaviour of  $\Gamma$  near the transition point.

This work was supported by the Swiss National Science Foundation.

## References

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