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CHAOTIC DYNAMICS OF FLUXONS IN LARGE-AREA JOSEPHSON JUNCTIONS

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Last years the investigation of the fluxon dynamics in the Josephson junctions attracts much interest of scientists. It is connected with a possibility to control the fluxon parameters with the help of external effects or the changes of junction parameters [1-3]. In this paper we consider the fluxon motion in large-area inhomogeneous Josephson junction at the presence of external variable current.

Let us consider a motion of the radial-symmetric vortex in inhomogeneous junction under the action of high frequency external force. In this case the equation for the phase difference between two superconducting films in standard dimensionless variables [1] takes the form

$$\phi_{tt} - \phi_{rr} + \sin\phi = f \sin \Omega t + 1/r \phi_r - \varepsilon \alpha(\phi, r). \quad (1)$$

where α describes of the inhomogeneity influence.

Following [4-5] let us consider the dynamics of kink-shaped bubble satisfying (1) in which the energy is concentrated in a single well. We assume that the radius of such a bubble is large enough to be considered as a perturbation. For $\varepsilon=0, f=0$, and $\rho \gg 1$, one obtains the fluxon solution in the form

$$\phi_0(z) = 4 \tanh^{-1} \exp\left(\frac{z}{\rho}\right), \quad z = \frac{r-\rho}{(1-v^2)^{1/2}}, \quad \rho = vt + \rho_0,$$

where v is fluxon velocity and ρ is the radius.

We apply perturbation theory to the fluxon dynamics by taking right hand side in (1) as a small perturbation. Then one obtains

$$\frac{dv}{dt} = \frac{1}{\rho} + \varepsilon \alpha(\rho) + \pi f / [8 \cdot (1-\Omega^2)] \cdot \sin \omega t, \quad (2)$$

$$\frac{d\rho}{dt} = v(t), \quad (3)$$

where

$$a(\rho) = \frac{1}{4} \int \kappa(z, \rho) \operatorname{sech}^4 z \, dz.$$

Hereby we came to nonrelativistic limit $v^2 \ll 1$ and take into account the renormalization of the external field following [2].

Let us investigate the case when $\varepsilon \gg f$, and $1/\rho \sim f$.

In this case the system (2),(3) can be studied with the account of radiality and external current as a perturbation. Let us consider the truncated system (2),(3)

$$dv/dt = \varepsilon a(\rho), \quad (2')$$

$$d\rho/dt = v. \quad (3')$$

Let us consider concrete models of the inhomogeneity

$$\kappa(r) = \delta(r - r_1) + \delta(r - r_2). \quad (4)$$

This is a model of the Josephson junction with micro short-circuit the regions of strong superconducting currents, the sizes of which are much less than Josephson depth of penetration λ_J . The separatrix solution of the system (2'-3') given in non explicit form, is found from the equation [6]

$$2 H_c(t - t_0) = \rho' - \frac{\alpha + y_0}{2(y_0^2 - 1)^{1/2}} \ln \left[\frac{y_0^{-1} + (y_0^2 - 1)^{1/2} \tanh \rho'}{y_0^{-1} - (y_0^2 - 1)^{1/2} \tanh \rho'} \right], \quad (5)$$

$$y_0 = \alpha - 2/\alpha, \quad \alpha = \cosh(r_2 - r_1), \quad \rho' = \rho - \frac{r_1 + r_2}{2},$$

$$H_c = \frac{\varepsilon}{4} \frac{\alpha^2}{(\alpha^2 - 1)}.$$

Taking into account the separatrix solution one can find the criteria of the chaotic motion near the separatrix with the help of the Melnikov method [7]. Let us calculate the Melnikov integral $D(t_0)$. If $\omega_1 \gg 1$, we obtain

$$D(t_0) \approx \varepsilon \cdot a \cdot \sin(k+L) \cdot \sin \omega t_0 / [2\omega_1 \cdot (1 - \Omega^2)] - \ln(r_1/r_2), \quad (6)$$

where $\omega_1 = \Omega / 2H_c$, $r_2 > r_1 \gg 1$, $L = r_2 - r_1$. For $1 \ll \rho_0$ we have

$$\ln(r_1/r_2) = 1/\rho_0.$$

It is known that if $D(t_0)$ changes a sign, then the separatrix is intersect and a motion becomes chaotic in this region. Having studied the result presented in formulae (6) we came to conclusion that in general case the fluxon dynamics in large-area Josephson junctions under the action of variable external field can have a stochastic character and the account of radial symmetry results in narrowing of the chaos region, i.e it acts as a dumping. This conclusion should be taken into account while designing the Josephson oscillators with variable frequency, in that case when we apply the alternating current to change the frequency.

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