

Zeitschrift: Helvetica Physica Acta
Band: 65 (1992)
Heft: 2-3

Artikel: Statistical properties of 2D periodic Lorentz gas with infinite horizon
Autor: Bleher, P.M.
DOI: <https://doi.org/10.5169/seals-116489>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 07.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

STATISTICAL PROPERTIES OF 2D PERIODIC LORENTZ GAS WITH INFINITE HORIZON

P.M. Bleher

Raymond and Beverly Sackler Faculty of Exact Sciences, School of Mathematical
Sciences, Tel Aviv University, Ramat Aviv 69978, Israel

Abstract. We show that in the periodic 2D Lorentz gas with infinite horizon the limit distribution of the particle displacement is Gaussian but the normalization factor is $\sqrt{t \log t}$ and not \sqrt{t} as in the classical case. We find explicitly the covariance matrix of the limit distribution.

The problem we are interested in can be described in the following way. Let a point particle move freely with the unit velocity in the plane outside of a periodic set of circles of a radius $a < 1/2$ centered at the sites of a square lattice with the unit space. It is assumed that the particle reflects elastically from any circle when it reaches the circle. Denote $\vec{x}(t)$ the position of the particle at the moment t . The main problem we are interested in is: Assuming that at $t = 0$ the position of the particle in the phase space is uniformly distributed with respect to the Liouville measure of the periodic problem, what is the right normalization $N(t)$ such that there exists a (non-trivial) limit distribution of $\frac{\vec{x}(t) - \vec{x}(0)}{N(t)}$ as $t \rightarrow \infty$ and what is the limit distribution?

Let t_n be the time of the n -th collision of the particle with the circles. A discrete version of the problem is: What is N_n such that there exists a limit distribution of $\frac{\vec{x}(t_n) - \vec{x}(0)}{N_n}$ as $n \rightarrow \infty$ and what is the limit distribution? As a generalization of the problem one may think of a particle moving outside of a periodic set of non-intersecting strictly convex scatterers with the condition of elastic reflection under collisions with the scatterers.

An important feature of the original problem with the square lattice of circular scatterers is that in that case the length of free motion of the particle between subsequent collisions is unbounded. Such a configuration of scatterers is called a configuration with an infinite horizon. If the length of the particle free motion is bounded by a constant it is called a configuration with a finite horizon.

The problem on the statistical behavior of $\vec{x}(t) - \vec{x}(0)$ and the related one on the asymptotics of the velocity autocorrelation function $\langle \vec{v}(0) \vec{v}(t) \rangle$ in the periodic Lorentz gas were studied both theoretically and numerically in works of Sinai; Bunimovich, Sinai; Bunimovich; Kramli, Szasz; Casati, Comparin, Guarneri; Machta; Machta, Zwanzig; Friedman, Oono, Kubo; Friedman, Martin; Bouchad, Le Doussal; Zacherl, Geisel, Nierwetberg; Bleher and others. In particular, it was proved in [1] that if a periodic configuration of scatterers has a finite horizon then the central limit theorem holds: The limit distribution of $\frac{\vec{x}(t) - \vec{x}(0)}{\sqrt{t}}$ is Gaussian.

We report here the following main result (see [2]). Let us consider a periodic configuration of strictly convex non-intersecting scatterers. Assume that it has an infinite

horizon. Let $\Omega_1, \dots, \Omega_N$ be basic scatterers which generate the whole periodic configuration by means of shifts in the periods of the lattice. Let us call a corridor an open infinite strip in the plane which has no intersection with the scatterers and which is tangent from the both sides to an infinite number of scatterers. Let us enumerate C_{k1}, \dots, C_{kN_k} all the corridors which touch the scatterer Ω_k , $k = 1, \dots, N$. Denote d_{kl} the width of the corridor C_{kl} and h_{kl} the period of the lattice in the direction of C_{kl} . Then we state that the limit in distribution,

$$\frac{\vec{x}(t_n) - \vec{x}(0)}{\sqrt{n \log n}} = \vec{\xi}$$

exists, where $\vec{\xi}$ is a Gaussian random vector variable with zero mean and the covariance matrix $(D_{lm})_{l,m=1,2}$, which can be calculated in the following way. Let $(D\vec{z}, \vec{z}) = \sum_{l,m=1,2} D_{lm} z_l z_m$. Then

$$(D\vec{z}, \vec{z}) = \frac{1}{2 \sum_{k=1}^N |\partial\Omega_k|} \sum_{k=1}^N \sum_{l=1}^{N_k} h_{kl} d_{kl}^2 (\vec{\omega}_{kl}, \vec{z})^2 \quad (1)$$

where $|\partial\Omega_k|$ is the length of the boundary of the scatterer Ω_k and $\vec{\omega}_{kl}$ is the unit vector parallel to the boundary of C_{kl} . A related formula is valid for the continuous time dynamics:

$$\lim_{t \rightarrow \infty} \frac{\vec{x}(t) - \vec{x}(0)}{\sqrt{t \log t}} = \frac{\vec{\xi}}{\sqrt{\tau}}$$

where τ is the mean time of free motion. It is noteworthy that the covariance matrix D in (1) is positive if there exist two non-parallel corridors.

For the case of the square (or triangular) lattice of circular scatterers the formula (1) can be further specified. Namely, in that case D is proportional to the unit matrix, $D_{lm} = \frac{D_0}{2} \delta_{lm}$, and for small radius a we have that

$$D_0 = \frac{1}{8\pi^2 a^2} (1 + O(a^{1/4}))$$

for the square lattice and

$$D_0 = \frac{25\sqrt{3} - 36}{64\pi^2 a^2} (1 + O(a^{1/4}))$$

for the triangular lattice.

References

- [1] Bunimovich, L.A., Sinai, Ya.G.: Commun. Math. Phys. **78**, 247–280 (1981)
- [2] Bleher, P.M.: to appear in Journ. Statist. Phys.