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## Low temperature properties of two dimensional frustrated quantum antiferromagnets

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**Abstract.** We extend Chakravarty, Halperin and Nelson's analysis of low temperature  $2D$  quantum antiferromagnets to systems which have classical canted ground states, such as helical or triangular lattice systems. In particular, we calculate the quantum correlation length  $\xi$  at two loop order and argue that the measure of  $\xi$  could be a test of the existence of long range order in these models.

Partly motivated by the study of superconductivity, many investigations have been recently devoted to the nature of the order in  $D = 2$  quantum frustrated antiferromagnets. Most of these works assume a disordered phase at  $T = 0$  and propose the existence of non trivial ground states. However, numerical simulations as well as analytical studies suggest that Néel order persists, even for spin  $1/2$ , in many canted systems[1,2,3]. We propose to test the nature of the order at  $T = 0$  in frustrated canted systems by measuring the correlation length at low temperature. The comparison between experimental results and a theoretical calculation based upon the assumption of Néel order at low temperature, was already successfully performed by Chakravarty, Halperin and Nelson for collinear magnets[4]. We have calculated the quantum correlation length at two loop order for a wide class of  $2D$  canted systems having zero net magnetization, such as the triangular and helical models. In contrast with the collinear case, the  $O(3)$  rotation group is completely broken in the classical ground state of canted systems so that we expect three spin-waves in the quantum case, if the long range order is not destabilized by the quantum fluctuations. The hydrodynamical theory describing the long distance physics of frustrated systems with zero net magnetization, predicts a linear spectrum with a priori different velocities for these spin-waves[5]. The simplest model that reproduces the spectrum and the interaction among spin waves is the quantum non-linear sigma ( $NL\sigma$ ) model. When two spin-waves have the same velocity (triangular, helical models, etc...), the symmetry is not only  $O(3)$ , but  $O(3) \otimes O(2)$ , where the  $O(2)$  indicates that two spin-waves are equivalent[6]. The quantum  $NL\sigma$  model is thus  $O(3) \otimes O(2)/O(2)$  and its action is[7]:

$$S = -\frac{1}{2} \int_0^\beta d\tau \int d^2x \left[ \text{Tr} \left( P_0 \left( g^{-1} \partial_0 g \right)^2 + P_\perp \left( g^{-1} \partial_i g \right)^2 \right) \right] \quad (1)$$

where  $g(x, \tau) \in SO(3)$ ,  $i = 1, 2$ ,  $\beta$  is the inverse temperature and  $P_0, P_\perp$  are two diagonal matrices containing the 4 coupling constants, i.e. the 2 spin-waves velocities and the 2 spin-stiffnesses. Since the three spin-waves do not have the same velocities, i.e.  $P_0$  is not proportional to  $P_\perp$ , the model is not Lorentz invariant. Nevertheless, we have been able to obtain the one loop recursion relations for  $\beta$  and the 4 parameters in eq.(1)[6]. These equations show that a non trivial, Lorentz and  $O(3) \otimes O(3)/O(3) = O(4)/O(3)$  symmetric fixed point exists, at  $T = 0$ . The hypersurface associated to this fixed point divides the coupling constant space into a disordered phase for small  $S$  and a Néel ordered phase for large  $S$ . There also exists a whole fixed hypersurface at  $T = 0$  and

$S = \infty$ . This surface controls the long distance physics of the spin-waves in the Néel phase since the renormalization group flow drives the system towards it. After a sufficient number of iterations,  $e^{l^*} \sim T^{-1}$ , the quantum fluctuations become negligible. We end up with an effective quasi classical  $2D$ ,  $O(3) \otimes O(2)/O(2)$  NL $\sigma$  model with effective parameters  $P_0(l^*)$ ,  $P_\perp(l^*)$ . The whole effect of quantum fluctuations is to renormalize the coupling constants. The quantum correlation length can then be calculated from the classical one, in which  $P_0, P_\perp$  are replaced by  $P_0(l^*)$  and  $P_\perp(l^*)$ . We have carefully calculated at one loop order the effective parameters  $P_0(l^*)$ ,  $P_\perp(l^*)$  by integrating the quantum fluctuations in eq.(1). We have also calculated at two loop order the classical correlation length from which follows the quantum one. We find finally, for the quantum correlation length in the Pauli-Villars regularization scheme[6a]:

$$\xi = C_\xi^{PV} \frac{\hbar c_1}{k_B T} \sqrt{\frac{2(1-\alpha)}{\rho_1}} \exp \left( \left[ (1-\alpha)G(\alpha) - G(0) \right] \left[ -\frac{1}{8} - \frac{1-\alpha}{16\alpha} \left( 1 + \frac{\log(c_3/c_1)^2}{1-(c_3/c_1)^2} \right) \right] \right) \exp \left( 2\pi\rho_1 G(\alpha)/2k_B T \right) \quad (2)$$

where:

$$\begin{aligned} G(\alpha) &= 2 + 2(1-\alpha)(\arg \tanh \sqrt{\alpha})/\sqrt{\alpha} \quad ; \quad 0 \leq \alpha < 1 \\ G(\alpha) &= 2 + 2(1-\alpha)(\arctan \sqrt{-\alpha})/\sqrt{-\alpha} \quad ; \quad \alpha \leq 1 \end{aligned} \quad (3)$$

and where  $C_\xi^{PV}$  is a constant,  $\rho_1$  and  $\rho_3$  are spin-stiffnesses,  $\alpha = 1 - \rho_3/\rho_1$  and  $c_1$  and  $c_3$  are spin-wave velocities. This is the general expression for the correlation length of frustrated quantum antiferromagnets with zero net magnetization, when two spin-waves have the same velocity. It contains no adjustable parameter. As argued by Chakravarty et al.[4], the constants  $c_1, c_3, \rho_1$  and  $\alpha$  are phenomenological input parameters that can be obtained, for instance, by a spin-wave calculation. Let us finally emphasize that our expression for  $\xi$  can be, in principle, experimentally tested on  $^3\text{He}$  adsorbed on graphite substrate[8], since this system is thought to be modeled by a Heisenberg  $S = 1/2$  model on the triangular lattice, with interactions either ferromagnetic or antiferromagnetic, depending on coverage.

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