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Critical Behaviour at the Metal-Insulator Transition in 2D Systems with Spin Orbit Scattering

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Abstract

In this paper it is argued that the ideal systems for studying numerically various phenomena associated with the disorder induced metal-insulator transition (MIT) are 2D ones with symplectic symmetry. A few results are summarized.

Introduction. In spite of its being the subject of intense experimental, numerical and purely theoretical investigations for over a decade, the nature of the MIT is still far from being understood. Even fundamental questions such as those of the type of transition as well as universality and the numerical values of the critical indices remain unresolved, or are at least still controversial. Numerical calculations are usually performed for 3D systems which require enormous computer effort since the size of the matrices involved grows exponentially with the spatial dimension. Therefore reduction of the spatial dimensionality is strongly desired for numerical purposes, but the only 2D symmetry class showing extended metallic and localized phases in the independent electron approximation is the symplectic one.

A lattice model. We study a model recently proposed by Ando to simulate n-channel inversion layers on the surface of III-IV compound semiconductors[1]. It consists of a layer of p-type orbitals and a second one of s-type orbitals with energies $E_s > E_p$. Hopping from one s-orbital to the next is only possible via p-orbitals. Thus, the model takes on the following form:

$$H = \sum_{j\sigma} \varepsilon_j c^\dagger_{j\sigma} c_{j\sigma} + \sum_{ij\sigma\sigma'} V^{\sigma\sigma'}_{ij} c^\dagger_{i\sigma} c_{j\sigma'}, \tag{1}$$

the matrix V being time reversal invariant: $V^{\sigma\sigma'}_{ij} = \sigma\sigma' V^{-\sigma'-\sigma}_{ji}$. The numerical tools employed are the transfer matrix method and finite size scaling. The data analysis so far focused on...

Universality[2]. It was possible to reduce size effects by examining systems 7 times wider than the largest ones studied in 3D up to now. Critical exponents of the localization length have been determined for 5 different distribution functions for the diagonal disorder ε_j (box, Gaussian, triangle, Cauchy, 2-level). All of these agree within error but disagree considerably from the results of previous works[1,3,4] for different reasons. (Ando uses an inexact value for the critical disorder and an unstable fit method, MacKinnon's model belongs to a sublattice symmetry class, and Zirnbauer uses an uncontrollable approximation.) The value we propose for the critical exponent of the localization length of 2D symplectic Anderson models with diagonal disorder is $\nu = 2.75 \pm 0.15$.

Phase diagrams and anisotropy effects. Starting from the extended phase of our model, a MIT can be triggered by varying the disorder W, the energy E, the ratio of transversal and longitudinal

hopping integrals $\frac{J_\perp}{J_\parallel}$, or the spin-orbit coupling strength. The complete phase diagrams have been obtained (a) in the $E-W$ plane, showing that the somewhat puzzling reentrant behaviour observed in 3D does not occur in 2D; (b) in the $S-W$ plane, suggesting that the MIT occurs for arbitrarily weak SO-coupling, with a mobility edge at $W_c \propto S_c^{1/4}$; (c) in the $W - J_\perp$ plane. Unexpectedly, in the latter case the conductance calculated from Landauer's formula first *increases* with decreasing transversal coupling. Although difficult, this behaviour deserves to be tested experimentally. There is no evidence for a second scaling parameter[5].

Spectral properties. The assumption of spectral rigidity is the essential ingredient when calculating the conductivity from the density of lyapunov exponents $\rho(\gamma)$ in a random matrix model[6]. It has now explicitly been demonstrated that (a) after spectral unfolding the level repulsion of lyapunov exponents tends to infinity even close to the transition; (b) $\rho(\gamma \to 0)$ vanishes at the critical point as γ^{D-2} in 3D, as $\gamma^{1/r}$ in 2D and develops a gap at $\gamma = 0$ in the localized phase.

Conductivity. Confirming the relation $s = (D-2)\nu$, where $\sigma = \mid W - W_c \mid^s$, a minimum metallic conductivity has been found quite close to the Mott value of $\frac{1}{2}\frac{e^2}{h}$: $\sigma_{\min} = (1.4 \pm 0.1)\frac{e^2}{h}$.

A continuum model. Some aspects of the problem are easier to analyze using a model of free electrons scattering off localized impurities. We start from the model

$$H = \frac{p^2}{2m} + V(\vec{r}) - \vec{\sigma} \cdot (\nabla U_{SO} \times \vec{p}), \quad V(\vec{r}) = \sum_a \lambda_a \delta(\vec{r} - \vec{R}_a), \tag{2}$$

characterized by a microscopic spin flip length $\ell_{SO} \propto \mid \nabla U_{SO} \mid^{-1}$. The Hilbert space is discretized using plane waves satisfying $k_{\max}^{-1} < \ell_{SO} \ll k_{\min}^{-1}$. After choosing the impurity strengths and positions at random, the Hamiltonian is diagonalized exactly. Eigenvalues and eigenvectors serve as the raw data for...

Multifractal analysis. At the MIT, the multifractal language[7] is consistently applicable to the system. Inverse participation ratios, generalized dimensions, $f(\alpha)$ spectra and scaling exponents have been calculated checking the underlying assumptions of the scaling hypothesis. Anomalous diffusion has been observed. Close to the band edges, one parameter scaling fails.

Level statistics. 1D and 2D Anderson models have recently been investigated as examples for quantum chaos having a continuous set of eigenvalues as opposed to the kicked quantum rotator, but all indications for level repulsion have been traced back to finite size effects[8], since orthogonal Anderson models are always localized in $D \leq 2$, whereas the extended parts of the spectrum of our symplectic model exhibit level repulsion over a wide range of system parameters. The repulsion exponent is 2 instead of 4 because, apart from time reversal invariance, additional constraints have been put on the Hamiltonian.

References

1. T. Ando, *Phys. Rev. B* **40** (1989), 5325.
2. U. Fastenrath, G. Adams, R. Bundschuh, T. Hermes, B. Raab, I. Schlosser, T. Wehner & T. Wichmann, *Physica A* **172** (1991), 302.
3. A. MacKinnon, in *Localization and Confinement of Electrons in Semiconductors, Springer Series in Solid State Sciences, Vol. 97*, F. Kuchar, ed., 1990.
4. M. R. Zirnbauer, *4. DFG-Rundgespräch über den Quanten-Hall-Effekt*, Schleching (1989).
5. A. Kawabata, *Physica A* **167** (1990), 279.
6. J. T. Chalker & M. Bernhardt, "The Transfer Matrix Approach to Anderson Localization", *5. DFG-Rundgespräch über den Quanten-Hall-Effekt*, Schleching (1991).
7. W. Pook & M. Janßen, *Z. Phys. B* **82** (1991), 295.
8. M. P. Sörensen & T. Schneider, *Z. Phys. B* **82** (1991), 115.