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Corrections to Fermi Liquid Behaviour in D=2: DOS

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Abstract: Fermi liquid theory (FLT) relies on specific expansion parameters. It breaks down in low dimensions and if one or more expansion parameters fail to be small. We calculate leading order corrections to FLT, in particular the density of states (DOS), with the aim of understanding recently observed anomalies in high T_c superconductors.

The most successful microscopic theory of superconductivity is “fermi liquid superconductivity” [1]. It covers traditional weak-coupling superconductivity (Al, Nb,...), superconductivity of metals with strong electron-phonon interaction (Hg, Pb,...), strongly correlated electrons (heavy-fermion superconductivity,...), systems with unconventional pairing (superfluid ^3He ,...), disordered metals (amorphous superconductors,...), metals with complex band structures (A15 superconductors,...), etc. The central equation of fermi liquid theory (FLT) is Landau’s quasiclassical transport equation for quasiparticle excitations, generalized to the superconducting state [2]:

$$[\epsilon\hat{\tau}_3 - \hat{\sigma}_c - \hat{\sigma}_{ph} - \hat{\sigma}_{imp} - \hat{v}_{ext}, \hat{g}]_{\otimes} + i\vec{v}_F \cdot \vec{\nabla} \hat{g} = 0. \quad (1)$$

The transport equation of FLT leads to various types of low-frequency collective modes (sound, spin waves, pair-fluctuations, etc.). They appear as non-equilibrium solutions of eq.(1) but do not affect equilibrium properties of the fermi liquid in leading order in the expansion parameters of FLT, which include, for example, the ratio k_F^{-1}/ξ_0 of the fermi wavelength and the coherence length. The quality of the leading order FLT depends significantly on the dimension d of the system. Corrections to FLT related to collective modes carry an additional factor $(k_F^{-1}/\xi_0)^{d-1}$. Hence, FLT is inconsistent in $d=1$ (neglected corrections are of the same order as the terms that are kept!), but a good first approximation in $d=2,3$ provided the expansion parameters are, indeed, small. Clearly, these corrections are more important in $d=2$ than $d=3$.

There is not yet a generally accepted interpretation of the anomalous physical properties of high- T_c superconductors. We are interested in studying how far these properties can be understood in terms of FLT with leading corrections included. The short coherence length and two-dimensionality of high- T_c superconductors should make these corrections important. A classification in orders of k_F^{-1}/ξ_0 is done most easily by analyzing the diagram expansion for the generating functional Φ [3] of interacting electrons in a solid. FLT is generated by the diagrams of order $(k_F^{-1}/\xi_0)^2$ shown in Fig.1a-c. We are specifically interested in corrections to FLT coming from the coupling of quasiparticles to low q modes. These leading corrections to Φ are of order $(k_F^{-1}/\xi_0)^{1+d}$, and are given by the ring diagrams [4] shown in Fig.1d.



Figure 1: Leading order diagrams of fermi liquid theory (FLT), and first corrections. Leading order terms describe electron-electron interactions (a), electron-phonon interactions (b), and impurity scattering (c). Diagrams (d) give the first corrections to FLT in $d=2$. The vertex in (d) stands for any of the basic interaction vertices in (a), (b) and (c).

In our explicit calculations we concentrate on a clean system with strong electron-phonon interaction (Migdal-Eliashberg model [5]). Here, eq.(1) holds because of Migdal’s theorem. We go

beyond Migdal's class of processes, and discuss the leading corrections to FLT coming from a coupling to low- q particle-particle modes. Above T_c these effects are traditionally interpreted as consequences of "fluctuating Cooper pairs", and have been studied extensively in the region $|T - T_c| \ll T_c$ of enhanced fluctuations [6]. We consider the wider range $|T - T_c| \approx T_c$, and thus include also non-enhanced effects which would not be important in traditional fluctuation theory. They gain importance with increasing expansion parameter k_F^{-1}/ξ_0 which is the same as Migdal's parameter Θ_D/T_F for $T_c \approx \Theta_D$.

A specific quantity of interest is the "tunnelling DOS", $N(\epsilon) \sim \int d^2k \text{Im}G(k, \epsilon)$. It is energy independent, $N(\epsilon) = N_0$, in FLT. The leading energy dependent corrections have the form $\delta N(\epsilon, T)/N_0 = k_F^{-1}/\xi_0 \nu(\epsilon, T)$, and can be calculated from corrections to the electron Green's function generated by the ring diagrams of Fig.1d. Typical results for ν are shown in Fig.2. They are obtained by a direct numerical evaluation of diagrams. The integrations and summations are performed at imaginary Matsubara frequencies, and the analytic continuation is done numerically by Padé approximants.

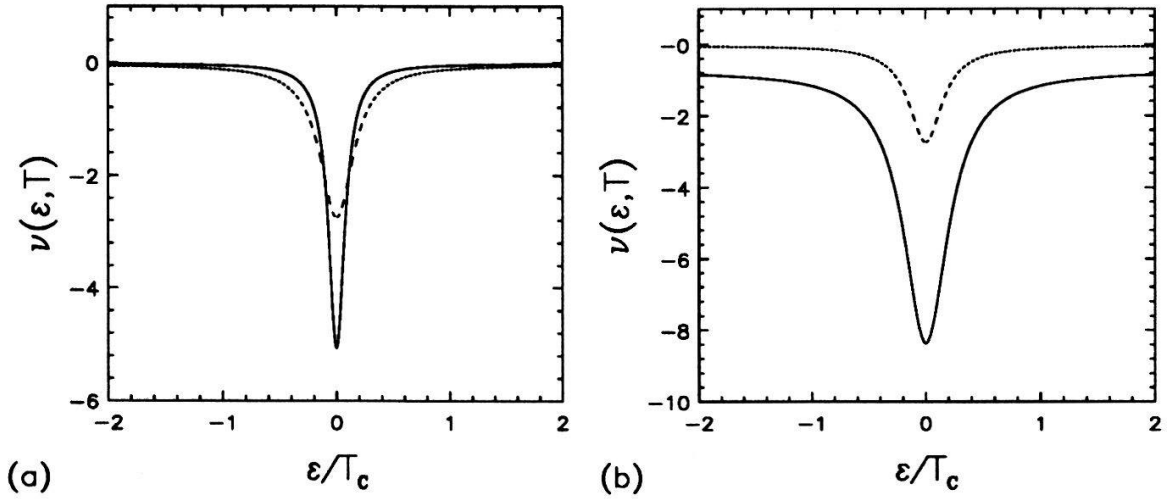


Figure 2: Leading corrections to Migdal's theory. The correction $\nu(\epsilon, T)$ to the DOS is calculated for Einstein spectra with coupling constants $\lambda_1=1.6$ ($T_c/\Theta_D=0.18$), and $\lambda_2=3.2$ ($T_c/\Theta_D=0.29$). Results are presented for [λ_1 , $T/T_c=1.5$, solid line in (a)], [λ_1 , $T/T_c=1.8$, dashed lines in (a) and (b)], and [λ_2 , $T/T_c=1.8$, solid line in (b)].

The tunnelling density of states is a specific example of a measurable quantity which is featureless in standard FLT, but develops pronounced temperature and energy dependences (a narrow "dip" near E_F) if leading corrections to FLT are taken into account. We speculate that this kind of anomalous corrections might explain deviations from FLT observed in high T_c superconductors. An analysis of leading corrections in T_1T and $\rho(T, \omega)$, and a detailed comparison with experimental data will probably verify or falsify our conjecture in the near future.

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