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# Flux creep: the stochastic approach

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**Abstract.** Considering the magnetic and structural fluctuations in superconductors as a stochastic force acting over the flux lines and making use of the theory of stochastic processes some features of the 2-D motion of fluxons in superconductors are derived. Some new parameters are defined and their main characteristics given. Finally a physical discussion of the method is presented.

## Introduction

There are many papers devoted to the again popular flux creep models first pioneering papers of which were the papers by Anderson [1] and Kim *et al.* [2] but surprisingly none of them (to the best of our knowledge) have explored sufficiently the clear stochastic characteristic that should be present in the two dimensional motion of fluxons in superconductors. This work is devoted to such a kind of treatment.

## Theory

Let us assume that the motion equation that the flux lines obey in the superconductor is:

$$m\ddot{x}(t) + \gamma\dot{x}(t) = \alpha \quad (1)$$

where  $m$  is an effective mass,  $\gamma$  is the linear viscosity coefficient for the motion of fluxons in the superconductor,  $x$  the coordinate of the flux tube, the dot means derivation respect the time  $t$  and  $\alpha$  is a constant that later will be equated to zero.

The solution of (1) with initial conditions  $x(0) = x_0$  and  $\dot{x}(0) = 0$  that is a flux tube initially at rest (for example in a pinning state) is:

$$x(t) = \frac{\alpha}{\gamma} t + \frac{\alpha m}{\gamma^2} \left( \exp\left(-\frac{\gamma t}{m}\right) - 1 \right) + x_0 \quad (2)$$

Taking into account that the solution of (1) coincides by definition with the centre of the fluxon assumed to be cylindrically symmetric we can use the equation describing the dependence of the mean square deviation of coordinates on time [3]:

$$\overline{(x^t - x^0)^2} = 2\theta \left( \frac{\partial x_\alpha}{\partial \alpha} \right)_{\alpha=0} \quad (3)$$

where  $\theta = kT$ .

Using (3) in (2) and with the fact that the important time  $\tau$  is in which the fluxon is out of a pinning well that must be greater than the relaxation time  $m/\gamma$  we obtain after a naive deduction:

$$\overline{(x - x^0)\tau^{-1}} = 2 \frac{\theta}{\gamma} \quad (4)$$

## Results and discussion

The left hand side of equation (4) can be interpreted as the product of a “mean velocity” and a “mean free path”  $v$  and  $l$  respectively, this is

$$vl = 2 \frac{\theta}{\gamma} \quad (5)$$

what makes possible to relate equation (5) to the Einstein relation for the diffusion processes.

When the external magnetic field is not too close to  $B_{c1}$  the Lorentz force causes the vortex lattice to drift with a velocity

$$V = P/n \quad (6)$$

where  $n$  is the vortex lattice viscosity that in some way must be related to the previously used  $\gamma$  and  $P = J \times B$  for perpendicular current  $J$  and external magnetic field  $B$ . In a first crude approximation the mean velocity of a single flux line and the drift velocity of the flux line lattice will be considered the same and equating equations (5) and (6) the following expression for the current as a function of the external magnetic field is obtained:

$$J = (2n\theta/(l\gamma))/B \quad (7)$$

result that strongly remember the classical Bean's assumption [4] (actually is not Bean's but Kim *et al.*'s [2]).

From (7) considering the two viscosity coefficients to be proportional (which is not too difficult to imagine) is possible taking into account the fact that the critical current decrease with increasing the temperature a limit dependence  $I(T) \sim T$  for the mean free path of fluxons.

In the previous discussion almost no mention has been done to the possibility of trapping centers although is a well established fact its existence. The equation that the “particle” density obey when the particles are diffusing in a medium with trapping centers is [5]:

$$\frac{\partial P(x, t)}{\partial t} = D \nabla_d^2 P(x, t) - \lambda V(x) P(x, t) \quad (8)$$

where  $d$  is the dimension,  $\nabla_d^2$  the laplacian,  $D$  the diffusion constant and  $V(x)$  the random trapping potential.

The possibility of deppining must add another term to the right hand side of equation (8) but this must not affect the non self averaging behavior in the

asymptotic regime [5] and such a situation can induce phase transitions opening a possible way to explain the now popular flux line lattice melting and related processes.

## Conclusions

The possibility of a stochastic approach to flux creep in type II superconductors appears to be a viable one although comparison with experiments should have the last word. The results presented above were found in the simplest possible way and as a consequence of this is that there are not quantitative but only qualitative results. Further and deeper approach is necessary: it should be the subject of a next work.

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