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## VORTEX DYNAMICS IN SUPERCONDUCTING FRACTAL NETWORKS

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**Abstract.** We report experiments measuring the ac response of a Sierpinski Gasket (SG) of Al wires to a weak driving ac field, as a function of dc magnetic field and driving frequency. Genuine fractal behavior becomes particularly manifest in the network's ac magnetoconductance, where a very rich fine structure reflecting flux quantization phenomena related to the multiconnected self-similar geometry of the gasket is observed. The analysis of the frequency dependence of the fine structure suggests that thermally activated fluctuations have to be taken into account in order to explain the dynamical properties of the system.

Current interests in superconductivity concentrate on granular and percolating materials. The Sierpinski Gasket (SG) is a system originally proposed by Gefen et al. [1] to model the geometrical features of the percolating cluster's backbone.

So far, experimental studies focused on the  $T_c(H)$  phase boundary [2], which was found to agree well with spectrum calculations of the linearized Ginzburg-Landau (GL) theory [3].

Another approach based on the non-linear GL equation in the London limit ( $|\psi| = cst$ ) was made by Alexander and Halevi (AH) [4]. Using the SG scaling properties, they expressed the magnetic energy of an  $n^{th}$  order gasket for different symmetrical vortex configurations (lowest energy solutions [5]). A major result of this study is that for a given vortex configuration, the dominant energy contribution comes from loops surrounding one flux quantum.

In a Josephson junction model, it can also be shown [6] that, due to the loops of various sizes forming the gasket, the pinning potential experienced by a vortex is highly hierarchical. This idea, combined with the AH result, makes the SG very attractive for studying vortex dynamics.

Using an inductive technique, we measured the complex ac response of a network of interconnected 6<sup>th</sup> order SG as a function of frustration  $f$ , expressing the number of flux quanta per elementary triangular cell of the SG, over a wide range of driving frequencies  $\omega/2\pi$  (100Hz - 10MHz). We extracted the array's inverse kinetic inductance  $L_k^{-1}$  with a numerical method [7].

The sample was photolithographically patterned from a 1000Å thick Al film. Length and width of the elementary links were  $a = 4.9\mu m$  and  $w = 1\mu m$ . All the measurements presented here were performed 7mK below the network's mean-field transition temperature  $T_c(0) = 1.718K$ , a region where the wire network can be considered a Josephson junction array (JJA) [8] and where vortex pinning is sufficiently weak [8] to observe the oscillations in  $L_k^{-1}$  shown in Fig. 1. As expected, the data are periodic (with period 1) and even in the frustration  $f$ . The fine structures reflect the change of phase coherence through the array due to flux redistribution in the different loops of the SG. This idea is illustrated by the prediction of the AH theory [4].

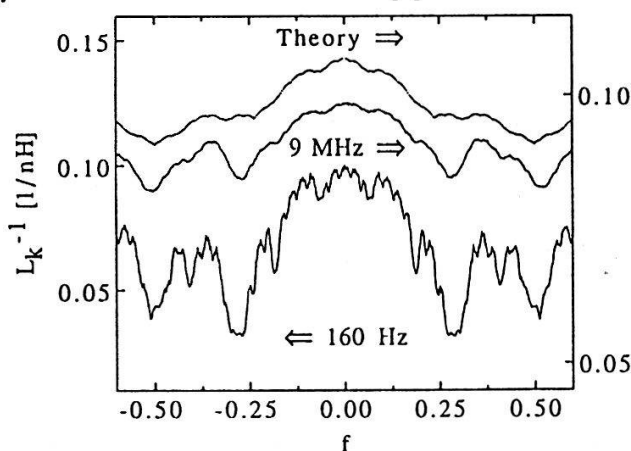


Figure 1: Inverse kinetic inductance at two different frequencies and prediction of the AH theory (slightly shifted for clarity) as a function of frustration.

We recall that in a JJA with sinusoidal current-phase relation, if we neglect pinning and fluctuations, the non-renormalized inverse kinetic inductance  $L_{ko}^{-1}$  is proportional to the second derivative of the array's ground-state energy with respect to the vector potential. Using the AH recursive relation for the SG ground-state energy [4] and relying on a quadratic expansion of the cosine, we obtain the following expression for the frustration induced change of the inverse kinetic inductance,  $\Delta L_{ko}^{-1}(T, f) = L_{ko}^{-1}(0) - L_{ko}^{-1}(f)$

$$\Delta L_{ko}^{-1}(T, f) = L_{ko}^{-1}(T, 0)(4\pi^2/18)[f^2 + (1/2) \sum_{h=0}^{n-1} (2^{2h+1} f - p_h)^2 / 5^h] \quad (1)$$

where the sum runs over the  $n$  families of loops forming the  $n^{th}$  order gasket. The integers  $p_h$  describe the vortex occupation and are chosen to minimize the energy.

An example for a  $2^{nd}$  order gasket is shown in Fig. 2. Notice that this curve has to be imagined even in  $f$ , and symmetric with respect to  $f=1/2$ .

In order to compare this prediction with the experimental results, we made a calculation for a  $3^{rd}$  order gasket which, as shown in Fig. 1, displays approximately the same structure as the 9 MHz measurement. At low frequency, we also see that the fine structure is much sharper and richer. The features of the 160 Hz curve cannot be reproduced with this simple fluctuation-free model, even with calculations of higher order gaskets. This observation, supported by the facts that the oscillations were observed only in a very narrow region ( $\sim 15mK$ ) close to  $T_c(0)$  and by a strong temperature dependence of the structure, suggests that thermally activated fluctuations play a dominant role. Other arguments supporting this idea will be presented elsewhere [6].

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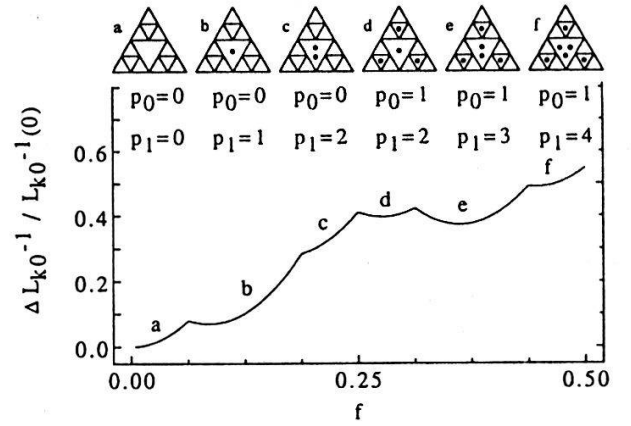


Figure 2: Relative change of kinetic inductance and corresponding vortex configurations for a  $2^{nd}$  order gasket according to the AH theory.

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