

**Zeitschrift:** Helvetica Physica Acta  
**Band:** 65 (1992)  
**Heft:** 2-3

**Artikel:** Charge dynamics in junction arrays  
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**DOI:** <https://doi.org/10.5169/seals-116444>

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## Charge Dynamics in Junction Arrays

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**abstract:** 2D Junction arrays with nearest neighbour capacitance larger than the self capacitance are a physical realization of the 2D Coulomb gas. One therefore expects a Kosterlitz-Thouless-Berezinskii (KTB) transition, where charge dipoles dissociate. We performed simulations of the charge dynamics in such arrays, based on tunnel rates for single electrons and Cooper pairs. The resulting I-V curves clearly show the Coulomb gap at low temperatures, while at higher temperatures one finds effects of the KTB transition.

2D arrays of Josephson junctions have long been studied as a realization of the *XY*-model, which can be mapped onto a 2D Coulomb gas (CG) of vortices. Nowadays it is possible to fabricate arrays in which the charging energy  $E_C$  exceeds the Josephson coupling  $E_J$  [1]. The electrostatic energy  $E_{elst}$  of a configuration of charges  $Q_i$  on the  $i^{th}$  island is:  $E_{elst} = \frac{1}{2} \sum_{i,j} Q_i (C^{-1})_{i,j} Q_j$ . If the nearest neighbour capacitance  $C$  is much larger than the self capacitance  $C_0$ , the inverse capacitance matrix  $C_{i,j}^{-1}$  behaves logarithmically up to a scale set by the screening length  $\Lambda = \sqrt{C/C_0}$ . Therefore the electric charges in these arrays provide a *direct* physical realization of a 2D CG.

The dynamics, incoherent tunneling of charges between neighbouring islands  $i$  and  $j$ , is governed by tunnel rates. For quasi-particles (QP) and Cooper pairs (CP) they are [2]:

$$\begin{aligned} \Gamma_{QP}^{ij} &= (\Delta E_{ij}/e^2 \hbar R_t) [\exp(\Delta E_{ij}/k_b T) - 1]^{-1} \\ \Gamma_{CP}^{ij} &= \frac{\pi E_J^2}{4\alpha_s \hbar \Delta E_{ij}} \frac{1 - \exp(-\Delta E_{ij}/k_b T)}{(\pi/\alpha_s)^2 + \sinh^2(\frac{1}{2}\Delta E_{ij}/k_b T)}, \text{ for } \alpha_s \gg 1, \end{aligned} \quad (1)$$

where  $\alpha_s = R_q/R_s$  measures the effective shunt resistance  $R_s$  seen by the junction ( $\alpha_s \gg 1$  for a generic array),  $\alpha_t = R_q/R_t$  measures the tunnel resistance  $R_t$  and  $R_q = \frac{\hbar}{4e^2}$  is the quantum resistance.  $\Delta E_{ij}$  is the difference in energy of the *whole* array before and after a tunnel event (global rule). Tunneling of a charge in the direction of the voltage drop will generally cost energy if the external voltage is lower than the Coulomb gap voltage  $V_g \sim N_s \frac{e}{C}$  ( $N_s$  is the number of junctions in series)[3]. At low temperatures the rates (1) vanish for  $\Delta E_{ij} > 0$  and for  $V \leq V_g$  no current will flow, i.e. Coulomb blockade. At higher voltages the rate  $\Gamma_{QP}$  leads to diffusive motion for QP; the rate  $\Gamma_{CP}$  is peaked at  $\Delta E_{ij} = 0$ , reflecting the resonant nature of CP tunneling for  $\alpha_s \gg 1$ .

As the system is a 2D CG, there is a charge unbinding transition for QP at  $k_b T_c = \frac{E_C}{4\pi\epsilon}$  ( $k_b T_c = \frac{E_C}{\pi\epsilon}$  for CP), where  $\epsilon$  is a dielectric constant slightly larger than one and  $E_C = e^2/2C$ . For  $T < T_c$  the conductance is nonlinear for small voltages, i.e.  $I \sim V^a(T)$  [4]. At the transition the exponent  $a$  jumps from 3 to 1. For  $T > T_c$  conductance is linear, i.e.  $I \sim V$ , and proportional to the density  $n$  of free (unpaired) charges, which follows a square root cusp relation:  $n \sim \exp\{-b(T/T_c - 1)^{-\frac{1}{2}}\}$ ;  $b$  is a constant of order unity.

Real time simulations based on the rates (1) yield I-V curves for QP and CP as shown in figs. 1a and 2. For low temperatures a Coulomb gap indeed develops; fig. 2 shows the resonant structure for CP. The presence of a fixed thermal distribution of QP on the islands weakens the Coulomb gap for CP (fig. 2b). The density of charges  $n$  follows the theoretical prediction (fig. 1b) if the applied voltage is low. Since a finite voltage  $V_c = \frac{e}{\pi C}$  is needed to separate pairs with the largest separation (the system size), there is only a restricted range of voltages where the jump in the

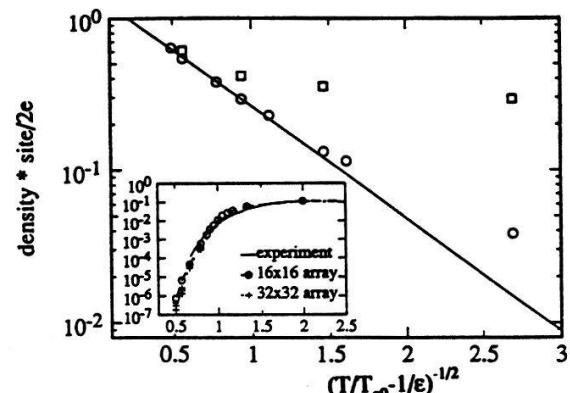
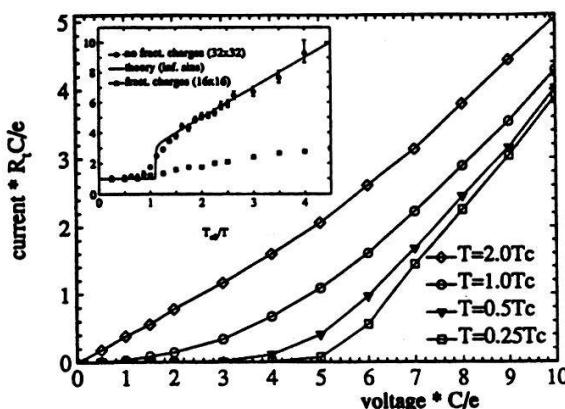


Figure 1: a: I-V curves for QP in a 32x32 array, the inset shows the exponent  $a$  in  $I \sim V^a$  vs. inverse temperature. b: Charge density for  $T > T_c$  vs. temperature for CP in 16x16 array,  $\alpha_s = 25$ , 'boxes': same with fixed QP background; the inset shows conductance  $\times R_t$  vs. temperature  $/\epsilon T_c$  for CP.

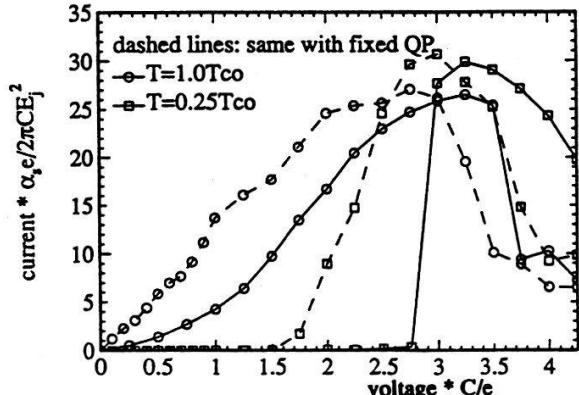
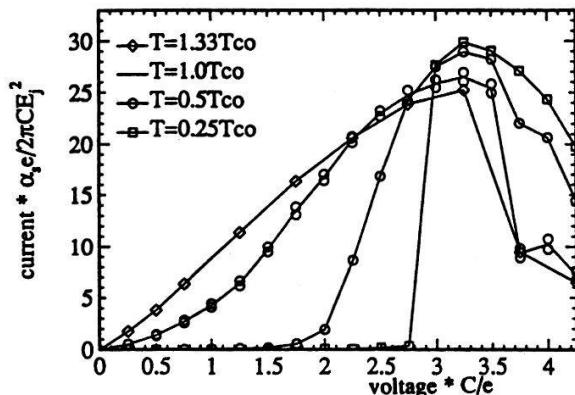


Figure 2: a: IV curves for CP in a 16x16 array,  $\alpha_s = 10$ . b: same, dashed lines denote fixed QP background.

exponent of the nonlinear conductance can be observed. The exponent  $a$  is shown as a function of temperature in the inset of fig. 1a. The correspondence to the theoretical prediction is good for QP in a 32x32 system. Random fractional offset-charges wash out the transition.

In order to make closer contact to experiment, it would be interesting to obtain IV-curves for both CP and QP tunneling at the same time. If the ratio of the tunnel rates for QP and CP is varied, one expects a transition as a function of  $R_t$  or  $T$  from QP dominated to CP dominated behaviour [5].

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