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## Magnetically Catalyzed Superconductivity in 2D.

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**Abstract.** We discuss various *parity conserving* 2D models with magnetically catalyzed superconductivity which have recently been proposed in [1,2]. The common mechanism by which they become superconducting is shown to hinge on the presence of a massless mode associated with the spontaneous breaking of a topological U(1) symmetry. A possible failure of the perturbative argument due to a nonperturbative anomaly in the topological symmetry that destroys the massless mode and thus superconductivity is pointed out. We put forward a scenario that evades the effect of the anomaly on the massless mode so that superconductivity is restored.

Superfluidity in a condensed matter system arises when the vacuum breaks particle-number symmetry  $U(1)^Q$ . The symmetry may either be spontaneously broken as in Bose systems, or dynamically via the formation of Cooper pairs as in Fermi systems. Most startling phenomena of a superfluid can be understood as arising from a massless mode whose presence is assured by Goldstone's theorem. When coupled to an electromagnetic field it triggers the Higgs mechanism through which the system becomes superconducting.

The recent study of anyon systems has revealed a new class of superfluids in which superfluidity is triggered by the breaking of a topological symmetry whose current is "trivially" conserved. More specifically, in anyon superfluids the symmetry group  $U(1)^Q \otimes U(1)^\Phi$  breaks down to a diagonal  $U(1)$  subgroup. Here,  $U(1)^\Phi$  is the topological symmetry generated by the flux  $\Phi = \int d^2x \tilde{f}^0$ , with  $\tilde{f}^\mu \equiv \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda$  the dual field strength of the statistical gauge field  $a^\mu$ . The massless mode resulting from the symmetry breaking was first found by Fetter, Hanna, and Laughlin [3].

In this note we consider models in which superconductivity is intimately related to 2D antiferromagnetism. Antiferromagnets are commonly described by the  $CP^1$  Lagrangian

$$\mathcal{L} = \frac{2}{f} |(\partial_\mu - ia_\mu)z|^2, \quad z^\dagger z = 1 \quad (1)$$

where  $z$  is a scalar doublet. The model has two phases, a weak-coupling phase describing the Néel-ordered state and a strong-coupling phase where  $a^\mu$  is massless. The phase structure may be understood as different realizations of  $U(1)^\Phi$ . In the weak-coupling phase the topological symmetry is unbroken while in the strong-coupling phase it is broken, at least perturbatively, with the corresponding massless Goldstone mode the "photon"  $\gamma$  (with  $\tilde{f}^\mu = \partial^\mu \gamma$ ) [4].

Lagrangian (1) does not describe electric charge carriers. Let us introduce these by coupling the model to an isospin doublet of (relativistic) fermions  $\psi^\sigma$  ( $\sigma = 1, 2$ ) in a way that preserves  $U(1)^\Phi$  symmetry. This may be achieved either via minimal coupling [1]

$$\mathcal{L} = \frac{2}{f} |(\partial_\mu - ia_\mu)z|^2 + \bar{\psi}^\sigma (\delta_{\sigma\tau} i \not{\partial} - ga_\mu \tau_{\sigma\tau}^3) \psi^\tau - m \bar{\psi}^\sigma \tau_{\sigma\tau}^3 \psi^\tau, \quad (2)$$

where  $\tau^a$  ( $a = 1, 2, 3$ ) are Pauli matrices in isospin space, or via a Yukawa coupling [2]

$$\mathcal{L} = \frac{2}{f} |(\partial_\mu - ia_\mu)z|^2 + \bar{\psi}^\sigma i \not{\partial} \psi^\sigma - g' n^a \bar{\psi}^\sigma \tau_{\sigma\tau}^a \psi^\tau, \quad n^a n^a = 1 \quad (3)$$

where  $n^a \equiv z^\dagger \tau^a z$ . Both models are manifestly parity invariant unlike anyonic theories. Lagrangian (2) was derived (under some simplifying assumptions) from the Hubbard model by Dorey and Mavromatos [1]. Model (3) at low energies is essentially equivalent to (2).

In the strong-coupling phase of the interacting models the gauge boson  $a_\mu$  remains massless (at least in mean-field theory) but the symmetry breaking is now  $U(1)^Q \otimes U(1)^\Phi \rightarrow U(1)^{Q-\Phi}$  as in anyon systems. Since fermion-number symmetry is spontaneously broken the models are in the superfluid state. The physical picture is that in the weak-coupling phase there is a bound state of the  $CP^1$  soliton and two fermions. The phase transition in the  $CP^1$  model is driven by the condensation of solitons. In the interacting theory this leads to appearance of a superconducting condensate.

It should be noted that these conclusions hinge on the presence of the topological symmetry in the  $CP^1$  model. Although perturbatively this symmetry exists, recent lattice and analytic calculations [5] indicate that a nonperturbative effect renders the symmetry anomalous in the strong-coupling phase. More specifically, it turns out that monopole configurations contribute to the partition function. Since (anti-) monopoles (annihilate) create “magnetic” flux  $\Phi$ , a finite density ( $\rho_-$ )  $\rho_+$  of (anti-) monopoles leads to the anomaly  $\partial_\mu \tilde{f}^\mu \propto \rho_+ - \rho_-$ . If this phenomenon persists after the coupling to fermions, the interacting models display no superfluidity.

However, coupling to fermions can provide a natural mechanism to evade this conclusion. In the presence of fermions the monopole contribution is formally proportional to  $\det(i\not{\partial} - g\tau_3 \mathcal{A} - m\tau_3)$ , where  $\mathcal{A}_\mu$  is the monopole vector potential. As is well known [6] this formal expression is not well defined due to the core singularity of the potential and specification of the boundary condition of the wave function at the core is necessary. For a particular choice of boundary condition the spectrum of the Dirac operator contains two zero modes. In this case the anomaly equation changes to:  $\partial_\mu \tilde{f}^\mu \propto \rho_+ \bar{\psi}_1 \psi_2 - \rho_- \psi_1 \bar{\psi}_2$ . Hence, only Green functions containing an access of two fermionic fields feel this anomaly, and the “photon”  $a_\mu$  remains massless. In fact, it is the Goldstone boson of the broken  $U(1)^Q$  symmetry. In this phase there is a nonzero Cooper-pair condensate  $\langle \psi_1 \psi_2 \rangle \propto e^{-S_{\text{monopole}}}$ . Whether this choice of boundary condition is the right one is determined by the microscopic theory. It is interesting to note that it appears naturally if (2) is considered as a low-energy limit of the 2D Georgi-Glashow model [7].

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