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FQHE in the Disk Geometry : Exact Diagonalization for few Electrons and Extrapolation to the Bulk Limit

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Abstract. We diagonalize exactly the Hamiltonian of electrons which move in two dimensions in a strong magnetic field and interact via Coulomb forces. We work in the disk geometry. The size of the system is defined by the area of the neutralizing background potential. The multiparticle energy spectra are calculated numerically in the vicinity of filling factor 1/3. We propose an extrapolation from finite particle number N to the infinite system. This yields, already with data for up to seven electrons, a value for the ground state energy, which agrees well with the known result of a Monte Carlo simulation using Laughlin's variational wavefunction. The total angular momentum of the ground state is also in accordance with Laughlin's ansatz. Moreover, we find an indication for a cusp in the extrapolated energy per particle at filling factor 1/3. Finally, we demonstrate how the low-lying eigenenergies move as the filling factor is decreased. We argue that the quasiparticle energies in the disk geometry can be extracted from this behavior by extrapolation.

Introduction

The fractional quantum Hall effect (FQHE) is well described by the concept of quasiparticles above incompressible ground states. The variational wavefunctions for the ground state and the quasiparticle states given originally by Laughlin [1] and later by Jain [2] have been checked against numerical calculations in few electron systems; for a review see [3]. Much work, numerical and analytical, has been devoted to the model in the spherical geometry introduced by Haldane [4], in which one can identify states with one, two, or four quasiparticles already in energy spectra of systems with few electrons. Recently, in the context of high temperature superconductivity, anyons have attracted a lot of interest [5], while they are also the candidates for the quasiparticles with fractional charge and statistics in the FQHE. After anyons in a uniform magnetic field have been studied [6], one should now include an anyon-anyon interaction to describe the quasiparticles of the FQHE. It would be desirable to determine quantitatively these anyon-anyon interaction coefficients directly from the original electron Hamiltonian. However, in a spherical geometry, the statistics of anyons is constrained to specific values, which depend on the number of anyons in the system [7]. This constraint can be formulated as the condition that all the flux lines, which an anyon sees originating from the other anyons, should carry an integer total flux. This is similar to Dirac's condition which fixes the strenght of the magnetic monopole producing the magnetic field on the sphere. Thus, two anyons on the sphere are either Bosons or Fermions and, i. e. , the case of statistics 1/3 (this corresponds to a 1/3 filled lowest Landau level, filling factor ν =1/3 in the FQHE) is excluded. This is one of the reasons why we study a planar geometry which does not yield such a topological constraint.

Model and Results

Here we describe our first results. We work in the geometry of a finite disk, keep only states of the lowest Landau level up to a maximum angular momentum, and use the Coulomb interaction 1/r. The background is made up from positive charges of strength N/g in the states with angular momenta $0,\ldots,g-1$. Thus, the size of the disk is $2\pi g$. Lengths and energies are in units of the magnetic length l_c and e^2/l_c, respectively. In the numerical calculations, the multiparticle space is decomposed into blocks with fixed total angular momentum $M, M_{min} \leq M \leq M_{max}$. Within each block, the matrix elements are calculated from the two particle matrix elements of the Coulomb interaction. Then the spectra in the blocks are obtained with the Householder and QR method ($N \leq 6$) and with the Lanczos algorithm ($N \geq 7$).
The symmetry against a particle hole transformation, which yields a single particle term in the Hamiltonian for finite g, is checked. For a filled Landau level, the energy approaches its known exact bulk limit with a finite size correction of order $1/\sqrt{N}$. For $\nu = 1/3$, we determine a ground state energy per particle (E) of -0.4105 by extrapolating E for $g = 3N$ linearly in a graph of E versus $1/\sqrt{N}$. We find good agreement with the well known Monte Carlo result [8]. The finite size corrections are well described by the term $1/\sqrt{N}$. Extrapolating E in a similar way at ν near 1/3, we find indications for a downward cusp in the graph of E versus ν.
Unfortunately, in the planar geometry under consideration, it is more difficult to obtain the properties of quasielectrons and quasiholes than it is in the spherical geometry. Here, the spectra with $g = 3N$ do not show the largest gaps between ground state and excited states. Consequently, one cannot determine properties of states with one quasielectron or with one quasihole simply from calculations with $g = 3N - 1$ or with $g = 3N + 1$. First, one needs to determine which g gives the best finite size approximation to the incompressible ground state. Here, we suggest the following. As g increases in the vicinity of $g = 3N$, the behavior of the total angular momenta of the ground state (M_0) and of the first excited state (M_1) is studied. As the spectra develop with increasing g, the largest gaps are formed for the two g's, between which M_1 jumps from $M_1 < M_0$ to $M_0 < M_1$. These "stable states" have an angular momentum $M_0 = M^* = \frac{3N(N-1)}{2}$, where M^* is the angular momentum of Laughlin's variational wavefunction. Extrapolating the energies E to the bulk limit, we need, in addition to the corrections of $1/\sqrt{N}$, terms of order $1/N$ to find better agreement with the above result. A corresponding extrapolation of the quasihole energy is in progress. In a similar way, we intend to obtain the interactions of the quasiparticles. To this end, larger systems have to be studied.
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