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Incompressible Quantum Liquid versus Two-Dimensional Electron Solid

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Abstract. A simple but rigorous approach of sum-rule techniques being applied to the density and displacement fluctuations of the centers of cyclotron orbits of Coulomb-interacting electrons provides a study of the nature of two respective condensed phases: an incompressible quantum liquid of Bose condensed charge-vortex composites and a two-dimensional quantum solid with a lattice-periodic structure due to broken magnetic translational invariance.

Introduction

Interacting electrons of charge $-e$ being confined to a plane perpendicular to a magnetic field are considered to compete between two condensed phases: an incompressible quantum liquid [1] and a two-dimensional (2D) quantum solid, respectively, if the magnitude B of the field is sufficiently large that the mean number ν^{-1} of magnetic flux quanta ch/e per electron exceeds one. For a given area A of that plane this then corresponds to a fractional filling factor of the lowest Landau level $\nu = 2\pi r_L^2 n_e < 1$, i.e., a fractional ratio of the mean areal density of electrons, $N_e/A = n_e$, and of flux quanta, $N_s/A = (2\pi r_L^2)^{-1}$, respectively, if $r_L = (c\hbar/eB)^{1/2}$ denotes the Larmor radius. For certain rational values of $\nu = p/q < 1$, with p and q denoting mutual primes and q usually being odd, the incompressible quantum liquid (IQL) phase of Bose condensed charge-vortex composites (p charges and q flux quanta) may be stabilized because of a novel broken symmetry [2]. This IQL exhibits a quantized Hall conductance $\sigma_{12} = \nu e^2/h$, being accompanied by minima in the longitudinal conductance σ_{11} [3]. Below a critical filling factor ν_c , the expectation values $R_j \equiv \langle X_j \rangle$ of the centers of the cyclotron orbits (guiding centers) with non-commuting Cartesian components are assumed to possibly form a lattice-periodic structure with a non-vanishing shear modulus $\mu > 0$ due to broken magnetic translational invariance. Particularly, in the dilute electron ($\nu \rightarrow 0$) or high-magnetic-field ($r_L \rightarrow 0$) limit, the triangular lattice of the classical 2D Wigner crystal [4] may be identified with the ground-state of this sort of a 2D quantum solid, to be called quantum Hall crystal (QHC) [5]. Reentrance phenomena of the correlated electron system may finally allow the IQL-phase to exist even at certain rational values $p/q < \nu_c$.

Outline of Approach and Summary of Results

Differences and similarities in the IQL-phase and the QHC-phase can now be associated quite generally with differences in their respective symmetries and various results are obtained by the use of sum-rule techniques without employing approximate wave functions as having been discussed in detail previously [5]. First, the finite-gap versus gapless behavior of collective modes $\omega(k)$ was shown to exist rigorously within the lowest Landau level in the $k \rightarrow 0$ limit of these two phases [6]. An important role in obtaining these results is played by the respective restoring forces of the non-commuting density fluctuations $\Delta(k,t) = \sum_j \exp(-ik \cdot X_j(t))$ of the guiding centers $X_j(t)$ and the fluctuations $u(k,t) = N_e^{-1/2} \sum_j [X_j(t) - R_j] \exp(ik \cdot R_j)$ of their displacements $X_j(t) - R_j$. The mentioned restoring forces are related to frequency moments which determine the non-dispersive parts in the spectral representations of the inverse of the response functions χ^{-1} . By the use of sum rules, these collisionless terms of χ^{-1} can therefore be calculated via those frequency moments of the spectral functions $\chi''(k,\omega)$ without making reference to the nature of the single-particle excitations.

In the IQL-phase of the uniform system of interacting electrons that restoring force at $k=0$ induces a k -independent ratio of the third and first frequency moment of the spectral function of guiding center density fluctuations $\chi''_{\Delta\Delta}$, giving rise to the gap at $k=0$ in the excitation spectrum $\omega_L(k=0) \neq 0$, derived from $\text{Re } \chi_{\Delta\Delta}^{-1}(k, \omega_L(k)) \equiv 0$. Combined with a sum rule, the k^4 -behavior actually obtained for both frequency moments in the $k \rightarrow 0$ limit provides a rigorous proof that also the static susceptibility $\chi_{\Delta\Delta}(k, 0) \sim k^4$, i.e., the longwave length density fluctuations are strongly suppressed in that IQL-phase. In single-mode approximation [7], this then implies moreover the magneto-roton minimum to exist at some finite k of ω_L , whose softening may induce the transition to the QHC-phase.

In the QHC-phase, combinations of the first and zeroth frequency moment of the spectral function of guiding center displacements play a similar role for the quasi-transverse restoring force being related to the dynamical matrix of conventional lattice dynamics. For the long-wavelength limit of the gapless Goldstone mode, being isotropic for a triangular lattice, the exact expression $\omega_s^2(k) = (2\pi e^2/m^* \omega_c^2 \epsilon) \mu k^3$ is then obtained from $\text{Re } \chi_{uu}^{-1}(k, \omega_s(k)) \equiv 0$ (m^* : effective mass, ϵ : background dielectric constant, $\omega_c = \hbar/m^* r_L^2$: cyclotron frequency). The isothermal shear modulus μ is rigorously related to the second derivative of the free energy with respect to displacement deformations according to the generalized elastic sum rule [8]. A non-vanishing shear modulus $\mu > 0$, therefore, reveals the crystalline-like nature of that phase rather generally.

A minimum, found in the ν -dependence of μ in an approach allowing for quantum statistical features and anharmonicities to one-loop order [6],[9], is possibly indicating a competition between the QHC-phase and the IQL-phase in the vicinity of fractional filling factors exhibiting quantized Hall conductance as revealed by experiment [10]. This, finally, would also be consistent with a diverging longitudinal resistivity, having been observed experimentally in a narrow region around $\nu=1/5$ [11], as well as with our recent estimate for the critical filling factor ν_c slightly higher than $1/5$, obtained from comparing total energy calculations in both phases, i.e., $E_L(\nu_c) = E_S(\nu_c)$ [12].

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