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Autor: Huckestein, B. / Schweitzer, L.
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Multifractality and Scaling in Disordered 2-D Landau Levels

B. Huckestein and L. Schweitzer

Physikalisch-Technische Bundesanstalt,
Bundesallee 100, W-3300 Braunschweig, F. R. Germany

Abstract. The structure of the eigenstates of disordered 2-D systems of size $M \times M$ in strong perpendicular magnetic fields is investigated. A multifractal behaviour is found in the centres of the Landau bands, which is described in terms of the $f(\alpha(q))$ vs. $\alpha(q)$ spectrum. The eigenstates $\psi_{E,M}(\mathbf{r})$ and the equilibrium current density $\mathbf{j}_{E,M}(\mathbf{r})$ show similar $f(\alpha(q))$ spectra which supports the view of a universal one-parameter scaling behaviour.

Introduction

Electronic states in disordered two-dimensional systems, subject to a strong perpendicular magnetic field, exhibit complex self similar structures on length scales l , which are larger than both the magnetic length and the correlation length of the potential, but are smaller than the system size M [1, 2, 3]. These states appear at energies near the centres E_n of the Landau bands, where the localization length $\xi(E)$ diverges as $|E - E_n|^{-\nu}$, with $\nu = 2.35$ [4, 5, 6]. To characterize the multifractal states, the $f(\alpha(q))$ vs. $\alpha(q)$ spectrum has been calculated [3, 6], which can be fitted to a parabola, $f(\alpha) = D_0 - (\alpha - \alpha_0)^2 / (4(\alpha_0 - D_0))$. This reflects the log-normal distribution of the box-probabilities $p_i(l, E, M) = \sum_{\mathbf{r} \in \Omega_i(l)} |\psi_{E,M}(\mathbf{r})|^2$. With $D_0 = 2$, we find for the lattice model $\alpha_0 = 2.29 \pm 0.02$, which is in accordance with the result for the continuum model [3].

In this contribution a multifractal analysis of the equilibrium current density $\mathbf{j}_{E,M}(\mathbf{r})$ is presented. This quantity shows a similar behaviour as found for the eigenstates. Thus, a universal $f(\alpha(q))$ -spectrum for a variety of physical observables seems likely.

Model and Method

A one band tight binding model on a simple cubic lattice with periodic boundary conditions is considered to describe non-interacting electrons in disordered 2-D systems in strong perpendicular magnetic fields. The site energies are independent random numbers distributed with probability $1/W$ over the interval $[-W/2, W/2]$ and the magnetic field B enters the phase factors of the transfer matrix elements. A Lanczos algorithm is applied to calculate the eigenvalues and eigenvectors of the corresponding sparse hermitian matrices.

The $f(\alpha)$ -spectrum is calculated according to [7] from

$$\alpha(q) = \lim_{\lambda \rightarrow 0} \sum_i \mu_i(q, \lambda) \ln P_i(\lambda) / \ln(\lambda) \quad (1)$$

and

$$f(q) = \lim_{\lambda \rightarrow 0} \sum_i \mu_i(q, \lambda) \ln \mu_i(q, \lambda) / \ln(\lambda), \quad (2)$$

with $\mu_i(q, \lambda) = P_i^q(\lambda) / \sum_j P_j^q(\lambda)$ and $\lambda = l/M$.

For the investigation of the equilibrium current density of an eigenstate with energy E , $\mathbf{j}_{E,M}(\mathbf{r}) = \sum_{\mathbf{r}'} \langle E | \mathbf{r}' \rangle \langle \mathbf{r}' | \hat{\mathbf{j}} | \mathbf{r} \rangle \langle \mathbf{r} | E \rangle$, a normalized box-probability was defined as $P_i(\lambda) = |\mathbf{j}_i(\lambda)| / \sum_k |\mathbf{j}_k(\lambda)|$ [3], because the current density is, in contrast to the eigenstates, not normalized, and $\mathbf{j}_i(\lambda) = \sum_{\mathbf{r} \in \Omega_i(\lambda)} \mathbf{j}_{E,M}(\mathbf{r})$.

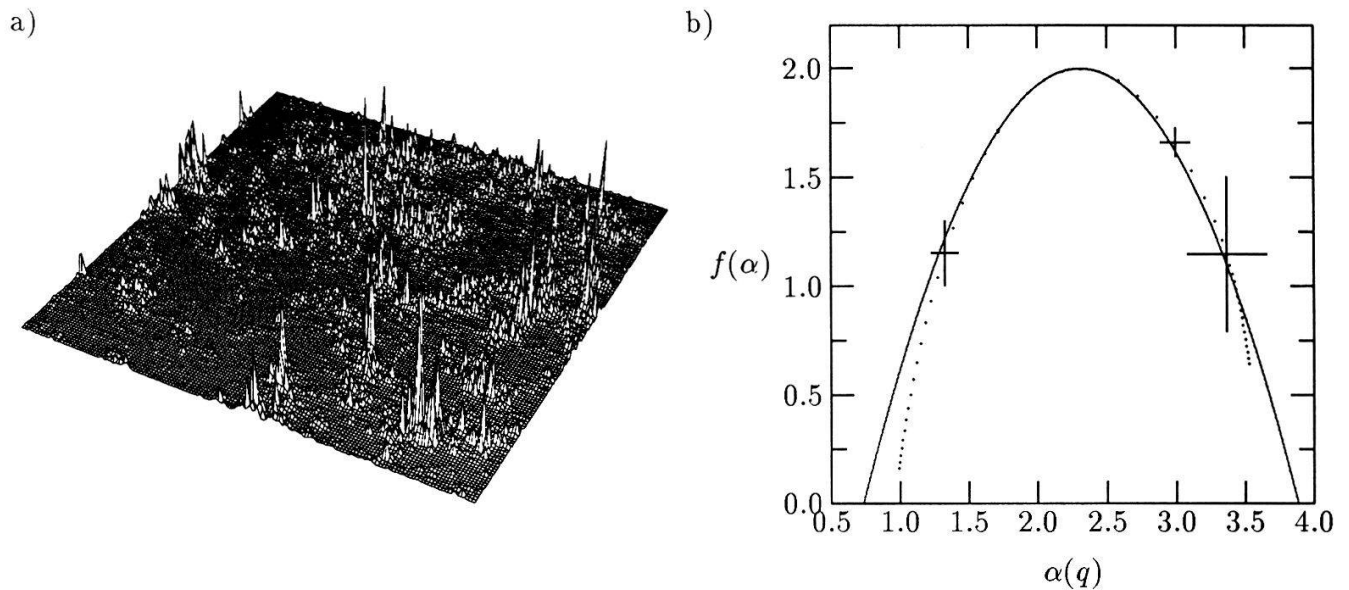


Fig. 1: a) The density $|\psi_{E,M}(\mathbf{r})|^2$ of the most extended state in the centre of the lowest Landau band. b) $f(\alpha)$ -spectrum of the current density of the state shown on the left. Crosses give the errors in $f(q)$ and $\alpha(q)$ for $q = 2, 1, -1, -2$ (from left to right).

Results and Discussion

In fig.1a the modulus of the most extended state, $|\psi_{E,M}(\mathbf{r})|^2$, of the lowest Landau band for a system of size $M = 150a$, is shown as a function of the position. The magnetic field strength is equivalent to $1/5$ flux-quanta h/e per unit lattice cell a^2 . A strongly peaked structure of the wavefunction is observed. However, this must not be regarded as a sign of localization, but reflects the fact that the logarithm of the density $|\psi_{E,M}(\mathbf{r})|^2$ is normally distributed [3]. The $f(\alpha(q))$ -spectrum of the corresponding equilibrium current density $\mathbf{j}_{E,M}(\mathbf{r})$ is plotted in fig.1b together with a parabolic fit from which a value of $\alpha_0 = 2.31 \pm 0.03$ is obtained. Therefore, the $f(\alpha(q))$ -spectrum of the current density is, within the current uncertainty, the same as for the eigenstates [6].

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