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Violation of Ehrenfest's Theorem in Particle Dynamics of Quantum Hall Systems

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Abstract. We investigate a large class of two-dimensional systems of electrons under quantum Hall conditions. Scattering between the localized Landau functions leads to time-dependent single-particle wave functions, which spread in the direction perpendicular to the macroscopic Hall field. The average velocity of these states does not in general correspond to a classical orbit according to Ehrenfest's theorem. We find that in addition to the usual classical term the Hall velocity contains a nonclassical part, which does not contribute to the Lorentz force. This nonclassical velocity term (which has been shown to play a crucial role for the integer quantum Hall effect in our systems) disappears, when the disorder potential is absent.

Introduction

We consider independent electrons on a two-dimensional strip of width L_y subject to a perpendicular strong magnetic field (0,0,B), a Hall electric field $(0,E_y,0)$ and a static disorder potential V(x,y), created by homogeneously distributed impurities. We chose the gauge $A_x = 0$, $A_y = Bx - cE_yt$ together with periodic boundary conditions in y-direction and $|\psi| = 0$ for large $|x|$. This model describes bulk states in a quantum Hall system.

The solutions of the time-dependent Schroedinger equation can be expanded into Landau functions $\psi_p(x,y,t) = L_y^{-1/2}\exp(i2\pi py)u_p(x,t)$, which are solutions in the absence of V(x,y). Here $u_p(x,t)$ is the product of a Hermit polynomial and a Gaussian centered at $x_p(t) = chp/(qBL_y) + cE_yt/B$. (We consider a one-band approximation.)

The *Hall velocity* of a (normalized) state $|j\rangle \equiv \psi^j(x,y,t) = \Sigma_p c_p^j(t)\psi_p(x,y,t)$ has the form

$$v_x^j(t) = d\langle j|x|j\rangle/dt = d[\Sigma_p |c_p^j(t)|^2 x_p(t)]/dt = cE_y/B + \Sigma_p x_p(t) d|c_p^j(t)|^2/dt. \quad (1)$$

The first term cE_y/B on the right hand side is the classical Hall velocity in the field E_y. The second term $\Sigma_p x_p(t) d|c_p^j(t)|^2/dt$ is a nonclassical Hall velocity v_{ncl}, which originates from the time-dependent scattering between the localized Landau functions. This process is induced by the disorder potential V(x,y) and the Hall field E_y [1].

The Schroedinger solutions $\psi^j(x,y,t)$ of our system have the following general properties [1]: At sufficiently low E_y all states are adiabatic [2] solutions. These adiabatic solutions have a modulus $|\psi^j(x,y,t)|$, which is periodic in time with period $\tau = h/(qE_yL_y)$. Since τ is very small (e.g. $\tau = 4\times10^{-12}$s for E_yL_y= 1mV), only time averages over $\Delta t = \tau$ are relevant for macroscopic purposes (these averages will be denoted by a bar $\overline{..}$). Adiabatic solutions in our system have the property, that $\bar{v}^j_x(t) = \bar{v}^j_y(t) = 0$. This means that adiabatic states do not contribute to the macroscopic Hall current. They are *insulating*.

Conducting states occur at higher (non-infinitesimal) values of the Hall field E_y, when nonadiabatic transitions between adiabatic states become possible. In general different adiabatic states become conducting at different values (threshold fields) of E_y.

Violation of Ehrenfest's theorem

We consider now a *conducting* state $|j\rangle$, which has developed according to the time-dependent Schroedinger equation during a time much larger than τ. We claim, that such a state violates Ehrenfest's theorem, which says, that the quantum mechanical mean values of the position and the velocity obey the classical equations of motion (see e. g. chapter VI of ref. [2]). Indeed, if this theorem was true, we would have

$$m d\bar{v}^j_y(t)/dt = q\overline{\langle j| E_y(x,y)| j\rangle} - (q/c)\bar{v}^j_x B. \qquad (2)$$

(Here $E_y(x,y)$ denotes the y-component of the total electric field $E_y - \partial V(x,y)/\partial y$ at (x,y).) This is Ehrenfest's relation after a time average over the short period τ. Now for times sufficiently longer than τ the wave function of a conducting state is spread in x-direction (delocalized) as a result of the time-dependent process of non-adiabatic transitions, which leads to scattering into Landau functions localized at different sites x_p on the x-axis (see [3] for explicit calculations). Hence $\bar{v}^j_y(t)$ tends to zero for these states (the local Hall velocities proportional to $\partial V(x,y)/\partial y$ are averaged out) and $\overline{\langle j| E_y(x,y)| j\rangle}$ becomes just equal to the homogeneous field component E_y (the average over the homogeneous disorder potential with respect to the delocalized orbital vanishes). Therefore equation (2) becomes

$$0 = qE_y - (q/c)\bar{v}^j_x B. \qquad (3)$$

According to (1) the velocity $\bar{v}^j_x(t)$ is composed of a constant, classical part cE_y/B and of a nonclassical part v_{ncl}, i. e., relation (3) is already fulfilled by the classical velocity part alone. There is no contradiction in this result, since the Ehrenfest theorem is valid only if the fluctuations of position and momentum are sufficiently small (see e. g. chapter VI-2 of [2]), and this condition is not fulfilled in our case.

Equations (2) and (3) formally define the y-component of the (average) Lorentz force (which is a classical concept). These equations are identical with the corresponding *classical* equation of motion, i. e., they are valid only for those quantum mechanical velocities, which are associated with *classical* trajectories. But this is not the case for the nonclassical velocity v_{ncl}, which vanishes in the classical limit. Since the velocity dependent part of the Lorentz force (which cancels the average electric force qE_y imposed on the particle and therefore keeps it on its equilibrium position in y-direction) is created entirely by the classical part cE_y/B of the average particle velocity $\bar{v}^j_x$, we therefore conclude, that *the nonclassical part v_{ncl} of the particle velocity does not give rise to a Lorentz force*!

Discussion

The nonclassical velocity parts represent the so-called compensating currents, which lead to integer quantization of the Hall conductance [1]. Our results provide a microscopic picture for the IQHE, which differs from arguments developed in the literature [4], according to which current compensation is thought to originate from effective Hall fields $\mathbf{E}_{eff}$, which are different for so-called localized and extended states, and therefore lead to different Hall velocities for these states according to the *classical* formula $c\mathbf{E}_{eff}/B$, i. e., the totality of each particle velocity is thought to create a Lorentz force, in opposition to our present result.

References

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