Zeitschrift: Helvetica Physica Acta

Band: 65 (1992)

Heft: 2-3

Artikel: Experiments on melting in classical and quantum two dimensional electron systems

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DOI: https://doi.org/10.5169/seals-116411

EXPERIMENTS ON MELTING IN CLASSICAL AND QUANTUM TWO DIMENSIONAL ELECTRON SYSTEMS

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Extended abstract.

"Two dimensional electron system" (2DES) here refers to electrons whose dynamics is free in 2 dimensions but blocked in the third and which interact through a threee dimensional Coulomb interaction $e^2/\epsilon|r_{ij}|$. Experiments have been performed in two limiting situations: the classical, low density, limit realised by electrons deposited on a liquid helium surface and the quantum, high density, limit realised by electrons at an interface between two epitaxially matched semiconductors. The first is five orders of magnitude below, the second two orders of magnitude above, the zero temperature, zero magnetic field Wigner transition which occurs at the density $n_s = n_W$ at which the zero-point kinetic energy

$$T_Q = \hbar^2/ma^2$$

where $a^2 = 1/\pi n_s$, overwhelms the Coulomb energy

$$V_c = e^2/\epsilon a$$

responsable for stabilising the crystalline configuration in the classical limit[1].

In the classical system, where $T_Q \ll V_c$ so that the thermodynamic state is determined simply by the competition between the temperature and the Coulomb interaction, melting is induced either by raising the temperature at constant density or by lowering the density at finite temperature. In the quantum system, however, it is not possible to lower the density below about $100n_W$ without the Coulomb interaction losing out to the random field representing the extrinsic disorder imposed by the semiconductor host. Instead one has to induce crystallisation with the help of the Lorentz force, by applying a perpendicular magnetic field B[2]. As the quantum magnetic length $\ell_c = (\hbar c/eB)^{1/2}$ is reduced with respect to the interelectronic spacing a, usually expressed by the filling factor $\nu = 2\ell_c^2/a^2$, the system exhibits the quantum Hall effect (QHE), first for integer then for fractional values of ν. The fractional quantum Hall effect (FQHE) is already a result of Coulomb induced correlation in the quantum liquid, but as ν is decreased still further the correlations are expected to take on long-range crystal-like periodicity accompanied by elastic shear rigidity. Such a state can nonetheless be destroyed by the disordering effect of temperature, giving rise to a phase boundary in a (T, B) plane.

The aim of experiment is first to determine the phase diagram and then to help elucidate the mechanism of the melting.

The Classical Limit

The first experiment[3] to show the solid-liquid transition detected the appearance of a new set of longitudinal modes arising from the enhancement of the electron-substrate interaction which accompanies the crystallization of the 2DES. In experiments, the electrons are held onto the liquid helium surface by a perpendicular electric field so that each electron exerts a force on the helium substrate which is resisted by its surface tension. If an electron were immobile, it would thereby create a dimple under itself[4], but in the liquid state it moves too quickly for the helium surface deformation to follow; in the crystalline state, on the other hand, the mean position of an electron is determined by that of the crystal, a much heavier, slower moving, object and the helium surface deforms to form an image of the electron lattice: the dimple under each lattice point is then a function of the mean fluctuation $\langle u^2 \rangle \propto T/\mu$ about the lattice site. The electronic vibrations become strongly coupled to the dimple lattice, the inertia of which profoundly modifies the low frequency portion of the dispersion relation as shown in figure 1[5]. Subsequent experiments were aimed specifically at determining the nature of the phase transition by measuring quantities like the shear modulus $\mu(T)$, the reciprocal lattice vectors, the thermodynamics and the internal kinetics. Nearly all these experiments make use of the dynamical modes illustrated on fig.1.

Crystallisation was first detected by observing the longitudinal "acoustic" modes where electrons and dimples move in phase (phason modes from the point of view of the helium deformation, indicated by 1 on fig.1)[2]. The frequency of the transverse "optical" mode, ω_0, where the electrons move in anti-phase to the dimples (point 2 in fig.1) was measured to deduce the mean square fluctuations which reflect the variation of the shear modulus[6,7]. This central quantity was subsequently determined more precisely by measuring a point (ω, q) on the transverse sound branch in the region where it decouples from the helium deformation and its frequency approaches $\omega = (\mu/mn_s)^{1/2}q$ (point 3 in fig.1)[8]. The peaks in $S(|Q|)$, giving the moduli of succesive stars of the reciprocal lattice, were got by observing the ripplon modes resonantly coupled to the electron lattice: $\Omega_Q = (\alpha/\rho)^{1/2}\ |Q|^{3/2}$ where α is the surface tension and ρ the density of the liquid helium (points 4 of fig.1)[3,9]. The specific heat in the solid phase and the entropy change on melting were also measured; that experiment employed the transverse optic mode as an electron thermometer and the magneto-plasmon modes to introduce a heat pulse[10].

The results from these experiments point to the formation of a triangular lattice for temperatures $T < \Gamma_{\rm cm}^{-1}e^2/\epsilon a$ ($\epsilon = 1.0023$) with $\Gamma_{\rm cm} = 127 \pm 3$. The shear modulus was found to diminish linearly with the tempeature by about 25% from $T = 0$ to $0.9T_m$, at which point it rounds off before dropping precipitously towards zero at $T = T_m^+$, much as had been predicted by the analytical and simulational estimates[11,12]; the $T = 0$ value was found to be about 10% lower than that calculated on a purely classical model [13,14]; within the experimental accuracy (about 10%), its value just prior to melting satisfies $\mu(T_m^-) = 4\pi\ T_m/a_o^2$, the criterion for the Kosterlitz-Thouless-Berezinskii[15,16] instability to free dislocations (a_o is the lattice constant) in an incompressible system. The specific heat, measured up to $0.9T_m$, is well accounted for by phonons using the measured value of $\mu(T)$; an

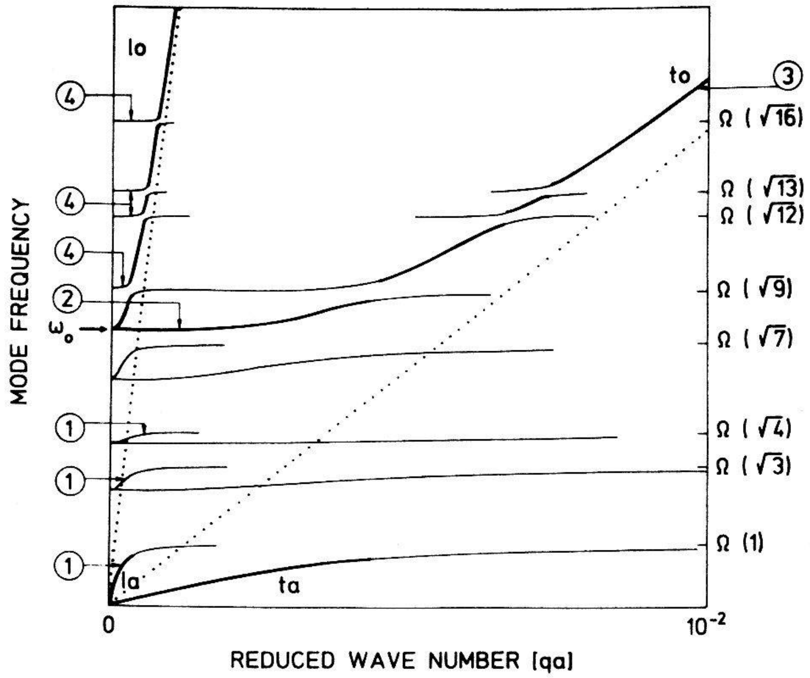

Fig.1: Small wavevector $(0 < qa < 10^{-2})$ portion of the frequency dispersion of vibrational modes of electron lattice coupled to liquid helium substrate. The vibrations of the electrons and the capillary waves progressively decouple as the wavevector increases and the electron excitation frequencies become those of a free lattice shown by the dotted asymptotes. The free capillary wave frequencies at the reciprocal lattice wavenumbers pg_0 are denoted by $\Omega(p)$ on the right hand scale. t and l refer to transverse and longitudinal modes, a and o to "acoustic" — helium phason and electron displacements in phase — and "optical" or antiphase motion. The points referenced by encircled numbers indicate the regions in which the experiments referred to in the text were performed: 1 ref.(3); 2 ref.(6,7); 3 ref.(8); 4 ref.(9). A theoretical account of the coupling is given in ref.(5).

upper bound of $0.2k_B$ per electron was established for any entropy change on melting.

Experiments thus point very strongly towards melting driven by the dissociation of dislocation pairs, the detailed statistics of which have been worked out by Nelson, Halperin and Young[17,18].

Experiments on the internal thermodynamic equilibration rate at melting, presented at this conference, show it to be $\sim 10^{-7}\ \omega_D$ (ω_D is the Debye frequency), in rather good accord with a model of dislocation dynamics mediated by interstitials[19,20]. Although it is of obvious interest to see if the intermediate hexatic liquid phase predicted for this type of continuous melting transition occurs, no experiment has yet been performed to check this.

The Quantum Limit

The degree of perfection in fabricating 2DES at the GaAs/GaAlAs heterojunction now permits meaningful experiments to be done in samples with densities $n_s \gtrsim 5 \times 10^{10}$ cm^{-2}, still some 200 times higher than the Wigner transition density, but nonetheless in the range where one can reach strong Coulomb correlation conditions in laboratory magnetic fields. Several different types of experiment now indicate that a qualitatively new phase, intrinsic to the 2DES, occurs for filling factors below $\nu = 1/5$ and temperatures below some fraction of the classical melting temperature $T_{cm} = e^2(\pi n_s)^{1/2}/\epsilon\Gamma_{cm}$, the measure of the available Coulomb correlation energy (for the density cited, these two marks are at $\sim$ 10 Tesla and 400 mK).

The simplest of experiments is to measure the electrical resistance, which might be reasonably expected to reveal the appearance of any qualitatively new state as it does for the FQHE. More specific, and complementary, information is obtained by looking at the dynamics of the 2DES, much in the same spirit as for the classical 2DES. Yet another approach consists in monitoring the internal energy of the system with increasing correlation.

Unlike electrons on liquid helium, the present system can be ohmically connected to the outside three-dimensional measuring world. The resistance tensor is usually measured by sending a constant current I through the sample and measuring the potential drops across it: $R_{xx} = V_L/I$ and $R_{xy} = V_T/I$ where V_L and V_T are the potential differences along and transverse to the current flow. The QHE manifests itself by a plateau in R_{xy} and a strong minimum in R_{xx} with magnetic field around integer or certain fractional, $\nu = p/q$, filling factors. Measurements of V_L at constant current show that its low-temperature thermal variation at a given magnetic field is qualitatively different according to the value of the filling factor. Metal-like around FQHE points $\nu = 1/5$ and $2/9$ ($V_L \propto \exp(-E(\nu)/T)$, $E > 0$), it becomes semiconductor-like ($E < 0$) for $\nu > 1/5$ and in an interval in $1/5 < \nu < 2/9$[21,22]. However if the current-voltage relation is non-ohmic, R_{xx} is not independent of current and loses its usual meaning; much more information can be extracted by determining $I(V)$ at each temperature. This latter type of experiment has shown that the I-V relation is linear in the "metallic" regions but strongly non-linear in the "semi-conducting" regions revealing a threshold field at which the differential resistance changes sharply[23,24]. The disappearance of this threshold defines boundaries in the (ν, T) plane (see fig.2).

The dynamics of the electrons, as opposed to the stationary conditions of a usual transport experiment, give different information; as for the classical system, new modes appear for a system with elastic shear rigidity. Experimentally one can get this information by measuring response to a space and time periodic electric field, $E = E\hat{x}\cos(\omega t)\cos(qx)$. This is expressed by the electric susceptibility $\chi_{xx}(\omega, q)$, the poles of which indicate the normal modes of vibration. In the absence of restoring forces, the 2D electron dynamics in a perpendicular magnetic field is described by simple cyclotron motion and the susceptibility shows a single finite frequency pole (resonance) at $\omega = \omega_c = eB/m^*c$ for all q. If there is a restoring force in the longitudinal (x) direction only, there is still a single finite frequency pole for given q, displaced to $\omega_+(q) = (\omega_c^2 + \omega_l^2(q))^{1/2}$ where $\omega_l(q)$ is the frequency characterising the restoring potential. But if, as would be the case in the presence of shear rigidity, there is also a restoring force in the transverse (y) direction, characterised by $\omega_t(q)$, a new low, but finite, frequency pole appears at $\omega_- \simeq \omega_t\omega_l/\omega_c$. Experiments have been performed which measure the frequency dependence of the susceptibility over a continuous interval $0.01 < \omega/2\pi < 2GHz$ at discrete spatial periodicities (fixed by electrode geometry), defining lines in (ω, q) space for $q = pq_o$, $p = 1, 2... \sim 10$ with $q_o a \sim 10^{-2}$. These measurements show low

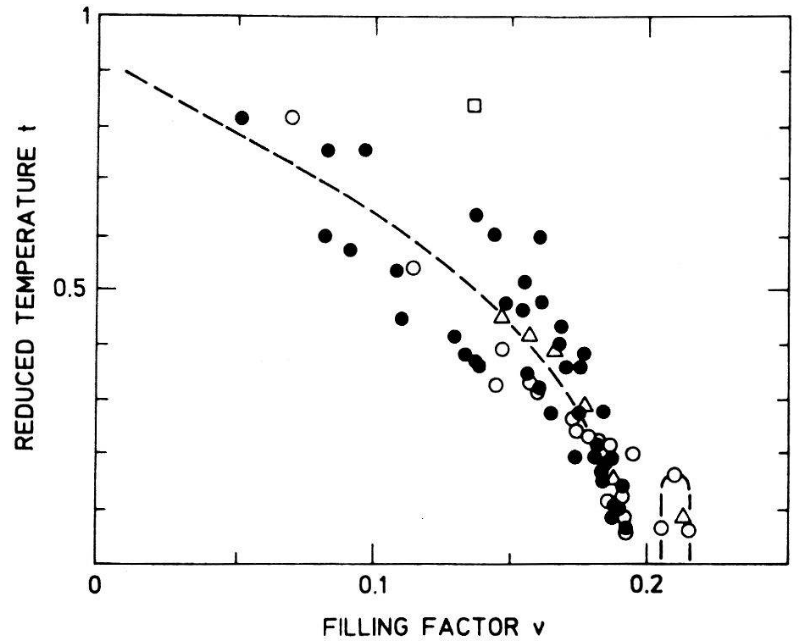

Fig.2: Comparison of experiments on the quantum 2DES. Radiofrequency resonances were found to appear in $\chi_{xx}(\omega, q)$ to the lower left of the points • (ref.(25,26,27)); non-linearity showing a threshold field was observed to appear in $I(V)$ on the low t side of the points Δ (ref.(23)) and the points o (ref.(24)); a step intensity variation with temperature was seen in the intensity for fixed field of the additional high field luminescence line at the point □ (ref.(31)). A semiconductor-like temperature variation has been observed in the longitudinal potential drop at constant current for filling factors lower than about 0.19 and between 0.21 and 0.22, whereas metal-like variation is seen about $\nu = \frac{1}{5}$ and $\frac{2}{9}$. Steps in the luminescence energy are seen for $\nu = \frac{1}{3}, \frac{1}{5}, \frac{1}{7}$ and $\frac{1}{9}$. The reduced variable representation employed, $t = T/T_{\rm cm}$ and filling factor $\nu = n_s \hbar c/eB$, should be "universal" (i.e. independent of sample and density) in the high density, zero extrinsic disorder, limit if the phenomenon is intrinsic to the 2DES. The densities involved are: $3.5 < n_s < 12 \times 10^{10}$ cm^{-2} for •, $n_s = 4.0$ and 8.9×10^{10} cm^{-2} for o, $n_s = 5.6 \times 10^{10}$ cm^{-2} for Δ and $n_s = 5.2 \times 10^{10}$ cm^{-2} for □. The zero field Wigner transition is expected at $n_s \approx 2.5 \times 10^{8}$ cm^{-2} from the quantum Monte-Carlo result of ref.(36). The laboratory scale can be appreciated from the example of $n_s = 10^{11}$ cm^{-2}, for which $\nu = \frac{1}{5}$ occurs at about 20 Tesla and $t = 1$ at about 560 mK.

frequency resonances to appear below a characteristic filling factor which is a decreasing function of temperature (inscribed within $\nu < 1/5$, $T < T_{\rm cm}$)[25,26,27]. Surface acoustic waves (SAW) are also sensitive to the electric susceptibility, by virtue of the electric field accompanying the acoustic field in the piezo-electric GaAs host. In principle they could be used to measure χ along a line $\omega = v_s q$ in the (ω, q) plane (v_s is the SAW propagation velocity), but in practice one is limited to a set of points on this dispersion line because of the resonant launching and receiving structures. Such measurements, reported at this conference, confirm the presence of finite frequency modes, but do not allow one to situate them precisely, because unlike the electromagnetic modes in the previously described experiment, the SAW dispersion appears not to intersect the electron mode dispersion[28].

Experiments to monitor energy variation have utilized the luminescence spectrum from the 2DES after excitation of electron-hole pairs in the GaAs. These again indicate qualitatively new behaviour in high B, low T conditions. In one experiment, where the luminescence is thought to arise from direct recombination of an electron in the 2DES with a hole in the valence band, a new luminescence energy, higher than those observed for the FQHE states, appears in the $\nu < 1/5$, low temperature region[29]. In another experiment, where a layer of Be acceptor sites was incorporated close to the 2DES to favourize recombination luminescence to neutralized acceptor levels, a new line also appears, but this time at lower energy; its intensity undergoes a step-like reduction at a temperature lower than T_{cm}, but it does not disappear completely before about $2T_{\mathrm{cm}}$[30,31].

The appearence of each of these qualitatively new behaviours can be regarded as a sort of "litmus test" and we must first see if they give a common indication. This demands comparing results between samples of different densities and disorders. To do so, it is convenient to choose a representation in the reduced variables appropriate to the high density, zero-disorder, limit: filling factor ν and temperature nomalised to the Coulomb interaction by $t = T/T_{\mathrm{cm}}$. If in such a representation one finds a common locus for all the litmus tests, one can be reasonably sure that they indicate the same underlying phenomenon and that it is intrinsic to the 2DES in involving only Landau level degeneracy and Coulomb interaction. Scatter might be attibuted to the effect of the random field. Such a common plot is made in fig.2 for all the experiments to date on the appearance of low-lying dynamical resonances and for the onset of nonlinear conduction. Despite the different types of experiment, the factor of 3.5 variation in densities and the wide range of samples and thereby of disorder potentials, the plot gives a quite reasonably universal locus. Although the transport thermal activation analysis does not yield a limiting temperature, the filling factors at which the Arrhenius temperature vanishes are compatible with the diagram. The lower temperature intensity reduction step of the new Be recombination luminescence line, however, occurs at somewhat higher temperature than the average locus (open square on fig.2)[31], and while the other luminescence experiment[29] shows that the new line accompanies non-linear conduction, there is not yet enough data to check the disappearance boundary. FQHE states are clearly observed in transport at $\nu = 1/5$ and $\nu = 2/9$; shallow minima in the constant current $R_{xx}(= V_L/I)$ at $\nu = 2/11$ and $1/7$ have been observed, but without corresponding plateaux in R_{xy}[24,27,32,33]. The luminescence with Be for $T > T_{\mathrm{cm}}$ indicates the presence of special $\nu = \frac{1}{7}$ and $\frac{1}{9}$ states by the sequence of steps in luminescence energy for $\nu = 1/3$, $1/5$, $1/7$ and $1/9$[34]. It may be important in comparing these experiments to bear in mind that luminescence is a very local probe (length scale $L \sim a_B$) whereas transport and dynamics are more global ($L \sim$ sample size, domain size or measuring wavelength scales).

We associate the points on Fig.2 with a boundary separating qualitatively different phases of the 2DES whose domain of existance is relatively insensitive to the random field parameters, at least for these high quality samples. The new phase wins out over the FQHE phase at high fields and low temperatures. One could intepret the diagram as showing a single new phase which competes with the FQHE liquid resulting in its domain of existance being pierced by the FQHE liquid state by virtue of the latter's stabilising cusp in energy at $\nu = 1/5$[35,22]. The upper existance temperature approaches the classical melting temperature as $\nu \longrightarrow 0$ and the critical filling factor for $t \longrightarrow 0$ is compatible with the a priori theoretical estimates made for the crossover from liquid to solid state stability. It seems that we are in the presence of a new phase; we must now establish its basic properties and understand the phase transition.

The presence both of the new dynamical mode and of the threshold driving field for conduction indicate that the new phase offers elastic, as opposed to viscous, opposition to shear. The values of the threshold field and of the resonance frequencies are not universal from sample to sample, but experiment has shown that, *within a given sample,* the values of these apparently very different quantities are related to one another[24]. Furthermore this relationship can be quantitatively accounted for by a model of a crystalline electron solid broken into domains by a random field. The actual values of these quantities, in this model, depend on the ratio of Coulomb energy to random field fluctuations, but the relationship between them is independent of this. Beyond the universality and the existance region of the new phase, this link is a very strong indication that the underlying phenomenon is magnetically induced Wigner crystallisation of the 2DES. In the samples examined so far, however, the domain size or coherence area is limited to a few hundred electrons. The present evolution of the work on the quantum 2DES makes it reasonable to think that we shall soon reach the same level of understanding of this quantum transition as we have of the classical melting problem.

Any of this work in which I have been personally involved was done in collaboration with at least two of the following: J. Poitrenaud, D. Marty, G. Deville, F. Gallet, A. Valdes, C. Glattli, E. Andrei, B. Etienne, E. Paris, R. Clark, P. Wright, T. Foxon, J. Harris, V. Duburcq, O. Probst, C. Dorin. Technical help from G. Delmas, C. Chaleil, J. Verrier and P. Pari is acknowledged with pleasure. Institutional collaborations with the joint Centre National de la Recherche Scientifique (CNRS)/Max Planck Institut laboratory for high magnetic fields at Grenoble, the CNRS Laboratory for Microelectronics and Microstructures at Bagneux and the Philips Research Laboratory at Redhill have been indispensible.

References

[1] E.P Wigner, *Phys. Rev.* **46**, 1002 (1934).

[2] J. Durkan, R.J. Elliott and N.H. March, *Rev. Mod. Phys.* **40**, 812 (1968).

[3] C.C. Grimes and G. Adams, *Phys. Rev. Lett.* **42**, 795 (1979).

[4] V.B. Shikin, *Zh. Eksp. Teor. Fiz.* **60**, 713 (1971) (*Sov. Phys.-JETP* **33**, 387 (1971)).

[5] D. Fisher, B.I. Halperin and P.M. Platzman, *Phys. Rev. Lett.* **42**, 798 (1979).

[6] F. Gallet, G. Deville, A. Valdes and F.I.B. Williams, *Phys. Rev. Lett.* **49**, 212 (1982).

[7] A. Valdes, *Thèse de Troisième cycle*, Université de Paris Sud, Orsay, France (1982).

[8] G. Deville, A. Valdes, E. Andrei and F.I.B. Williams, *Phys. Rev. Lett.* **53**, 588 (1984).

[9] G. Deville, *J. Low Temp. Phys.* **72**, 135 (1988).

[10] C. Glattli, E. Andrei and F.I.B. Williams, *Phys. Rev. Lett.* **60**, 420 (1988).

[11] D. Fisher, *Phys. Rev.* **B26**, 5009 (1982).

[12] R. Morf, *Phys. Rev. Lett.* **43**, 931 (1979).

[13] G. Meissner, H. Namaizawa and M. Voss, *Phys. Rev.* **B13**, 1370 (1976).

[14] L. Bonsall and A.A. Maradudin, *Phys. Rev.* **B15**, 1959 (1977).

[15] J.M. Kosterlitz and D.J. Thouless, *J. Phys.* **C6**, 1181 (1973).

[16] V.L. Berezinskii, *Zh. Eksp. Teor. Fiz.* **61**, 1144 (1971) (*Sov. Phys. JETP* **34**, 610 (1971)).

[17] D.R. Nelson and B.I. Halperin, *Phys. Rev.* **B19**, 2457 (1979).

[18] A.P. Young, *Phys. Rev.* **B19**, 1855 (1979).

[19] G. Deville, contributed paper, these proceedings

[20] D.S. Fisher, B.I. Halperin and R. Morf, *Phys. Rev.* **B20**, 4692 (1979).

[21] R.L. Willett, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. West and K.W. Baldwin, *Phys. Rev.* **B38**, 7881 (1988).

[22] H.W. Jiang, R.L. Willet, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer and K.W. West *Phys. Rev. Lett.* **65**, 633 (1991).

[23] V.J. Goldman, J.E. Cuningham, M. Shayegan and M. Santos, *Phys. Rev. Lett.* **65**, 2188 (1990).

[24] F.I.B. Williams, P.A. Wright, R.G. Clark, E.Y. Andrei, G. Deville, D.C. Glattli, O. Probst, B. Etienne, C. Dorin, C.T. Foxon and J.J. Harris, *Phys. Rev. Lett.* **66**, 3285 (1991).

[25] E.Y. Andrei, G. Deville, D.C. Glattli, F.I.B. Williams, E. Paris and B. Etienne, *Phys. Rev. Lett.* **60**, 2765 (1988).

[26] D.C. Glattli, G. Deville, V. Duburcq, F.I.B. Williams, E. Paris, B. Etienne and E. Andrei, *Surf. Sci.* **229**, 344 (1990).

[27] F.I.B. Williams, E.Y. Andrei, R.G. Clark, G. Deville, B. Etienne, C.T. Foxon, D.C. Glattli, E. Paris and P.A. Wright in "Localization and Confinement of Electrons in Semiconductors" p.192, Springer-Verlag 1990.

[28] R.L. Willet, invited paper, these proceedings

[29] R.G. Clark, R.A. Ford, S.R. Haynes, J.F. Ryan, A.J. Turberfield, P.A. Wright, F.I.B. Williams, G. Deville, D.C. Glattli, J.R. Mallet, M. van der Burgt, P.M.W. Oswald, F. Herlach, C.T. Foxon and J.J. Harris, *Physica* **B169**, 336 (1991).

[30] H. Buhmann, W. Joss, K. von Klitzing, I.V. Kukushkin, A.S. Plaut, G. Martinez, K. Ploog and V.B. Timofeev, *Phys. Rev. Lett.* **66**, 926 (1991).

[31] I.V. Kukushkin, N.J. Pulsford, K. von Klitzing, K. Ploog and V.B. Timofeev, Conference Proceedings, Electron properties of two dimensional systems, Nara, July 1991 (to be published in *Surf. Sci.*).

[32] J.R. Mallet, R.G. Clark, R.J. Nicholas, R. Willett, J.J. Harris and C.T. Foxon, *Phys. Rev.* **B38**, 2200 (1988).

[33] V.J. Goldman, M. Shayegan and C.C. Tsui, *Phys. Rev. Lett.* **61**,881 (1988).

[34] H. Buhmann, W. Joss, K. von Klitzing, I.V. Kukushkin, A.S. Plaut, K. Ploog, G. Martinez and V.B. Timofeev, *Phys. Rev. Lett.* **65**, 1056 (1990).

[35] B.I. Halperlin, *Phys. Rev. Lett.* **52,** 1583 (1984).

[36] B. Tanatar and D.M. Ceperley, *Phys. Rev.* **B39**, 5005 (1989).