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Josephson Junction Arrays and Superconducting Wire Networks

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Abstract. Techniques used to fabricate integrated circuits make it possible to construct superconducting networks containing as many as 10^6 wires or Josephson junctions. Such networks undergo phase transitions from resistive high-temperature states to ordered low-resistance low-temperature states. The nature of the phase transition depends strongly on controllable parameters such as the strength of the superconductivity in each wire or junction and the external magnetic field. This paper will review the physics of these phase transitions, starting with the simplest zero-magnetic field case. This leads to a Kosterlitz-Thouless transition when the junctions or wires are weak, and a simple mean-field transition when the junctions or wires are strong. Rich behavior, resulting from frustration, occurs in the presence of a magnetic field.

Introduction: Superconducting Wires and Josephson Junctions

A great deal of research on two-dimensional physics has been made possible by advances in materials preparation and microfabrication. Observation of the quantum Hall effect was made possible by the ability to make good inversion layers--by having good control over the properties of a material as a function of depth. By contrast, the systems to be discussed here rely on control of materials in the lateral directions. Networks of Josephson junctions or of superconducting wires are basically just integrated circuits.

In order to understand these integrated circuits, it is first necessary to understand their basic elements, which are single superconducting wires and Josephson junctions. In a superconductor, there is a complex order parameter, $\psi(\mathbf{r})$, whose magnitude squared gives the local density of superconducting electrons, $n_s(\mathbf{r})$ [1]:

$$|\psi(\mathbf{r})|^2 = n_s(\mathbf{r}) \tag{1}$$

Just below the transition temperature of the bulk material, T_{co}, $\psi(\mathbf{r})$ obeys a Schrödinger-like equation, the linearized Ginzburg-Landau equation,

$$\frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - \frac{q^*}{c}\mathbf{A}\right)^2 \psi = \frac{\hbar^2}{2m^*\xi^2(T)}\psi \tag{2}$$

where a non-linear term has been dropped because ψ is small. The superconducting "particles" are Cooper pairs, with charge q^*=-2e and mass $m^*=2m_o$, twice the mass of the electron, and $\mathbf{A}$ is the vector potential. The coherence length $\xi(T)$ diverges as T approaches

T_{co} from below, as

$$\xi(T) = \xi(0)\left(1 - \frac{T}{T_{co}}\right)^{-\frac{1}{2}} \quad . \tag{3}$$

In addition to Eqs. (2) and (3), the standard quantum-mechanical equation for the current holds:

$$\mathbf{J}_s = \frac{q^*}{2m^*}\psi^*\left(\frac{\hbar}{i}\nabla - \frac{q^*}{c}\mathbf{A}\right)\psi + \text{c.c.} \tag{4}$$

which can be written in a useful form, by letting $\psi = |\psi| e^{i\phi}$, as

$$\mathbf{J}_s = \frac{\hbar q^*}{m^*}|\psi|^2\left(\nabla\phi - \frac{q^*}{\hbar c}\mathbf{A}\right) \quad . \tag{5}$$

Eq. (5) shows that supercurrents are caused by phase gradients.

A Josephson junction consists of two superconductors which are weakly coupled by a non-superconducting region[1]. This non-superconducting region may be an insulator or vacuum, through which superconducting electrons can tunnel, or a normal metal, through which the superconducting electrons diffuse, as shown schematically in Fig. 1.

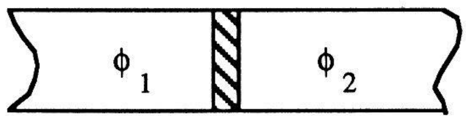

Fig. 1. Schematic picture of a Josephson junction. Superconductors on the left and right have phases ϕ_1 and ϕ_2, and are separated by a small non-superconducting region, indicated by the cross-hatching.

In either case, a small supercurrent, i_S, can be carried through this device, given by the well-known Josephson equation,

$$i_s = i_c \sin\left(\phi_2 - \phi_1 - \frac{q^*}{\hbar c}\int_1^2 \mathbf{A}\cdot d\mathbf{l}\right) \quad . \tag{6}$$

Here ϕ_1 and ϕ_2 are the phases of the wavefunctions of the two superconductors which make up the Josephson junction, and i_C is the maximum supercurrent which the junction can carry. In the limit where the gauge-invariant phase difference in Eq. (6) is small, the sine can be replaced by its argument, and Eq. (6) is seen to be a finite-difference version of Eq. (5). The sine in Eq. (6) assures that changes in the phases of 2π will not have any measurable consequences.

Eqs. (5) and (6) both describe devices in which spatial variations in the phase cause a supercurrent to flow. In wires, the phase gradient is uniform, while in Josephson junctions, it is concentrated in the barrier (The wires leading up to the barrier in a junction have much higher critical currents than the junction, so their phase gradients can be neglected.) The physics of the two devices is otherwise very similar.

Another equation is needed to describe voltages in superconductors. This is the second Josephson equation,

$$V = \frac{\hbar}{q^*}\frac{d}{dt}\left(\phi_2 - \phi_1 - \frac{q^*}{\hbar c}\int_1^2 \mathbf{A}\cdot\mathbf{dl}\right) \tag{7}$$

which states that voltages between two points in a superconductor or across a Josephson junction are proportional to the time derivative of the gauge-invariant phase difference.

Finally, it will be useful to have an expression for the energy stored in a Josephson junction. The time integral of the voltage times the current gives this energy; combining Eqs. (6) and (7) in this way yields

$$E = -\frac{\hbar i_c}{2e}\cos\left(\phi_2 - \phi_1 - \frac{q^*}{\hbar c}\int_1^2 \mathbf{A}\cdot\mathbf{dl}\right) \tag{8}$$

Square Lattices of Josephson Junctions

Next, consider a square lattice of Josephson junctions[2,3], shown schematically in Fig. 2. Initially, the magnetic field is assumed to be zero, so that the vector potential terms in the previous equations may be neglected.

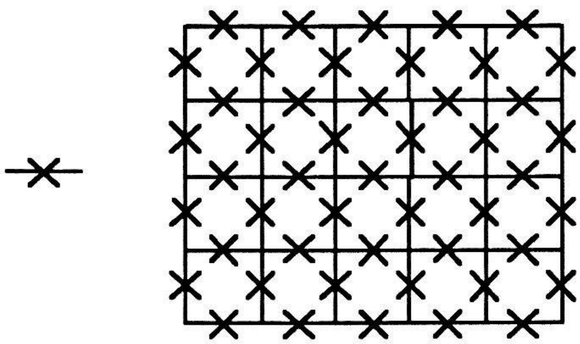

Fig. 2. Circuit symbol for a Josephson junction (left), where X represents the barrier which separates the left and right superconducting lines. A section of square lattice of Josephson junctions is represented on the right.

A Hamiltonian for the square lattice of junctions is written, for zero magnetic field, by summing all of the individual junction energies, using Eq. (8). This gives

$$H = -\sum_{<ij>} E_J \cos\left(\phi_j - \phi_i\right) \tag{9}$$

where the i^{th} node in the array has phase ϕ_i, and the sum is over nearest neighbors. The quantity, $E_J \equiv (\hbar i_c/2e)$, is the Josephson coupling energy.

Next, imagine representing each phase ϕ_i as a unit vector, $\mathbf{s}_i$, making an angle ϕ_i with the x-axis. The Hamiltonian of Eq. (9) can then be written as

$$H = -\sum_{<ij>} E_J \mathbf{s}_i \cdot \mathbf{s}_j \tag{10}$$

Eq. (10) is the Hamiltonian for a square lattice of planar (or XY) spins. A square Josephson junction array is thus isomorphic to the two-dimensional XY model.

Kosterlitz and Thouless showed [4] that the two-dimensional XY model undergoes a special kind of phase transition, now known as the Kosterlitz-Thouless (KT) transition. At zero temperature, all of the ϕ_i's are equal, which minimizes the total energy of Eq. (9); in the spin language, all of the $\mathbf{s}_i$'s are parallel.

As the temperature is raised, thermal excitations disrupt this ground state. The excitation which is important here is a *vortex*. A vortex is a whirlpool of supercurrent, which may either be clockwise (-) or counterclockwise (+). The phase configuration around a vortex is shown in Fig. 3.

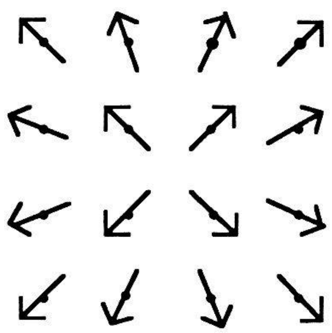

Fig. 3. Directions of phases ϕ_i near the center of a vortex. Points indicate nodes in the array of junctions. The magnitude of the resulting circulating currents [see Eq. (6)] varies as 1/r for large distance r from the center.

At temperatures below the Kosterlitz-Thouless transition temperature T_{KT}, + and - vortices exist as bound pairs. At T_{KT}, vortex pairs start to unbind. This is the Kosterlitz-Thouless transition.

To detect this transition in a Josephson junction array, a uniform current is sent from left to right through the array, and the voltage drop from left to right is measured. As long as the vortex pattern is stationary on the average, the phases will be constant on the average, so that the average voltage will be zero, from Eq. (7). The external current exerts a transverse force on vortices, with the forces on + and - vortices being in opposite directions. Thus, when the vortices are all bound pairs, resistance will be zero in the limit of zero current. When the vortices unbind, they will move in response to the current, changing the phases, which leads to resistance, via Eq. (7).

When the theory for the effect of vortex unbinding in arrays is worked out in detail[5], it is found that the voltage V is proportional to the current I above T_{KT}, while V varies as a power of I below T_{KT}. For both regimes,

$$V \propto I^{a(T)} \tag{11}$$

where $a(T)=1$ for $T>T_{KT}$, $a(T) = 3$ just below T_{KT}, and a(T) increases as T is lowered. The power a(T) *jumps* from 1 to 3 as the temperature is lowered; this is analogous to the jump in the superfluid density in a two-dimensional film of helium. Fig. 4 is based on data on an array containing more than a million junctions[6]; except for some smearing due to finite-current effects, a jump is present.

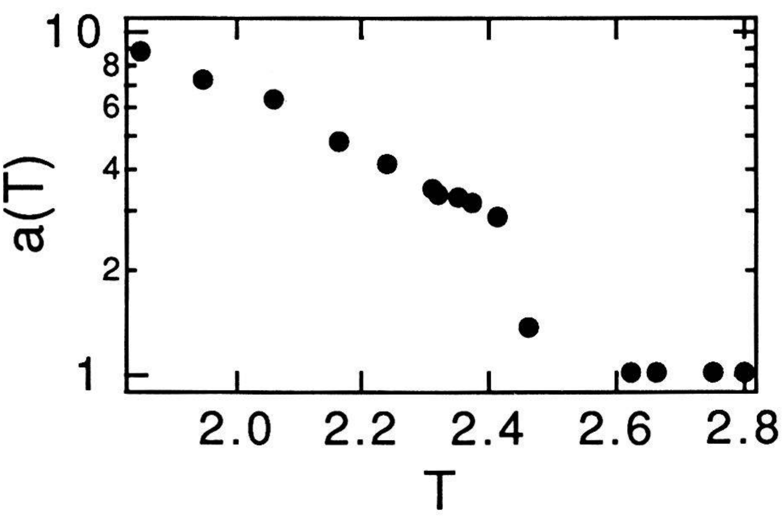

Fig. 4. The I-V exponent a(T) [defined by $V \propto I^{a(T)}$] for a 1000 by 1000-junction array, showing the jump from 1 to 3 at the Kosterlitz-Thouless transition. From Ref. 6.

When a magnetic field is applied perpendicular to the array, the full Hamiltonian

$$H = -\sum_{<ij>} E_J \cos\left(\phi_j - \phi_i - \frac{q^*}{\hbar c} \int_i^j \mathbf{A} \cdot \mathbf{dl} \right) \tag{12}$$

must be used instead of Eq. (9). The effect of the vector potential term in Eq. (12) is to induce *frustration* in the ground state[7,2,3]. An interesting consequence is that the experimenter can change the frustration by varying the field applied to the sample.

When the field is small, vortices of one sign are induced in the array. It is convenient to measure magnetic field in terms of the flux quantum Φ_0=hc/2e, via

$$f \equiv \frac{Ba^2}{\Phi_o} \tag{13}$$

for a square lattice, where a is the lattice spacing. For small f, there are fa^2 field-induced vortices per unit area. These vortices are not paired, so they cause a non-zero resistance, even at low temperatures[8].

When f=p/q, with p and q small integers (1/2,1/3,2/3, etc.), the ground states of Eq. (12) are quite complex[2,7]. One particularly interesting case is f=1/2, which gives maximum frustration. The ground state for f=1/2 has an alternating pattern of + and - vortices, so that there is an Ising-like symmetry in the problem, as well as the XY-like symmetry. Experiment indicates that the transition in this case is still a Kosterlitz-Thouless transition[9].

Square Lattices of Superconducting Wires

As was mentioned above, superconducting wires and Josephson junctions are very similar devices. A square lattice of wires--schematically, just Fig. 2 without the X's--also undergoes phase transitions. In the critical region, close enough to the transition temperature, wire arrays undergo the same types of transitions as junction arrays[10].

Until recently, however, this regime was not experimentally accessible. Wires of typical size (made with micrometer lithography) and pure materials have higher critical currents than junctions, so that thermal fluctuations have less effect on wire arrays than junction arrays. Wire arrays are thus very well-suited for studying *mean-field* phase transitions[2,11,12].

The mean-field approach to this problem is quite appealing. Eq. (4) is used to calculate the supercurrent flowing into the node, which must be zero. For the m^{th} node, this leads to

$$\sum_{n=1}^{4}\left[-\psi_m \cot\left(\frac{a}{\xi(T)}\right)+\psi_n \frac{e^{iA_{mn}}}{\sin\left(\frac{a}{\xi(T)}\right)}\right]=0 \tag{14}$$

where the sum is over the nearest neighbors of node m, ψ_m is the order parameter on the m^{th} node, and

$$A_{mn}=\frac{2\pi}{\Phi_O}\int_m^n \mathbf{A}\cdot\mathbf{dl} \tag{15}$$

These equations also describe a tight-binding model for charges on the same lattice with the same field, as discussed by Hofstadter[13].

Eq. (14) represents m coupled nonlinear equations for the order parameters at all the nodes of the network. For a given value of f, there is a *maximum temperature* that allows a solution to Eq.

(14), temperature entering the equation through the $[a/\xi(T)]$ term. This temperature defines the transition temperature $T_c(f)$ of the array as a function of magnetic field. This maximum temperature corresponds to a maximum value of the coherence length [Eq. (3)], or a minimum value of the multiplicative constant in the Schrödinger-like equation, Eq. (2). Finding $T_c(f)$ is thus equivalent to finding the minimum eigenvalue for the analogous tight-binding problem.

It is convenient to look at $[a/\xi(T_c)]^2$. Using Eq. (3), this can be written as

$$\left[\frac{a}{\xi(T_c)}\right]^2 = \left[T_{co} - T_c(f)\right]\left[\left(\frac{a}{\xi(0)}\right)^2 \frac{1}{T_{co}}\right] \equiv \delta T_c(f) \cdot \alpha \quad . \tag{16}$$

Note that $\delta T_c(f) \propto [\xi(0)/a]^2$, so that smaller lattices have larger shifts in T_c.

A plot of $\delta T_c(f)\ \alpha$ vs. f is shown in Fig. 5 [14]. The transition temperature is a complicated non-monotonic function of the magnetic field, with pronounced features at low-order rational fractions (f=1/2,1/3,2/3,1/4), and is periodic in f.

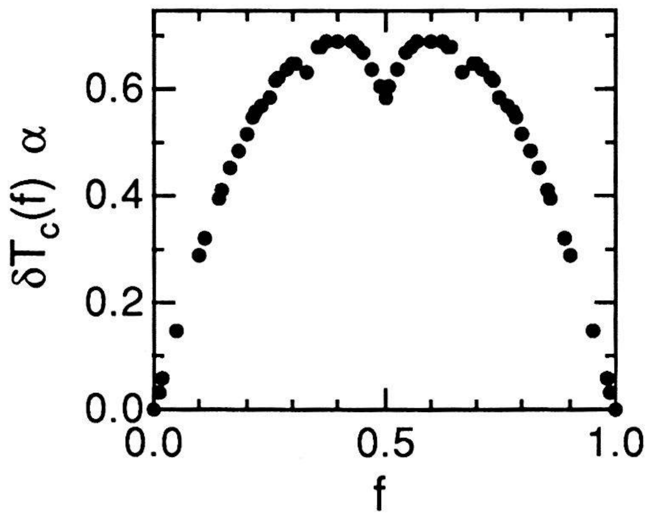

Fig. 5. Mean-field prediction for shift in transition temperature plotted against the number of flux quanta per unit cell in a square lattice.

In experiments, $\delta T_c(f)$ is determined by sending a fixed current through the sample, and varying the temperature in zero field until some fraction of the normal-state resistance, typically 1/2, is obtained. The sample voltage is then held constant by a feedback loop that varies the temperature as the field is varied[2,12,15]. In this way, $T_c(f)$ is measured directly. Excellent agreement is obtained between theory and experiment.

Other Physics in Wire Networks and Junction Arrays

The previous sections dealt with the simplest problems--the phase transitions in square lattices of junctions or wires. Many other types of networks have been studied, and aspects other than phase transitions have been explored[2,3,12]. Other geometries, such as triangular lattices, random networks, quasicrystalline networks, and fractal networks have all been studied[2,16].

As mentioned above, a wire array should also undergo an observable Kosterlitz-Thouless transition if the wires are weak enough[10]. Such transitions have recently been observed[17]. In the opposite case, arrays of strongly coupled Josephson junctions, a mean field transition is predicted to occur[18], although this aspect has apparently not been explored experimentally.

Recently, there has been a good deal of work on the dynamics of arrays which are driven far from equilibrium. The critical currents of arrays have been measured, and compared with theoretical predictions of the pinning potential that retards vortex motion[19]. In addition, the resonant response of arrays to large-amplitude ac signals has been measured, as well as the microwave power emitted by arrays biased above their critical currents[20]. From this perspective, arrays are systems of coupled non-linear oscillators, with possible chaotic and turbulent behavior, as well as potential device applications.

Finally, it should be mentioned that everything in this paper is classical, in spite of the occurrence of a Schrödinger-like equation, in the sense that the quantum fluctuations in the phase variables have been ignored. Arrays of very small junctions need a fully quantum-mechanical treatment; this is the subject of the paper by G. Schön in this volume.

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